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On (m, n)-Regularity of Γ-Semigroups

Wichayaporn Jantanan[†] **and Thawhat Changphas**[†,‡,1]

[†]Department of Mathematics, Faculty of Science
Khon Kaen University, Khon Kaen 40002, Thailand
[‡]Centre of Excellence in Mathematics
CHE, Si Ayuttaya Rd., Bangkok 10400, Thailand
e-mail : jantanan-2903@hotmail.com (W. Jantanan)
thacha@kku.ac.th (T. Changphas)

Abstract : In this paper, we introduce the notions of (m, n)-regularity and (m, n)-Γ-ideal in Γ-semigroups. Some characterizations of (m, n)-regular Γ-semigroups based on (m, n)-Γ-ideals will be given. Similar results on semigroups have been done by Dragica N. Krgović in [1].

Keywords : Semigroup; Γ-semigroup; Regular Γ-semigroup; Γ-ideal; Bi-ideal; (m, n)-regular; (m, n)-ideal; (m, n)-Γ-ideal
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1 Introduction

Let S be a semigroup and m, n non-negative integers. A subsemigroup A of S is called an *(m, n)-ideal* [2] of S if

$$A^m S A^n \subseteq A$$

(A^0 is defined as $A^0 S = S$ and $SA^0 = S$). If $m = 1$ and $n = 1$, then A is called a *bi-ideal* ([3], p.11) of S.

A semigroup S is called an *(m, n)-regular semigroup* [1] if for any $a \in S$ there exists $x \in S$ such that

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[1]Corresponding author.

$$a = a^m x a^n$$

(a^0 is defined as $a^0 x = x$ and $xa^0 = x$). For $m = 1$ and $n = 1$, S is said to be a *regular semigroup* ([4], p.10).

Ideal-characterizations of regular semigroups have been studied (see [5], [2], [3]). In [1], Dragica N. Krgović characterized (m, n)-regular semigroups based on (m, n)-ideals. In this paper, we introduce the concepts of (m, n)-regularity and (m, n)-Γ-ideal in a Γ-semigroup. We characterize (m, n)-regular Γ-semigroups using (m, n)-Γ-ideals.

2 Preliminaries

It is known that the concept of Γ-semigroup has been introduced by M. K. Sen in [6]. Thereafter, the definition defined by Sen was changed by M. K. Sen and N. K. Saha [7] as follows: let S and Γ be two non-empty sets. Then S is called a Γ-semigroup if

(1) $x\alpha y \in S$ and

(2) $(x\alpha y)\beta z = x\alpha(y\beta z)$

for all $x, y, z \in S$ and all $\alpha, \beta \in \Gamma$.

In [8], N. Kehayopulu defined Γ-semigroups by adding the uniqueness condition to the definition defined above as follows:

Definition 2.1. *Let S and Γ be two non-empty sets. Then S is called a Γ-semigroup if*

(1) *$x\alpha y \in S$ for all $x, y \in S$ and all $\alpha \in \Gamma$.*

(2) *If $x, y, z, w \in S$ and $\alpha, \beta \in \Gamma$ such that $x = z, y = w$ and $\alpha = \beta$, then $x\alpha y = z\beta w$.*

(3) *$(x\alpha y)\beta z = x\alpha(y\beta z)$ for all $x, y, z \in S$ and all $\alpha, \beta \in \Gamma$.*

In this paper, we follow Definition 2.1.

Let S be a Γ-semigroup. For nonempty subsets A, B of S, we let

$$A\Gamma B = \{a\alpha b : a \in A, b \in B, \alpha \in \Gamma\}.$$

If $x \in S$, let $A\Gamma x = A\Gamma\{x\}$ and $x\Gamma A = \{x\}\Gamma A$.

Let S be a Γ-semigroup and $A \subseteq S$. If n is a positive integer, we let

$$A^n = A\Gamma A\Gamma \cdots \Gamma A \text{ (n-times) and } x^n = \{x\}^n.$$

A non-empty subset A of a Γ-semigroup S is called a *sub*-Γ-*semigroup* of S if $x\alpha y \in A$ for all $x, y \in A$ and all $\alpha \in \Gamma$.

We define (m, n)-Γ-ideals and (m, n)-regularity of a Γ-semigroup as follows.

Definition 2.2. *Let S be a Γ-semigroup and m,n non-negative integers. A sub-Γ-semigroup A of S is called an (m,n)-Γ-ideal of S if*

$$A^m\Gamma S\Gamma A^n \subseteq A.$$

Here, A^0 is defined as $A^0\Gamma S = S$ and $S\Gamma A^0 = S$.

For each an element a of a Γ-semigroup S, it is easy to see that $a^m\Gamma S$ and $S\Gamma a^n$ are $(m,0)$-Γ-ideal and $(0,n)$-Γ-ideal of S, respectively.

Definition 2.3. *Let S be a Γ-semigroup and m,n non-negative integers. Then S is said to be (m,n)-regular if for any $a \in S$ there exists $x \in S$ such that*

$$a \in a^m\Gamma x\Gamma a^n.$$

Here, a^0 is defined as $a^0\Gamma x = \{x\}$ and $x\Gamma a^0 = \{x\}$.

Note that every Γ-semigroups is $(0,0)$-regular.

3 Main Results

Let S be a Γ-semigroup and m,n non-negative integers. It is easy to see that the intersection of all (m,n)-Γ-ideals of S containing an element a of S, denoted by $[a]_{m,n}$, is an (m,n)-Γ-ideal of S containing a.

Theorem 3.1. *Let S be a Γ-semigroup and let $a \in S$.*

(i) $[a]_{m,n} = \bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n$ *for any positive integers m,n.*

(ii) $[a]_{m,0} = \bigcup_{i=1}^{m}\{a^i\} \cup a^m\Gamma S$ *for any positive integers m.*

(iii) $[a]_{0,n} = \bigcup_{i=1}^{n}\{a^i\} \cup S\Gamma a^n$ *for any positive integers n.*

Proof. (i) We have $\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n \neq \emptyset$. Let $x,y \in \bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n$. If $x,y \in a^m\Gamma S\Gamma a^n$ or $x \in \bigcup_{i=1}^{m+n}\{a^i\}, y \in a^m\Gamma S\Gamma a^n$ or $x \in a^m\Gamma S\Gamma a^n, y \in \bigcup_{i=1}^{m+n}\{a^i\}$, then $x\Gamma y \subseteq a^m\Gamma S\Gamma a^n$, and thus $x\Gamma y \subseteq \bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n$. Suppose that $x,y \in \bigcup_{i=1}^{m+n}\{a^i\}$. Then $x = a^p, y = a^q$ for some $1 \leq p,q \leq m+n$. There are two cases to consider. If $1 \leq p+q \leq m+n$, then $x\Gamma y \subseteq \bigcup_{i=1}^{m+n} a^i$. If $m+n < p+q$, then $x\Gamma y \subseteq a^m\Gamma S\Gamma a^n$. Therefore, $x\Gamma y \subseteq \bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n$. This shows that $\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n$ is a sub-Γ-semigroup of S.

We have

$$\begin{aligned}
&\left(\bigcup_{i=1}^{m+n}\{a^i\}\cup a^m\Gamma S\Gamma a^n\right)^m \Gamma S\\
=\ &\left(\bigcup_{i=1}^{m+n}\{a^i\}\cup a^m\Gamma S\Gamma a^n\right)^{m-1} \Gamma\left(\bigcup_{i=1}^{m+n}\{a^i\}\cup a^m\Gamma S\Gamma a^n\right)\Gamma S\\
=\ &\left(\bigcup_{i=1}^{m+n}\{a^i\}\cup a^m\Gamma S\Gamma a^n\right)^{m-1} \Gamma\left(\bigcup_{i=1}^{m+n}\{a^i\}\Gamma S\cup a^m\Gamma S\Gamma a^n\Gamma S\right)\\
=\ &\left(\bigcup_{i=1}^{m+n}\{a^i\}\cup a^m\Gamma S\Gamma a^n\right)^{m-1} \Gamma(a\Gamma S)\\
=\ &\left(\bigcup_{i=1}^{m+n}\{a^i\}\cup a^m\Gamma S\Gamma a^n\right)^{m-2} \Gamma\left(\bigcup_{i=1}^{m+n}\{a^i\}\cup a^m\Gamma S\Gamma a^n\right)\Gamma(a\Gamma S)\\
=\ &\left(\bigcup_{i=1}^{m+n}\{a^i\}\cup a^m\Gamma S\Gamma a^n\right)^{m-2} \Gamma\left(\bigcup_{i=1}^{m+n}\{a^i\}\Gamma a\Gamma S\cup a^m\Gamma S\Gamma a^n\Gamma a\Gamma S\right)\\
=\ &\left(\bigcup_{i=1}^{m+n}\{a^i\}\cup a^m\Gamma S\Gamma a^n\right)^{m-2} \Gamma(a^2\Gamma S)\\
&\vdots\\
=\ &a^m\Gamma S.
\end{aligned}$$

Similarly, we get

$$S\Gamma\left(\bigcup_{i=1}^{m+n}\{a^i\}\cup a^m\Gamma S\Gamma a^n\right)^n = S\Gamma a^n.$$

Consequently,

$$\begin{aligned}
&\left(\bigcup_{i=1}^{m+n}\{a^i\}\cup a^m\Gamma S\Gamma a^n\right)^m \Gamma S\Gamma\left(\bigcup_{i=1}^{m+n}\{a^i\}\cup a^m\Gamma S\Gamma a^n\right)^n\\
=\ &a^m\Gamma S\Gamma a^n\\
\subseteq\ &\bigcup_{i=1}^{m+n}\{a^i\}\cup a^m\Gamma S\Gamma a^n.
\end{aligned}$$

Therefore, $\bigcup_{i=1}^{m+n}\{a^i\}\cup a^m\Gamma S\Gamma a^n$ is an (m,n)-Γ-ideal of S containing a, whence $[a]_{m,n}\subseteq\bigcup_{i=1}^{m+n}\{a^i\}\cup a^m\Gamma S\Gamma a^n$. For the reverse inclusion, let $x\in\bigcup_{i=1}^{m+n}\{a^i\}\cup a^m\Gamma S\Gamma a^n$. If $x\in\bigcup_{i=1}^{m+n}\{a^i\}$, then $x=a^j$ for some $1\leq j\leq m+n$, hence $x\in[a]_{m,n}$. If $x\in a^m\Gamma S\Gamma a^n$, by

$$a^m\Gamma S\Gamma a^n \subseteq ([a]_{(m,n)})^m\Gamma S\Gamma([a]_{(m,n)})^n \subseteq [a]_{(m,n)},$$

then $x \in [a]_{m,n}$. This proves that $\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n \subseteq [a]_{m,n}$. Hence $[a]_{m,n} = \bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n$ as required. That (ii) and (iii) are true can be proved similarly. □

Lemma 3.2. *Let S be a Γ-semigroup and let $a \in S$. Let m,n be positive integers.*

(i) $([a]_{m,0})^m\Gamma S = a^m\Gamma S$.

(ii) $S\Gamma([a]_{0,n})^n = S\Gamma a^n$.

(iii) $([a]_{m,n})^m\Gamma S\Gamma([a]_{m,n})^n = a^m\Gamma S\Gamma a^n$.

Proof. (i) Since $[a]_{(m,0)} = \bigcup_{i=1}^{m}\{a^i\} \cup a^m\Gamma S$, we have

$$\begin{aligned}
([a]_{(m,0)})^m\Gamma S &= \left(\bigcup_{i=1}^{m}\{a^i\} \cup a^m\Gamma S\right)^m \Gamma S \\
&= \left(\bigcup_{i=1}^{m}\{a^i\} \cup a^m\Gamma S\right)^{m-1} \Gamma \left(\bigcup_{i=1}^{m}\{a^i\} \cup a^m\Gamma S\right)\Gamma S \\
&= \left(\bigcup_{i=1}^{m}\{a^i\} \cup a^m\Gamma S\right)^{m-1} \Gamma \left(\bigcup_{i=1}^{m}\{a^i\}\Gamma S \cup a^m\Gamma S\Gamma S\right) \\
&= \left(\bigcup_{i=1}^{m}\{a^i\} \cup a^m\Gamma S\right)^{m-1} \Gamma(a\Gamma S) \\
&= \left(\bigcup_{i=1}^{m}\{a^i\} \cup a^m\Gamma S\right)^{m-2} \Gamma \left(\bigcup_{i=1}^{m}\{a^i\} \cup a^m\Gamma S\right)\Gamma(a\Gamma S) \\
&= \left(\bigcup_{i=1}^{m}\{a^i\} \cup a^m\Gamma S\right)^{m-2} \Gamma \left(\bigcup_{i=1}^{m}\{a^i\}\Gamma a\Gamma S \cup a^m\Gamma S\Gamma a\Gamma S\right) \\
&= \left(\bigcup_{i=1}^{m}\{a^i\} \cup a^m\Gamma S\right)^{m-2} \Gamma(a^2\Gamma S) \\
&\vdots \\
&= a^m\Gamma S.
\end{aligned}$$

Therefore $([a]_{(m,0)})^m\Gamma S = a^m\Gamma S$.
(ii)This can be proved similarly as (i).

(iii) Since $[a]_{(m,n)} = \bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n$, we obtain

$$\begin{aligned}
([a]_{(m,n)})^m\Gamma S &= \left(\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n\right)^m \Gamma S \\
&= \left(\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n\right)^{m-1} \Gamma \left(\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n\right)\Gamma S \\
&= \left(\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n\right)^{m-1} \Gamma \left(\bigcup_{i=1}^{m+n}\{a^i\}\Gamma S \cup a^m\Gamma S\Gamma a^n\Gamma S\right) \\
&= \left(\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n\right)^{m-1} \Gamma(a\Gamma S) \\
&= \left(\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n\right)^{m-2} \Gamma \left(\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n\right)\Gamma(a\Gamma S) \\
&= \left(\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n\right)^{m-2} \Gamma \left(\bigcup_{i=1}^{m+n}\{a^i\}\Gamma a\Gamma S \cup a^m\Gamma S\Gamma a^n\Gamma a\Gamma S\right) \\
&= \left(\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n\right)^{m-2} \Gamma(a^2\Gamma S) \\
&\vdots \\
&= a^m\Gamma S.
\end{aligned}$$

Similarly, we get

$$\begin{aligned}
S\Gamma([a]_{(m,n)})^n &= S\Gamma\left(\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n\right)^n \\
&= S\Gamma\left(\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n\right)\Gamma\left(\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n\right)^{n-1} \\
&= \left(\bigcup_{i=1}^{m+n} S\Gamma a^i \cup S\Gamma a^m\Gamma S\Gamma a^n\right)\Gamma\left(\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n\right)^{n-1} \\
&= (S\Gamma a)\Gamma\left(\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n\right)\Gamma\left(\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n\right)^{n-2} \\
&= \left(\bigcup_{i=1}^{m+n} S\Gamma a\Gamma a^i \cup S\Gamma a\Gamma a^m\Gamma S\Gamma a^n\right)\Gamma\left(\bigcup_{i=1}^{m+n}\{a^i\} \cup a^m\Gamma S\Gamma a^n\right)^{n-2}
\end{aligned}$$

$$
\begin{aligned}
&= (S\Gamma a^2)\Gamma\left(\bigcup_{i=1}^{m+n}\{a^i\}\cup a^m\Gamma S\Gamma a^n\right)^{n-2}\\
&\ \vdots\\
&= S\Gamma a^n.
\end{aligned}
$$

Therefore

$$
\begin{aligned}
([a]_{(m,n)})^m\Gamma S)\Gamma([a]_{(m,n)})^n &= (a^m\Gamma S)\Gamma([a]_{(m,n)})^n\\
&= a^m\Gamma(S\Gamma([a]_{(m,n)})^n)\\
&= a^m\Gamma S\Gamma a^n.
\end{aligned}
$$

This completes the proof. □

The following results: Theorem 3.3-3.5, are comparable with [1] Theorem 1-3 and proofs are modifications.

Theorem 3.3. *Let S be a Γ-semigroup and m,n positive integers. The set of all $(m,0)$-Γ-ideals and the set of all $(0,n)$-Γ-ideals of S will be denoted by $R_{(m,0)}$ and $L_{(0,n)}$, respectively.*

(i) *S is $(m,0)$-regular if and only if $R = R^m\Gamma S$ for all $R \in R_{(m,0)}$.*

(ii) *S is $(0,n)$-regular if and only if $L = S\Gamma L^n$ for all $L \in L_{(0,n)}$.*

Proof. (i) Assume that S is $(m,0)$-regular. That is, $a \in a^m\Gamma S$ for all $a \in S$. Let R be an $(m,0)$-Γ-ideal of S. Then $R^m\Gamma S \subseteq R$. If $a \in R$, then by assumption, $a \in a^m\Gamma S$. Hence $R \subseteq R^m\Gamma S$.

Conversely, assume that $R = R^m\Gamma S$ for all $R \in R_{(m,0)}$. To show that S is (m,n)-regular, let $a \in S$. Take an $(m,0)$-Γ-ideal $R = [a]_{(m,0)}$ of S. Then

$$[a]_{(m,0)} = ([a]_{(m,0)})^m\Gamma S.$$

According to Lemma 3.2, we obtain

$$[a]_{(m,0)} = a^m\Gamma S.$$

Since $a \in [a]_{(m,0)}$, so $a \in a^m\Gamma S$. Hence S is $(m,0)$-regular.

(ii) This can be proved analogously. □

Theorem 3.4. *Let S be a Γ-semigroup and m, n non-negative integers. The set of all (m,n)-ideals in S is denoted by $A_{(m,n)}$. Then*

$$S \textit{ is } (m,n)\textit{-regular} \Leftrightarrow \forall A \in A_{(m,n)}(A^m\Gamma S\Gamma A^n = A). \tag{3.1}$$

Proof. There are 4 cases to consider.

Case 1: $m = 0$, $n = 0$. Since S is the only $(0,0)$-Γ-ideal of S, it follows that $A \in A_{(0,0)}$ implies $A = S$. Thus (3.1) holds.

Case 2: $m = 0$, $n \neq 0$. We have to show that

$$S \text{ is } (0,n)\text{-regular} \Leftrightarrow \forall A \in A_{(0,n)}(S\Gamma A^n = A).$$

This is true using Theorem 3.3.

Case 3: $m \neq 0$, $n = 0$. This can be proved similarly as Case 2.

Case 4: $m \neq 0$, $n \neq 0$. Let S be an (m,n)-regular. Then $a \in a^m\Gamma S\Gamma a^n$ for all $a \in S$. Let $A \in A_{(m,n)}$. Then $A^m\Gamma S\Gamma A^n \subseteq A$. If $a \in A$, then by assumption, $a \in a^m\Gamma S\Gamma a^n$. Thus $A \subseteq A^m\Gamma S\Gamma A^n$.

Conversely, assume that $A^m\Gamma S\Gamma A^n = A$ for all $A \in A_{(m,n)}$. If $a \in S$, then by Lemma 3.2,

$$[a]_{(m,n)} = ([a]_{(m,n)})^m\Gamma S\Gamma([a]_{(m,n)})^n = a^m\Gamma S\Gamma a^n.$$

Since $a \in [a]_{(m,n)}$, $a \in a^m\Gamma S\Gamma a^n$, and thus a is (m,n)-regular. Therefore, S is (m,n)-regular. □

Theorem 3.5. *Let S be a Γ-semigroup and m,n non-negative integers. The set of all $(m,0)$-Γ-ideals and the set of all $(0,n)$-Γ-ideals of S will be denoted by $R_{(m,0)}$ and $L_{(0,n)}$, respectively. Then*

$$S \text{ is } (m,n)\text{-regular} \Leftrightarrow \forall R \in R_{(m,0)}\ \forall L \in L_{(0,n)}\ (R \cap L = R^m\Gamma L \cap R\Gamma L^n) \quad (3.2)$$

(Here $R^0\Gamma L = L$ and $R\Gamma L^0 = R$).

Proof. There are 4 cases to consider.

Case 1: $m = 0, n = 0$. Since S is $(0,0)$-regular, we have (3.2) holds.

Case 2: $m = 0, n \neq 0$. Since $R = S$, the equation $R \cap L = R^m\Gamma L \cap R\Gamma L^n$ be comes $L = L \cap S\Gamma L^n$. Thus $L \subseteq S\Gamma L^n$, and hence $L = S\Gamma L^n$. Then (3.2) becomes

$$S \text{ is } (0,n)\text{-regular if and only if } \forall L \in L_{(0,n)}(L = S\Gamma L^n).$$

This follows by Theorem 3.3.

Case 3: $m \neq 0, n = 0$. This can be proved as Case 2.

Case 4: $m \neq 0, n \neq 0$. We assume first that S is (m,n)-regular. Let $R \in R_{(m,0)}$ and $L \in L_{(0,n)}$. We have

$$R^m\Gamma L \subseteq R^m\Gamma S \subseteq R \text{ and } R\Gamma L^n \subseteq S\Gamma L^n \subseteq L.$$

Then $R^m\Gamma L \cap R\Gamma L^n \subseteq R \cap L$. For the reverse inclusion, let $a \in R \cap L$. By assumption,

$$a \in (a^m\Gamma S)\Gamma a^n \subseteq R\Gamma L^n \text{ and } a \in a^m\Gamma(S\Gamma a^n) \subseteq R^m\Gamma L.$$

Hence $R \cap L \subseteq R^m\Gamma L \cap R\Gamma L^n$.

Conversely, suppose that the expression on the right hand side of (3.2) holds. Then

$$\forall R \in R_{(m,0)}\ \forall L \in L_{(0,n)}\ (R \cap L \subseteq R\Gamma L).$$

Take $R = [a]_{(m,0)}$ and $L = S$, by Lemma 3.2, we obtain

$$[a]_{(m,0)} \subseteq ([a]_{(m,0)})^m\Gamma S \subseteq a^m\Gamma S.$$

Thus $[a]_{(m,0)} \subseteq a^m \Gamma S$. Similarly, we get $[a]_{(0,n)} \subseteq S\Gamma a^n$. Hence

$$[a]_{(m,0)} \cap [a]_{(0,n)} \subseteq a^m \Gamma S \cap S\Gamma a^n.$$

Since $a^m \Gamma S$ is an $(m, 0)$-Γ-ideal and $S\Gamma a^n$ is an $(0, n)$-Γ-ideal, by assumption, we have

$$a^m \Gamma S \cap S\Gamma a^n \subseteq a^m \Gamma S \Gamma S \Gamma a^n \subseteq a^m \Gamma S \Gamma a^n.$$

Hence

$$[a]_{(m,0)} \cap [a]_{(0,n)} \subseteq a^m \Gamma S \Gamma a^n.$$

Since $a \in [a]_{(m,0)} \cap [a]_{(0,n)}$, S is (m, n)-regular. □

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