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Homomorphisms and Derivations in Lie JC*-Algebras

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Abstract : We investigate isomorphisms between JC^*-algebras, homomorphisms between Lie JC^*-algebras and derivations on Lie JC^*-algebras associated with the functional inequality $\|f(\frac{x-y}{2}+z)-f(x)-2f(z)\| \leqslant \|f(\frac{x+y}{2}+z)\|$.

Keywords : Lie JC*-algebra homomorphism; Lie JC*-algebra derivation; JC*-algebra isomorphism; functional inequality.
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1 Introduction

The original motivation to introduce the class of nonassociative algebras known as Jordan algebras came from quantum mechanics (see [1]). Let $L(H)$ be the real vector space of all bounded self-adjoint linear operators on H, interpreted as the (bounded) *observables* of the system. In 1932, Jordan observed that $L(H)$ is the (nonassociative) algebra via the *anticommutator product* $x \circ y := \frac{xy+yx}{2}$. A commutative algebra X with product $x \circ y$ is called a *Jordan algebra*. A Jordan C^*-subalgebra of a C^*-algebra, endowed with the anticommutator product, is called a JC^*-algebra.

A C^*-algebra C, endowed with the Lie product $[x,y]=\frac{xy-yx}{2}$ on C, is called a Lie C^*-algebra. A C^*-algebra C, endowed with the Lie product $[.,.]$ and the

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anticommutator product $\circ$, is called a Lie JC^*-algebra if $(C, \circ)$ is a JC^*-algebra and $(C, [.,.])$ is a Lie C^*-algebra (see [2, 3, 4]). During the last decades several Lie theory arguments related to functional equations and functional inequalities have been investigated by a number of mathematicians; cf. [5, 6, 7, 8] and references therein.

In this paper we study Lie JC^*-algebra homomorphisms in Lie JC^*-algebras. Our results generalize the JC^*-algebra isomorphisms posed by Park, An and Cui [9] in JC^*-algebras. Moreover, we present the Lie JC^*-algebras derivations on Lie JC^*-algebras associated by the following functional inequality

$$\left\|f(\frac{x-y}{2}+z)-f(x)-2f(z)\right\| \leqslant \left\|f(\frac{x+y}{2}+z)\right\|. \tag{1.1}$$

2 Homomorphisms between Lie JC^*-algebras and Isomorphisms in JC^*-algebras

At the first of this section we would like to investigate Lie JC^*-algebra homomorphisms between two Lie JC^*-algebras and then ,as corollaries, result JC^*-algebra isomorphisms between two JC^*-algebras associated with the functional inequality (1.1). Throughout this section, assume that $\mathcal{A}$ and $\mathcal{B}$ are two Lie JC^*-algebra respectively with norm $\|.\|_{\mathcal{A}}$ and $\|.\|_{\mathcal{B}}$, and also assume that $\mathcal{X}$ and $\mathcal{Y}$ are two JC^*-algebra respectively with norm $\|.\|_{\mathcal{X}}$ and $\|.\|_{\mathcal{Y}}$. First we need the following proposition.

Definition 2.1. *[10] A $\mathbb{C}$-linear mapping $H : \mathcal{A} \to \mathcal{B}$ is called a Lie JC^*-algebra homomorphism if H satisfies*

$$H(x \circ y) = H(x) \circ H(y),$$
$$H([x, y]) = [H(x), H(y)],$$
$$H(x^*) = H(x)^*$$

for all $x, y \in \mathcal{A}$.

Definition 2.2. *[10, 11] For two JC^*-algebras $\mathcal{A}$ and $\mathcal{B}$, a bijective $\mathbb{C}$-linear mapping $H : \mathcal{A} \to \mathcal{B}$ is called a JC^*-algebra isomorphism if H satisfies*

$$H(x \circ y) = H(x) \circ H(y),$$

for all $x, y \in \mathcal{A}$.

Proposition 2.3. *Suppose $f : \mathcal{A} :\to \mathcal{B}$ be a mapping such that*

$$\left\|f(\frac{x-y}{2}+z)-f(x)-2f(z)\right\|_{\mathcal{B}} \leqslant \left\|f(\frac{x+y}{2}+z)\right\|_{\mathcal{B}} \tag{2.1}$$

for all $x, y, z \in \mathcal{A}$. Then f is Cauchy additive.

Proof. Assume that $x = y = z = 0$ in (2.1), we get

$$\| -2f(0)\|_{\mathcal{B}} \leqslant \|f(0)\|_{\mathcal{B}},$$

so $f(0) = 0$.

Let $y = x, z = -x$ in (2.1), it follows that

$$\|f(-x) - f(x) - 2f(-x)\|_{\mathcal{B}} = \| - f(x) - f(-x)\|_{\mathcal{B}} \leqslant \|f(0)\|_{\mathcal{B}} = 0$$

for all $x \in \mathcal{A}$. Hence $f(-x) = -f(x)$ for all $x \in \mathcal{A}$.

Let us suppose $x = 0, y = -2z$ in (2.1), we get

$$\|f(2z) - 2f(z)\|_{\mathcal{B}} \leqslant \|f(0)\|_{\mathcal{B}} = 0$$

for all $z \in \mathcal{A}$. Thus $f(2z) = 2f(z)$ for all $z \in \mathcal{A}$.

Let $z = -\frac{(x+y)}{2}$ in (2.1), it follows that

$$\Big\|f(-y) - f(x) - 2f(-\frac{x+y}{2})\Big\|_{\mathcal{B}} = \| - f(y) - f(x) + f(x+y)\|_{\mathcal{B}} \leqslant \|f(0)\|_{\mathcal{B}} = 0$$

for all $x, y \in \mathcal{A}$, which this proves that

$$f(x+y) = f(x) + f(y)$$

for all $x, y \in \mathcal{A}$ and so that f is Cauchy additive. □

Theorem 2.1. *Suppose $r \neq 1$ and θ be nonnegative real numbers, and let $f : \mathcal{A} \to \mathcal{B}$ be a mapping such that*

$$\Big\|f(\frac{\mu x - y}{2} + z) - \mu f(x) - 2f(z)\Big\|_{\mathcal{B}} \leqslant \Big\|f(\frac{\mu x + y}{2} + z)\Big\|_{\mathcal{B}}, \tag{2.2}$$

$$\|f([x, y]) - [f(x), f(y)]\|_{\mathcal{B}} \leqslant \theta(\|x\|_{\mathcal{A}}^{2r} + \|y\|_{\mathcal{B}}^{2r}), \tag{2.3}$$

$$\|f(x \circ y) - f(x) \circ f(y)\|_{\mathcal{B}} \leqslant \theta(\|x\|_{\mathcal{A}}^{2r} + \|y\|_{\mathcal{B}}^{2r}), \tag{2.4}$$

$$\|f(x^*) - f(x)^*\|_{\mathcal{B}} \leqslant \theta(\|x\|_{\mathcal{A}}^{r} + \|x\|_{\mathcal{A}}^{r}) \tag{2.5}$$

for all $\mu \in \mathbb{T}^1 := \{\lambda \in \mathbb{C} : \|\lambda\| = 1\}$ and all $x, y, z \in \mathcal{A}$. Then the mapping $f : \mathcal{A} \to \mathcal{B}$ is a Lie JC^*-algebra homomorphism.

Proof. Assume $r < 1$.
Suppose $\mu = 1$ in (2.2), then by Proposition 2.3, implies the mapping $f : \mathcal{A} \to \mathcal{B}$ is a Cauchy additive. So $f(0) = 0$. Assume $y = -\mu x$ and $z = 0$ in (2.2), so that

$$\|f(\mu x) - \mu f(x)\|_{\mathcal{B}} \leqslant \|f(0)\|_{\mathcal{B}} = 0$$

for all $x \in \mathcal{A}$ and all $\mu \in \mathbb{T}^1$. Therefore it is concluded that $f(\mu x) = \mu f(x)$ for all $x \in \mathcal{A}$ and all $\mu \in \mathbb{T}^1$. Now by Theorem 2.1 of [12], the mapping f is a $\mathbb{C}$-linear. So one can consider $f(x) = \lim_{n\to\infty} \frac{1}{2^n} f(2^n x)$ for all $x \in \mathcal{A}$.

It follows from (2.3) that

$$\begin{aligned}\|f([x,y])-[f(x),f(y)]\|_{\mathcal{B}} &= \lim_{n\to\infty}\frac{1}{4^n}\|f([2^nx,2^ny]-[f(2^nx),f(2^ny)]\|_{\mathcal{B}}\\ &\leqslant \lim_{n\to\infty}\frac{4^{nr}\,\theta}{4^n}(\|x\|_{\mathcal{A}}^{2r}+\|y\|_{\mathcal{A}}^{2r})=0\end{aligned}$$

for all $x,y\in\mathcal{A}$, which proves

$$f([x,y])=[f(x),f(y)],$$

for all $x,y\in\mathcal{A}$.

It follows from (2.4) that

$$\begin{aligned}\|f(x\circ y)-f(x)\circ f(y)\|_{\mathcal{B}} &= \lim_{n\to\infty}\frac{1}{4^n}\|f(2^nx\circ 2^ny)-f(2^nx)\circ f(2^ny)\|\\ &\leqslant \lim_{n\to\infty}\frac{4^{nr}\,\theta}{4^n}(\|x\|_{\mathcal{A}}^{2r}+\|y\|_{\mathcal{A}}^{2r})=0\end{aligned}$$

for all $x,y\in\mathcal{A}$. Then we obtain

$$f(x\circ y)=f(x)\circ f(y)$$

for all $x,y\in\mathcal{A}$.

And also from (2.5) is concluded that

$$\begin{aligned}\|f(x^*)-f(x)^*\|_{\mathcal{B}} &= \lim_{n\to\infty}\frac{1}{2^n}\left\|f\Big(2^nx^*\Big)-f\Big(2^nx\Big)^*\right\|_{\mathcal{B}}\\ &\leqslant \lim_{n\to\infty}\frac{2^{nr}\,\theta}{2^n}(\|x\|_{\mathcal{A}}^{r}+\|x\|_{\mathcal{A}}^{r})\end{aligned}$$

for all $x\in\mathcal{A}$. Thus we proved

$$f(x^*)=f(x)^*$$

for all $x\in\mathcal{A}$, which this completes the proof. Similarly, one can obtains the result for the case $r>1$. □

Theorem 2.2. *Suppose $r\neq 1$ and θ be nonnegative real numbers, and let $f:\mathcal{A}\to\mathcal{B}$ be a mapping satisfying (2.2) such that*

$$\|f([x,y])-[f(x),f(y)]\|_{\mathcal{B}}\leqslant\theta(\|x\|_{\mathcal{A}}^{r}.\|y\|_{\mathcal{B}}^{r}), \tag{2.6}$$

$$\|f(x\circ y)-f(x)\circ f(y)\|_{\mathcal{B}}\leqslant\theta(\|x\|_{\mathcal{A}}^{r}.\|y\|_{\mathcal{B}}^{r}), \tag{2.7}$$

$$\|f(x^*)-f(x)^*\|_{\mathcal{B}}\leqslant\theta(\|x\|_{\mathcal{A}}^{\frac{r}{2}}.\|x\|_{\mathcal{A}}^{\frac{r}{2}}) \tag{2.8}$$

for all $x,y,z\in\mathcal{A}$. Then the mapping $f:\mathcal{A}\to\mathcal{B}$ is a Lie JC^*-algebra homomorphism.

Proof. Assume $r > 1$.
By the same reasoning as in the proof of Theorem 2.1, the mapping f is a $\mathbb{C}$-linear. So one can consider $f(x) = \lim_{n\to\infty} 2^n f(\frac{x}{2^n})$ for all $x \in \mathcal{A}$.

It follows from (2.6) that

$$\begin{aligned}\|f([x,y]) - [f(x), f(y)]\|_{\mathcal{B}} &= \lim_{n\to\infty} 4^n \|f([\frac{x}{2^n}, \frac{y}{2^n}] - [f(\frac{x}{2^n}), f(\frac{y}{2^n})]\|_{\mathcal{B}} \\ &\leqslant \lim_{n\to\infty} \frac{4^n\,\theta}{4^{nr}}(\|x\|^r_{\mathcal{A}}.\|y\|^r_{\mathcal{A}}) = 0\end{aligned}$$

for all $x, y \in \mathcal{A}$, which proves

$$f([x,y]) = [f(x), f(y)],$$

for all $x, y \in \mathcal{A}$.

It follows from (2.7) that

$$\begin{aligned}\|f(x\circ y) - f(x)\circ f(y)\|_{\mathcal{B}} &= \lim_{n\to\infty} 4^n \|f(\frac{x}{2^n}\circ\frac{y}{2^n}) - f(\frac{x}{2^n})\circ f(\frac{y}{2^n})\| \\ &\leqslant \lim_{n\to\infty} \frac{4^n\,\theta}{4^{nr}}(\|x\|^r_{\mathcal{A}}.\|y\|^r_{\mathcal{A}}) = 0\end{aligned}$$

for all $x, y \in \mathcal{A}$. This implies

$$f(x\circ y) = f(x)\circ f(y),$$

for all $x, y \in \mathcal{A}$.

And also from (2.8) is derived that

$$\begin{aligned}\|f(x^*) - f(x)^*\|_{\mathcal{B}} &= \lim_{n\to\infty} 2^n \left\| f\left(\frac{x^*}{2^n}\right) - f\left(\frac{x}{2^n}\right)^*\right\|_{\mathcal{B}} \\ &\leqslant \lim_{n\to\infty} \frac{2^n\,\theta}{2^{nr}}(\|x\|^{\frac{r}{2}}_{\mathcal{A}}.\|x\|^{\frac{r}{2}}_{\mathcal{A}})\end{aligned}$$

for all $x \in \mathcal{A}$, and this proves

$$f(x^*) = f(x)^*$$

for all $x \in \mathcal{A}$. Therefore we conclude $f : \mathcal{A} \to \mathcal{B}$ is a Lie JC^*-algebra homomorphism. Similarly, one can obtains the result for the case $r < 1$. □

Now we investigate JC^*-algebra isomorphisms in the remaining of this section as the results of above Theorems.

Corollary 2.4. *Suppose $r \neq 1$ and θ be nonnegative real numbers, and let $f : \mathcal{X} \to \mathcal{Y}$ be a bijective mapping satisfying (2.2) such that*

$$\|f(x\circ y) - f(x)\circ f(y)\|_{\mathcal{Y}} \leqslant \theta(\|x\|^{2r}_{\mathcal{X}} + \|y\|^{2r}_{\mathcal{Y}}) \tag{2.9}$$

for all $x, y, z \in \mathcal{X}$. Then the mapping $f : \mathcal{X} \to \mathcal{Y}$ is a JC^*-algebra isomorphism.

Proof. Assume $r > 1$.
Similarly in the proof of Theorem 2.1, the mapping f is a $\mathbb{C}$-linear. So one can consider $f(x) = \lim_{n\to\infty} 2^n f(\frac{x}{2^n})$ for all $x \in \mathcal{X}$.

It follows from (2.9) that

$$\begin{aligned}\|f(x \circ y) - f(x) \circ f(y)\|_{\mathcal{Y}} &= \lim_{n\to\infty} 4^n \|f(\frac{x}{2^n} \circ \frac{y}{2^n}) - f(\frac{x}{2^n}) \circ f(\frac{y}{2^n})\|_{\mathcal{Y}} \\ &\leqslant \lim_{n\to\infty} \frac{4^n\,\theta}{4^{nr}} (\|x\|_{\mathcal{X}}^{2r} + \|y\|_{\mathcal{X}}^{2r}) = 0\end{aligned}$$

for all $x, y \in \mathcal{X}$. Thus

$$f(x \circ y) = f(x) \circ f(y)$$

for all $x, y \in \mathcal{X}$. Hence the mapping f is a JC^*-algebra isomorphism, as desired. Similarly, one can obtains the result for the case $r < 1$. □

Corollary 2.5. *Suppose $r \neq 1$ and θ be nonnegative real numbers, and let $f : \mathcal{X} \to \mathcal{Y}$ be a bijective mapping satisfying (2.2) such that*

$$\|f(x \circ y) - f(x) \circ f(y)\|_{\mathcal{Y}} \leqslant \theta(\|x\|_{\mathcal{X}}^r . \|y\|_{\mathcal{Y}}^r) \tag{2.10}$$

for all $x, y \in \mathcal{X}$. Then the mapping $f : \mathcal{X} \to \mathcal{Y}$ is a JC^*-algebra isomorphism.

Proof. Assume $r < 1$.
Similarly in the proof of Theorem 2.1, the mapping f is a $\mathbb{C}$-linear. So one can consider $f(x) = \lim_{n\to\infty} \frac{1}{2^n} f(2^n x)$ for all $x \in \mathcal{X}$.

It follows from (2.10) that

$$\begin{aligned}\|f(x \circ y) - f(x) \circ f(y)\|_{\mathcal{Y}} &= \lim_{n\to\infty} \frac{1}{4^n} \|f(2^n x \circ 2^n y) - f(2^n x) \circ f(2^n y)\| \\ &\leqslant \lim_{n\to\infty} \frac{4^{nr}\,\theta}{4^n} (\|x\|_{\mathcal{X}}^{2r} + \|y\|_{\mathcal{X}}^{2r}) = 0\end{aligned}$$

for all $x, y \in \mathcal{X}$. Thus

$$f(x \circ y) = f(x) \circ f(y)$$

for all $x, y \in \mathcal{X}$, which this completes the proof of this case. And by same reasons, we obtain the result for the case $r > 1$. □

3 Derivations on Lie JC^*-algebras

In this section, we are going to investigate Lie JC^*-algebra derivations on Lie JC^*-algebras associated with the functional inequality (1.1). Throughout this section, assume that $\mathcal{A}$ is a Lie JC^*-algebra with norm $\|.\|$.

Definition 3.1. *[10] A $\mathbb{C}$-linear mapping $D : \mathcal{A} \to \mathcal{A}$ is called a Lie JC∗-algebra derivation if D satisfies*

$$D(x \circ y) = (Dx) \circ y + x \circ (Dy),$$
$$D([x, y]) = [Dx, y] + [x, Dy],$$
$$D(x^*) = D(x)^*$$

for all $x, y \in \mathcal{A}$.

Theorem 3.1. *Suppose $r \neq 1$ and θ be nonnegative real numbers, and let $f : \mathcal{A} \to \mathcal{A}$ be a mapping satisfying (2.2) and (2.5) such that*

$$\|f([x, y]) - [f(x), y] - [x, f(y)]\| \leqslant \theta(\|x\|^{2r} + \|y\|^{2r}), \tag{3.1}$$
$$\|f(x \circ y) - f(x) \circ y - x \circ f(y)\| \leqslant \theta(\|x\|^{2r} + \|y\|^{2r}), \tag{3.2}$$

for all $x, y \in \mathcal{A}$. Then the mapping $f : \mathcal{A} \to \mathcal{A}$ is a Lie JC^*-algebra derivation.

Proof. Assume $r > 1$.
By the same reasoning as in the proof of Theorem 2.1, the mapping f is a $\mathbb{C}$-linear. So we can consider $f(x) = \lim_{n\to\infty} 2^n f(\frac{x}{2^n})$ for all $x \in \mathcal{A}$.

It follows from (3.1) that

$$\begin{aligned}\|f([x, y]) - [f(x), y] - [x, f(y)]\| &= \lim_{n\to\infty} \frac{1}{4^n}\|f([2^n x, 2^n y]) - [f(2^n x), 2^n y] - [2^n x, f(2^n y)]\| \\ &\leqslant \lim_{n\to\infty} \frac{4^{nr}\,\theta}{4^n}(\|x\|^{2r} + \|y\|^{2r}) = 0\end{aligned}$$

for all $x, y \in \mathcal{A}$. Therefore we obtain

$$f([x, y]) = [f(x), y] + [x, f(y)]$$

for all $x, y \in \mathcal{A}$.

It follows from (3.2) that

$$\begin{aligned}\|f(x \circ y) - f(x) \circ y - x \circ f(y)\| &= \lim_{n\to\infty} \frac{1}{4^n}\|f(2^n x \circ 2^n y) - f(2^n x) \circ 2^n y - 2^n x \circ f(2^n y)\| \\ &\leqslant \lim_{n\to\infty} \frac{4^{nr}\,\theta}{4^n}(\|x\|^{2r} + \|y\|^{2r}) = 0\end{aligned}$$

for all $x, y \in \mathcal{A}$. Then

$$f(x \circ y) = f(x) \circ y + x \circ f(y)$$

for all $x, y \in \mathcal{A}$. And from (2.5) by the same explanation in the proof of Theorem 2.1 we derive that $f(x^*) = f(x)^*$ for all $x \in \mathcal{A}$. Therefore we conclude $f : \mathcal{A} \to \mathcal{A}$ is a Lie JC^*-algebra derivation. Similarly, by the same arguments, we can obtain the result for the case $r < 1$. □

Theorem 3.2. *Suppose $r \neq 1$ and θ be nonnegative real numbers, and let $f : \mathcal{A} \to \mathcal{A}$ be a mapping satisfying (2.2) and (2.8) such that*

$$\|f([x,y]) - [f(x),y] - [x,f(y)]\| \leqslant \theta(\|x\|^r.\|y\|^r), \tag{3.3}$$

$$\|f(x \circ y) - f(x) \circ y - x \circ f(y)\| \leqslant \theta(\|x\|^r.\|y\|^r), \tag{3.4}$$

for all $x, y \in \mathcal{A}$. Then the mapping $f : \mathcal{A} \to \mathcal{A}$ is a Lie JC^*-algebra derivation.

Proof. Assume $r > 1$.
By the same reasoning as in the proof of Theorem 2.1, the mapping f is a $\mathbb{C}$-linear. So we can assume $f(x) = \lim_{n\to\infty} 2^n f(\frac{x}{2^n})$ for all $x \in \mathcal{A}$.

It follows from (3.3) that

$$\begin{aligned}\|f([x,y]) - [f(x),y] - [x,f(y)]\| &= \lim_{n\to\infty} 4^n \|f([\frac{x}{2^n}, \frac{y}{2^n}]) - [f(\frac{x}{2^n}), \frac{y}{2^n}] - [\frac{x}{2^n}, f(\frac{y}{2^n})]\| \\ &\leqslant \lim_{n\to\infty} \frac{4^n\,\theta}{4^{nr}}(\|x\|^r.\|y\|^r) = 0\end{aligned}$$

for all $x, y \in \mathcal{A}$. Hence

$$f([x,y]) = [f(x),y] + [x,f(y)]$$

for all $x, y \in \mathcal{A}$.

It follows from (3.4) that

$$\begin{aligned}\|f(x \circ y) - f(x) \circ y - x \circ f(y)\| &= \lim_{n\to\infty} 4^n \|f(\frac{x}{2^n} \circ \frac{y}{2^n}) - f(\frac{x}{2^n}) \circ \frac{y}{2^n} - \frac{x}{2^n} \circ f(\frac{y}{2^n})\| \\ &\leqslant \lim_{n\to\infty} \frac{4^n\,\theta}{4^{nr}}(\|x\|^r.\|y\|^r) = 0\end{aligned}$$

for all $x, y \in \mathcal{A}$. Therefore

$$f(x \circ y) = f(x) \circ y + x \circ f(y)$$

for all $x, y \in \mathcal{A}$. And from (2.8) by the same explanation in the proof of Theorem 2.2 it is obtained that $f(x^*) = f(x)^*$ for all $x \in \mathcal{A}$. Therefore we conclude $f : \mathcal{A} \to \mathcal{A}$ is a Lie JC^*-algebra derivation. Similarly, one can obtains the result for the case $r < 1$. □

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