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A Note on Tripled Fixed Point of w-Compatible Mappings in tvs-Cone Metric Spaces[1]

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Abstract : In this paper we have developed a new method of reducing tripled fixed point results in tvs-cone metric spaces to the respective results for mappings with one variable, even obtaining (in some cases) more general theorems. Our results generalize, extend, unify, enrich and complement recently tripled fixed point theorems established by Aydi et al. [1]. Also, by using our method several coupled fixed point results in tvs-cone metric spaces (hence and in usual metric case) can be reduced to the common fixed point results with one variable.

Keywords : tvs-cone metric spaces; tripled coincidence point; solid cone.

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1 Introduction and preliminaries

Ordered normed spaces, cones and topical functions have applications in applied mathematics, for instance, in using Newton's approximation method ([2–6]). and optimization theory ([7] and [8]). P -metric and P -normed spaces were introduced in the mid-20th century ([3], see also [5, 6]) by replacing an ordered Banach space instead of the set of real numbers, as the codomain for a metric. Huang and Zhang [9] re-introduced such spaces under the name of cone metric spaces, but went further, defining convergent and Cauchy sequences in the terms of interior points of the underlying cone. In such a way, nonnormal cones can be used as well (although they used only normal cones), paying attention to the fact that Sandwich theorem and continuity of the metric may not hold. They and other authors ([1, 6, 10–20]) proved some fixed point theorems for contractive-type mappings in cone metric spaces.

Consistent with [7] and [21] (see also [3, 5, 9, 22, 23]) the following definitions and results will be needed in the sequel.

Let E be a real topological vector spaces (tvs) with its zero vector θ. A nonempty subset P of E is called a convex cone if $P+P\subset P$ and $\lambda P\subset P$ for $\lambda\geq 0$. A convex cone P is said to be pointed (or proper) if $P\cap(-P)=\{\theta\}$; P is normal (or saturated) if E has a base of neighborhoods of zero consisting of order-convex subsets. For a given cone $P\subset E$, we can define a partial ordering $\preceq$ with respect to P by $x\preceq y$ if and only $y-x\in P$; $x\prec y$ to indicate that $x\preceq y$ but $x\neq y$, while $x\ll y$ will stand for $y-x\in intP$ (interior of P). If $intP\neq\emptyset$ then P is called a solid cone (see [5]).

For a pair of elements x,y in E such that
$x\preceq y$, put $[x,y]=\{z\in E: x\preceq z\preceq y\}$. A subset A of E is said to be order-convex if $[x,y]\subset A$, whenever $x,y\in A$ and $x\preceq y$.

An order topological vector space (E,P) is order-convex if it has a base of neighbourhoods of θ consisting of order-convex subsets. In this case, the cone P is said to be normal. If E is a normed space, this condition means that the unit ball is order-convex, which is equivalent to the condition that there is a number K such that $x,y\in E$ and $\theta\preceq x\preceq y$ implies that $\|x\|\leq K\|y\|$. A proof of the following assertion can be found, e.g., in [5].

Theorem 1.1. *If the underlying cone of an ordered tvs is solid and normal, then such tvs must be an ordered normed space.*

In the sequel, E will be a locally convex Hausdorff tvs with its zero vector θ, P a proper, closed and convex pointed cone in E with $intP\neq\emptyset$

and $\preceq$ a partial ordering with respect to P.

Definition 1.2 ([14, 22, 24, 25])**.** Let X be a nonempty set. Suppose that the mapping $d : X \times X \to E$ satisfies:

(**d1**) $\theta \preceq d(x,y)$ for all $x,y \in X$ and $d(x,y) = \theta$ if and only if $x = y$;

(**d2**) $d(x,y) = d(y,x)$ for all $x,y \in X$;

(**d3**) $d(x,y) \preceq d(x,z) + d(z,y)$ for all $x,y,z \in X$.

Then d is called a tvs-cone metric on X and (X,d) is called a tvs-cone metric or abstract metric space.

The concept of a tvs-cone metric space is more general than that of a cone metric space (see [14]).

Definition 1.3 ([24, 25])**.** Let (X,d) be a tvs-cone metric space. We say that $\{x_n\}$ is:

(**i**) a tvs-cone Cauchy sequence if for every c in E with $\theta \ll c$, there is an N such that for all $n,m > N$, $d(x_n,x_m) \ll c$;

(**ii**) a tvs-cone convergent sequence if for every c in E with $\theta \ll c$, there is an N such that for all $n > N$, $d(x_n,x) \ll c$ for some fixed x in X.

A tvs-cone metric space (X,d) is said to be complete if every tvs-cone Cauchy sequence in X is convergent in X.

The notation $\theta \ll c$ for c an interior point of the positive cone is used by Krein and Rutman [26]. See pages 10, 11 of their paper for a complete discussion.

Let (X,d) be a tvs-cone metric space. Then the following properties are often used, particularly when dealing with tvs-cone metric spaces in which the cone need not be normal (for details see [15, 27]):

$\mathbf{p}_1$) If $u \preceq v$ and $v \ll w$, then $u \ll w$.

$\mathbf{p}_2$) If $\theta \preceq u \ll c$ for each $c \in intP$, then $u = \theta$.

$\mathbf{p}_3$) If E is a real tvs with a cone P and if $a \preceq \lambda a$ where $a \in P$ and $0 \leq \lambda < 1$, then $a = \theta$.

$\mathbf{p}_4$) If $c \in intP$, $a_n \in E$ and $a_n \to \theta$ in locally convex Hausdorff tvs E, then there exists n_0 such that for all $n > n_0$ we have $a_n \ll c$. However, the converse is not true in general.

The metric fixed point theory is very important and useful in Mathematics. It can be applied in various areas, for instance: variational inequalities, optimization, approximation theory, etc.

The notion of a tripled fixed point in the context of partially ordered metric spaces was introduced and studied by [28]. Fixed point theory, coupled and tripled cases in ordered metric spaces was studied in ([1, 28–38]). We start out with listing some notation and preliminaries that we shall need to express our results.

Definition 1.4 ([1, 32])**.** Let X be a non-empty set and $F : X^3 \to X$ be a mapping. An element $(x, y, z) \in X^3$ is called a tripled fixed point of F if $F(x, y, z) = x, F(y, z, x) = y$ and $F(z, x, y) = z$.

Note that, Berinde and Borcut [28] defined differently the notion of a tripled fixed point in the case of ordered sets in order to keep true the mixed monotone property. For more details, see ([28, 32]).

Definition 1.5 ([1, 32])**.** Let X be a non-empty set, $F : X^3 \to X$ and $g : X \to X$ two mappings.

(**1**) An element $(x, y, z) \in X^3$ is called a tripled coincidence point of F and g if $F(x, y, z) = gx, F(y, z, x) = gy$ and $F(z, x, y) = gz$. Moreover, (x, y, z) is called a tripled common fixed point of F and g if

$$F(x, y, z) = gx = x, F(y, z, x) = gy = y \text{ and } F(z, x, y) = gz = z.$$

Note that if (x, y, z) is a tripled common fixed point of F and g then (y, z, x) and (z, x, y) are tripled common fixed points of F and g too.

(**2**) Mappings $F : X^3 \to X$ and $g : X \to X$ are called commutative if for all $x, y, z \in X$ holds $g(F(x, y, z)) = F(gx, gy, gz)$.

(**3**) [1] Mappings $F : X^3 \to X$ and $g : X \to X$ are called w-compatible if $g(F(x, y, z)) = F(gx, gy, gz)$ whenever $F(x, y, z) = gx, F(y, z, x) = gy$ and $F(z, x, y) = gz$. For some details see [11].

The proof of the following Lemma is immediately.

Lemma 1.6.

(1) Let (X, d) be a tvs-cone metric space. If $D : X^3 \times X^3 \to \mathbb{R}^+$ defined by

$$D(Y, V) = d(x, u) + d(y, v) + d(z, w), Y = (x, y, z), V = (u, v, w) \in X^3$$

then $\left(X^3, D\right)$ is a new tvs-cone metric space. It is not hard to see that tvs-cone metric space $\left(X^3, D\right)$ is complete if and only if (X, d) is a complete.

(2) *If $F : X^3 \to X$ and $g : X \to X$ then F and g are w-compatible if and only if the mappings $T_F : X^3 \to X^3$ and $T_g : X^3 \to X^3$ defined by*

$$T_F(Y) = (F(x,y,z), F(y,z,x), F(z,x,y))$$
$$\text{and } T_g(Y) = T_g(x,y,z) = (gx, gy, gz), \ Y = (x,y,z) \in X^3$$

are weakly compatible.

(3) *The mappings F and g are continuous if and only if T_F and T_g are continuous.*

(4) *$F\left(X^3\right)$ (resp. $g(X)$) is complete in tvs-cone metric spaces (X, d) if and only if $T_F\left(X^3\right)$ (resp. $T_g\left(X^3\right)$) is complete in tvs-cone metric space $\left(X^3, D\right)$.*

(5) *Mappings $F : X^3 \to X$ and $g : X \to X$ have a tripled coincidence point if and only if mappings T_F and T_g have a coincidence point in X^3.*

(6) *Mappings $F : X^3 \to X$ and $g : X \to X$ have a tripled common fixed point if and only if mappings T_F and T_g have a common fixed point in X^3.*

In [1], Aydi et al. proved the following theorems and formulated as Theorem 1 and 2.

Theorem 1.7. *Let (X, d) be a K-metric space with a cone P having non-empty interior and $F : X^3 \to X$ and $g : X \to X$ be mappings such that $F\left(X^3\right) \subseteq g(X)$. Suppose that for any $x, y, z, u, v, w \in X$, the following condition*

$$\begin{aligned} & d(F(x,y,z), F(u,v,w)) \\ & \le a_1 d(F(x,y,z), gx) + a_2 d(F(y,z,x), gy) + a_3 d(F(z,x,y), gz) \\ & \quad + a_4 d(F(u,v,w), gu) + a_5 d(F(v,w,u), gv) + a_6 d(F(w,u,v), gw) \\ & \quad + a_7 d(F(u,v,w), gx) + a_8 d(F(v,w,u), gy) + a_9 d(F(w,u,v), gz) \\ & \quad + a_{10} d(F(x,y,z), gu) + a_{11} d(F(y,z,x), gv) + a_{12} d(F(z,x,y), gw) \\ & \quad + a_{13} d(gx, gu) + a_{14} d(gy, gv) + a_{15} d(gz, gv) \end{aligned} \tag{1.1}$$

holds, where $a_i, i = 1, \ldots, 15$ are nonnegative real numbers such that $\sum_{i=1}^{15} a_i < 1$. Then F and g have a tripled coincidence point provided that $g(X)$ is a complete subspace of X.

Theorem 1.8. *Let $F : X^3 \to X$ and $g : X \to X$ be two mappings which satisfy all the conditions of Theorem 1.7. If F and g are w-compatible, then F and g have a unique common tripled fixed point. Moreover, a common tripled fixed point of F and g is of the form (u, u, u) for some $u \in X$.*

2 Main Results

Our first result is the following Lemma which generalizes Theorem 2.8 from [27]. In fact, this is very known Hardy-Rogers theorem ([39, 40]) in the context of tvs-cone metric space (the proof is the same as for cone metric case, see also [15, 24, 27]). After that we will formulate the theorem which is inspired by Theorem 1.7. and is more general than it.

Lemma 2.1. *Let (X, d) be a tvs-cone metric space and P be a solid cone. Suppose mappings $f, g : X \to X$ and that there exist nonnegative constants α_i satisfying $\sum_{i=1}^{5} \alpha_i < 1$ such that, for each $x, y \in X$*

$$d(fx, fy) \preceq \alpha_1 d(gx, gy) + \alpha_2 d(gx, fx) + \alpha_3 d(gy, fy) + \alpha_4 d(gx, fy) + \alpha_5 d(gy, fx). \tag{2.1}$$

If the range of g contains the range of f and one of $f(X)$ or $g(X)$ is a complete subspace of X, then f and g have a unique point of coincidence in X. Moreover if f and g are weakly compatible then f and g have a unique common fixed point.

The following results generalize and extend both Theorems 1.7 and 1.8.

Theorem 2.2. *Let (X, d) be a tvs-cone metric space with a solid cone P and $F : X^3 \to X$ and $g : X \to X$ be mappings such that $F(X^3) \subseteq g(X)$. Suppose that for any $x, y, z, u, v, w \in X$, the following condition*

$$\begin{aligned} &d(F(x,y,z), F(u,v,w)) + d(F(y,z,x), F(v,w,u)) \\ &+ d(F(z,x,y), F(w,u,v)) \\ &\quad \preceq b_1 (d(F(x,y,z), gx) + d(F(y,z,x), gy) + d(F(z,x,y), gz)) \\ &\qquad + b_2 (d(F(u,v,w), gu) + d(F(v,w,u), gv) + d(F(w,u,v), gw)) \\ &\qquad + b_3 (d(F(u,v,w), gx) + d(F(v,w,u), gy) + d(F(w,u,v), gz)) \\ &\qquad + b_4 (d(F(x,y,z), gu) + d(F(y,z,x), gv) + d(F(z,x,y), gw)) \\ &\qquad + b_5 (d(gx, gu) + d(gy, gv) + d(gz, gw)) \end{aligned} \tag{2.2}$$

holds, where $b_i, i = 1, \ldots, 5$ *are nonnegative real numbers such that* $\sum_{i=1}^{5} b_i < 1$. *Then* F *and* g *have a tripled coincidence point provided that one of* $F\left(X^3\right)$ *or* $g\left(X\right)$ *is a complete subspace of* X.

Proof. Putting $b_5 = \alpha_1, b_2 = \alpha_2, b_1 = \alpha_3, b_4 = \alpha_4$ and $b_3 = \alpha_5$, by Lemma 1.6. (**2**) the condition (2.2) become

$$\begin{aligned} D\left(T_F\left(Y\right), T_F\left(V\right)\right) \preceq \alpha_1 D\left(T_g\left(Y\right), T_g\left(V\right)\right) + \alpha_2 D\left(T_F\left(V\right), T_g\left(V\right)\right) \\ + \alpha_3 D\left(T_F\left(Y\right), T_g\left(Y\right)\right) + \alpha_4 D\left(T_F\left(Y\right), T_g\left(V\right)\right) \\ + \alpha_5 D\left(T_F\left(V\right), T_g\left(Y\right)\right) \end{aligned}$$

which is in fact the condition (2.1). Since $F\left(X^3\right) \subseteq g\left(X\right)$, it is clear that the range of T_g contains the range of T_F. Also, by Lemma 1.6 (**4**) one of $T_F\left(X^3\right)$ or $T_g\left(X^3\right)$ is a complete in tvs-cone metric space $\left(X^3, D\right)$. Hence, all conditions of Lemma 2.1 are satisfied. It means that the mappings T_F and T_g have a coincidence point in X^3, that is, by Lemma 1.6 (**5**) F and g have a tripled coincidence point. This completes the proof. □

Remark 2.3. *Theorem 2.2. is more general than Theorem 1.7, since the contractive condition (1.1) implies (2.2) with* $a_1+a_2+a_3 = b_1$,$a_4+a_5+a_6 = b_2$,$a_7 + a_8 + a_9 = b_3$,$a_{10} + a_{11} + a_{12} = b_4$ *and* $a_{13} + a_{14} + a_{15} = b_5$. *The following example shows that generalization is proper.*

Example 2.4. *Consider the Banach space* $E = C\left[0, 1\right]$ *of real-valued continuous functions with the max-norm and ordered by the cone* $P = \left\{f \in E : f\left(t\right) \geq 0 \text{ for } t \in [0, 1]\right\}$. *This cone is normal in the Banach-space topology on* E. *Let* τ^* *be the strongest locally convex topology on the vector space* E. *Then, the cone* P *is* τ^*-*solid, but it is not normal in the topology* τ^*. *Indeed, if this were the case, Theorem 1.1. would imply that the topology* τ^* *is normed, which is impossible since an infinite dimensional space with the strongest locally convex topology cannot be metrizable.*

Let $X = \mathbb{R}$ and $d : X \times X \to (E, \tau^*)$ be defined by $d\left(x, y\right)\left(t\right) := \left|x - y\right| e^t$. Then, (X, d) is a tvs-cone metric space which is not a cone metric space in the sense of [9]. Further, consider mappings $F : X^3 \to X$ and $g : X \to X$ given by $F\left(x, y, z\right) = \frac{x-3y}{5}, (x, y, z) \in X^3$ and $g\left(x\right) = x$, $x \in X$. Then all the conditions of Theorem 2.2. are satisfied. In particular, for $a_i = 0, i = 1, 2, \ldots, 12$; $a_{13} = a_{14} = a_{15} = \frac{4}{15}$ equivalently $b_1 = b_2 = b_3 =$

$b_4 = 0$ and $b_5 = \frac{4}{5}$ condition (2.2) reduces to

$$\begin{aligned}
&(d(F(x,y,z), F(u,v,w)) + d(F(y,z,x), F(v,w,u)) \\
&+d(F(z,x,y), F(w,u,v)))(t) \\
&\quad := |F(x,y,z) - F(u,v,w)| e^t + |F(y,z,x) - F(v,w,u)| e^t \\
&\qquad + |F(z,x,y) - F(w,u,v)| e^t \\
&\quad = \left|\frac{x-3y}{5} - \frac{u-3v}{5}\right| e^t + \left|\frac{y-3z}{5} - \frac{v-3w}{5}\right| e^t + \left|\frac{z-3x}{5} - \frac{w-3u}{5}\right| e^t \\
&\quad \leq \frac{4}{5}\left(|x-u| e^t + |y-v| e^t + |z-w| e^t\right)
\end{aligned}$$

and holds for all $x, y, z \in X$ and all $t \in [0,1]$. So, by Theorem 2.2. we obtain that mappings F and g have a tripled coincidence point $(x,y,z) = (0,0,0)$. The same conclusion cannot be obtained by Theorem 1.7. (Theorem 1. from [1]) since for $a_i = 0, i = 1, 2, \ldots, 12$; $a_{13} = a_{14} = a_{15} = \frac{4}{15}$ condition (2.1) reduces to

$$\left|\frac{x-3y}{5} - \frac{u-3v}{5}\right| e^t \leq \frac{4}{15}\left(|x-u| e^t + |y-v| e^t + |z-w| e^t\right)$$

for all $x, y, z \in X$ and all $t \in [0,1]$. However, for $x = u, z = w$ and $y \neq v$ we obtain a contradiction. It means that our Theorem 2.2 is a proper generalization of main result from [1].

Theorem 2.5. *Let $F : X^3 \to X$ and $g : X \to X$ be two mappings which satisfy all conditions of Theorem 2.2. If F and g are w-compatible, then F and g have a unique common tripled fixed point. Moreover, a common tripled fixed point of F and g is of the form (u,u,u) for some $u \in X$.*

Proof. According to Theorem 2.2 mappings F and g have a tripled coincidence point. Further, by Lemma 1.2 (**2**) corresponding mappings T_F and T_g are weakly compatible, from which by Lemma 2.1 follows that T_F and T_g have a unique common fixed point. Again, by Lemma 1.2 (**6**) mappings F and g have a unique common tripled fixed point. If (x,y,z) is a unique common tripled fixed point then by Lemma 1.2 (**1**) (y,z,x) and (z,x,y) are also tripled fixed point. Hence, $x = y = z$, that is, a common tripled fixed point of F and g is of the form (u,u,u) for some $u \in X$. This completes the proof. □

Remark 2.6. *Since Example 2.4. supports our Theorem 2.5 but not and Theorem 1.8 (Theorem 2 from [1]) again follows proper generalization of results from [1].*

Remark 2.7. *Putting* $E = \mathbb{R}, P = [0, +\infty)$ *in Theorems 2.2 and 2.5 one obtains the respective tripled coincidence point and common tripled fixed point theorems in metric spaces (we could not find explicit formulations for some of these assertions in literature).*

References

[1] H. Aydi, M. Abbas, W. Sintunavarat, P. Kumam, Tripled fixed point of w-compatible mappings in abstract metric spaces, Fixed Point Theory and Applications 2012, 2012:134.

[2] L.V. Kantorovich, The method of successive approximations for functional equations, Acta Math. 71 (1939) 63–77.

[3] L.V. Kantorovich, The majorant principle and Newton's method, Dokl. Akad. Nauk SSSR (N.S.) 76 (1951) 17–20.

[4] L.V. Kantorovich, On some further applications of the Newton approximation method, Vestnik Leningrad Univ. Ser. mat. Meh. Astr. 12 (7) (1957) 68–103.

[5] J. Vandergraft, Newton method for convex operators in partially ordered spaces, SIAM J. Numer. Anal. 4 (1967) 406–432.

[6] P.P. Zabreiko, K-metric and K-normed spaces: survey, Collect. Math. 48 (1997) 852–859.

[7] K. Deimling, *Nonlinear Functional Analysis,* Springer-Verlag, 1985.

[8] H. Mohebi, Topical functions and their properties in a class of ordered Banach spaces, Part II of the book "Continuous Optimization, Curent Trends and Modern Applications", 343–361, Springer, 2005.

[9] L.G. Huang, X. Zhang, Cone metric spaces and fixed point theorems of contractive mappings, J. Math. Anal. Appl. 332 (2) (2007) 1468–1476.

[10] M. Abbas, G. Jungck, Common fixed point results for noncommuting mappings without continuity in cone metric spaces, J. Math. Anal. Appl. 341 (2008) 416–420.

[11] M. Abbas, B.E. Rhoades, Fixed and periodic point results in cone metric spaces, Appl. Math. Lett. 22 (4) (2009) 511–515.

[12] M. Abbas, M. Ali Khan, S. Radenović, Common coupled fixed point theorems in cone metric spaces for w-compatible mappings, Appl. Math. Comput. 217 (2010) 195–202.

[13] H. Aydi, B. Samet, C. Vetro, Coupled fixed point results in cone metric spaces for w-compatible mappings, Fixed Point Theory and Applications 2011, 2011:27.

[14] I. Beg, A. Azam, M. Arshad, Common Fixed Point for Maps on Topological Vector Space Valued Cone Metric Spaces, International Journal of Mathematics and Mathematical Sciences, Volume 2009 (2009), Article ID 560264, 8 pages.

[15] Z. Kadelburg, S. Radenović, V. Rakočević, A note on the equivalence of some metric and cone metric fixed point results, Applied Mathematics Letters 24 (2011) 370–374.

[16] A. Kaewkhao, W. Sintunavarat, P. Kumam, Common fixed point theorems of c-distance on cone metric spaces, Journal of Nonlinear Analysis and Applications, Volume 2012 (2012), Article ID jnaa-00137, 11 Pages.

[17] S. Radenović, Common fixed points under contractive conditions in cone metric spaces, Comp. Math. Appl. 58 (2009) 1273–1278.

[18] W. Sintunavarat, P. Kumam, Common fixed points of $f-$weak contractions in cone metric spaces, Bull. Iranian Math. Soc. 38 (2) (2012) 293–303.

[19] W. Sintunavarat, Y.J. Cho, P. Kumam, Common fixed point theorems for c-distance in ordered cone metric spaces, Comp. Math. Appl. 62 (2011) 1969–1978.

[20] D. Turkoglu, M. Abuloha, Cone metric spaces and fixed point theorems in diametrically contractive mappings, Acta Mathematica Sinica, English Series 26 (3) (2010) 489–496.

[21] Y.C. Wong, K.F. Ng, *Partially Ordered Topological Vector Spaces*, Oxford Mathematical Monograph, Claredon Press, Oxford, UK, 1973.

[22] M. Djordjević, D. Djorić, Z. Kadelburg, S. Radenović, D. Spasić, Fixed point results under C-distance in tvs-cone metric spaces, Fixed Point Theory and Applications 2011, 2011:29.

[23] M.A. Krasnoseljski, P.P. Zabreiko, *Geometrical Methods in Nonlinear Analysis,* Springer, 1984.

[24] W.S. Du, A note on cone metric fixed point theory and its equivalence, Nonlinear Anal. 72 (2010) 1589–1593.

[25] Z. Kadelburg, S. Radenović, Coupled fixed point results under tvs-cone metric and w-cone-distance, Advances in Fixed Point Theory 2 (2012) 29–46.

[26] M.G. Krein, M.A. Rutman, Linear operators leaving invariant a cone in a Banach spaces, Uspehi Math. Nauk (N.S) 3 (1) (1948) 3–95.

[27] G. Jungck, S. Radenović, S. Radojević, V. Rakočević, Common fixed point theorems for weakly compatible pairs on cone metric spaces, Fixed Point Theory and Applications, Volume 2009 (2009), Article ID 643840, 13 pages.

[28] V. Berinde, M. Borcut, Tripled fixed point theorems for contractive type mappings in partially ordered metric spaces, Nonlinear Anal. 74 (2011) 4889–4897.

[29] M. Abbas, B. Ali, W. Sintunavarat, P. Kumam, Tripled fixed point and tripled coincidence point theorems in intuitionistic fuzzy normed spaces, Fixed Point Theory and Applications 2012, 2012:187.

[30] H. Aydi, E. Karapinar, M. Postolache, Tripled coincidence point theorems for $\varphi-$contractionsn in partially ordered metric spaces, Fixed Point Theory and Applications 2012, 2012:44.

[31] H. Aydi, E. Karapinar, B. Samet, Remarks on some recent fixed point theorems, Fixed Point Theory and Applications 2012, 2012:76.

[32] A. Amini-Harandi, Coupled and tripled fixed point theory in partially ordered metric spaces with applications to initial value problem, Math. Comput. Modell. 57 (9-10) (2013) 2343–2348.

[33] W. Sintunavarat, Y.J. Cho, P. Kumam, Coupled coincidence point theorems for contractions without commutative condition in intuitionistic fuzzy normed spaces, Fixed Point Theory and Applications 2011, 2011:81.

[34] W. Sintunavarat, Y.J. Cho, P. Kumam, Coupled fixed point theorems for weak contraction mapping under F-invariant set, Abstract and Applied Analysis, Volume 2012 (2012), Article ID 324874, 15 pages.

[35] W. Sintunavarat, P. Kumam, Coupled best proximity point theorem in metric spaces, Fixed Point Theory and Applications 2012, 2012:93.

[36] W. Sintunavarat, Y.J. Cho, P. Kumam, Coupled fixed-point theorems for contraction mapping induced by cone ball-metric in partially ordered spaces, Fixed Point Theory and Applications 2012, 2012:128.

[37] W. Sintunavarat, A. Petrusel, P. Kumam, Common coupled fixed point theorems for w^*-compatible mappings without mixed monotone property, Rend. Circ. Mat. Palermo 61 (2012) 361–383.

[38] W. Sintunavarat, P. Kumam, Y.J. Cho, Coupled fixed point theorems for nonlinear contractions without mixed monotone property, Fixed Point Theory and Applications 2012, 2012:170.

[39] G.E. Hardy, T.D. Rogers, A generalization of a fixed point of Reich, Canad. math. Bull. 16 (1973) 201–206.

[40] B.E. Rhoades, A comparison of Various Definitions of Contractive Mappings, Trans. Amer. Math. Soc. 226 (1997) 257–290.

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