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$(\tau_1, \tau_2)^*$-Semi Star Generalized Locally Closed Sets

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Abstract : The aim of this paper is to continue the study of $(\tau_1, \tau_2)^*$-semi star generalized closed sets by introducing the concepts of $(\tau_1, \tau_2)^*$-semi star generalized locally closed sets and study their basic properties in bitopological spaces.

Keywords : $(\tau_1, \tau_2)^*$-semi star generalized locally closed sets; $\tau_1\tau_2$-semi star generalized closed sets; $(\tau_1, \tau_2)^*$-generalized locally closed sets; $(\tau_1, \tau_2)^*$-semi generalized locally closed sets; $(\tau_1, \tau_2)^*$-generalized semi locally closed sets.
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1 Introduction

The study of generalization of closed sets [1, 2] has been found to ensure some new separation axioms which have been very useful in the study of certain objects of digital topology [3]. In recent years many generalizations of closed sets have been developed by various authors. Chandrasekhara Rao and Joseph [4] introduced the concepts of semi star generalized open sets and semi star generalized closed sets in unital topological spaces with the help of semi open sets [5].

Ganster and Reilly [6] introduced locally closed sets [7] in topological spaces and Stone called locally closed sets as FG sets. Maki, Sundaram and Balachandran [8] introduced the concept of generalized locally closed sets and obtained seven different notions of generalized continuities.

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Ganster, Arockiarani and Balachandran [9] introduced regular generalized locally closed sets and RGLC continuous functions and discussed some of their properties. and Palaniappan and Alagar [10] introduced regular generalized locally closed sets with respect to ideal. Chandrasekhara Rao and Kannan [11, 12] introduced the concepts of semi star generalized locally closed sets and s^*g-submaximal spaces in unital topological spaces.

Mean while Kelly [13] introduced the concept of bitopological spaces. Jelic [14] introduced locally closed sets and lc-continuity in bitopological settings. Chandrasekhara Rao and Kannan [15, 16] introduced the concepts of semi star generalized closed sets in bitopological spaces and then they introduced the concepts of $\tau_1\tau_2$-semi star generalized locally closed sets [26, 27] and pairwise s^*g-submaximal spaces with the help of s^*g-closed sets and studied their basic properties in bitopological spaces.

Kannan et al introduced $(\tau_1, \tau_2)^*$-semi star generalized closed sets [17] in bitopological spaces. In this sequel the aim of this paper is to introduce the concepts of $(\tau_1, \tau_2)^*$-semi star generalized locally closed sets and study their basic properties in bitopological spaces. In the next section some prerequisites and abbreviations are established.

2 Preliminaries

Let (X, τ_1, τ_2) or simply X denote a bitopological space. By $\tau_1\tau_2$-$S^*GO(X, \tau_1, \tau_2)$ {resp. $\tau_1\tau_2$-$S^*GC(X, \tau_1, \tau_2)$}, we shall mean the collection of all $\tau_1\tau_2$-s^*g open sets (resp. $\tau_1\tau_2$-s^*g closed sets) in (X, τ_1, τ_2). For any subset $A \subseteq X$, τ_i-int(A) and τ_i-cl(A) denote the interior and closure of a set A with respect to the topology τ_i respectively. A^C denotes the complement of A in X unless explicitly stated. We shall require the following known definitions.

Definition 2.1. A subset of a bitopological space (X, τ_1, τ_2) is called

(a) $\tau_1\tau_2$-semi open [18, 19] if there exists a τ_1-open set U such that $U \subseteq A \subseteq \tau_2$-cl (U).

(b) $\tau_1\tau_2$-semi closed [18, 19] if $X - A$ is $\tau_1\tau_2$-semi open.

Equivalently, a subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-semi closed if there exists a τ_1-closed set F such that τ_2-int $(F) \subseteq A \subseteq F$.

(c) $\tau_1\tau_2$-generalized closed ($\tau_1\tau_2$-g closed) [20] if τ_2-cl $(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-open in X.

(d) $\tau_1\tau_2$-generalized open ($\tau_1\tau_2$-g open) [20] if $X - A$ is $\tau_1\tau_2$-g closed.

(e) $\tau_1\tau_2$-semi star generalized closed ($\tau_1\tau_2$-s^*g closed) [15] if τ_2-cl $(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-semi open in X.

(f) $\tau_1\tau_2$-semi star generalized open ($\tau_1\tau_2$-s^*g open) [15] if $X - A$ is $\tau_1\tau_2$-s^*g closed in X.

(g) $\tau_1\tau_2$-semi generalized closed ($\tau_1\tau_2$-sg closed) [21] if τ_2-scl $(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-semi open in X.

(h) $\tau_1\tau_2$-semi generalized open ($\tau_1\tau_2$-sg open) [21] if $X - A$ is $\tau_1\tau_2$-sg closed in X.

(i) $\tau_1\tau_2$-generalized semi closed ($\tau_1\tau_2$-gs closed) [21] if τ_2-scl $(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_1-open in X.

(j) $\tau_1\tau_2$-generalized semi open ($\tau_1\tau_2$-gs open) [21] if $X - A$ is $\tau_1\tau_2$-gs closed in X.

(k) $(\tau_1, \tau_2)^*$-generalized closed $\{(\tau_1, \tau_2)^*$-g closed$\}$ [22] if $\tau_1\tau_2$-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_1\tau_2$-open in X.

(l) $(\tau_1, \tau_2)^*$-generalized open $\{(\tau_1, \tau_2)^*$-g open$\}$ [22] if $X-A$ is $(\tau_1, \tau_2)^*$-g closed.

(m) $(\tau_1, \tau_2)^*$-semi open [22] if $A \subseteq \tau_1\tau_2$-$cl[\tau_1\tau_2$-$int(S)]$.

(n) $(\tau_1, \tau_2)^*$-semi closed [22] if $X - A$ is $(\tau_1, \tau_2)^*$-semi open.

(o) $(\tau_1, \tau_2)^*$-semi generalized closed $\{(\tau_1, \tau_2)^*$-sg closed$\}$ [22] if $(\tau_1, \tau_2)^*$-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(\tau_1, \tau_2)^*$-semi open in X.

(p) $(\tau_1, \tau_2)^*$-semi generalized open $\{(\tau_1, \tau_2)^*$-sg open$\}$ [22] if $X - A$ is $(\tau_1, \tau_2)^*$-sg closed.

(q) $(\tau_1, \tau_2)^*$-generalized semi closed $\{(\tau_1, \tau_2)^*$-gs closed$\}$ [22] if $(\tau_1, \tau_2)^*$-$scl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_1\tau_2$-open in X.

(r) $(\tau_1, \tau_2)^*$-generalized semi open $\{(\tau_1, \tau_2)^*$-gs open$\}$ [22] if $X-A$ is $(\tau_1, \tau_2)^*$-gs closed.

(s) $(\tau_1, \tau_2)^*$-semi star generalized closed $\{(\tau_1, \tau_2)^*$-s^*g closed$\}$ [17] if $\tau_1\tau_2$-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_1\tau_2$-semi open in X.

(t) $(\tau_1, \tau_2)^*$-semi star generalized open $\{(\tau_1, \tau_2)^*$-s^*g open$\}$ [17] if $X - A$ is $(\tau_1, \tau_2)^*$-s^*g closed.

3 $(\tau_1, \tau_2)^*$-Semi Star Generalized Locally Closed Sets

Definition 3.1. A subset A of a bitopological space (X, τ_1, τ_2) is said to be

(a) $(\tau_1, \tau_2)^*$-s^*g locally closed set if $A = G \cap F$ where G is $\tau_1\tau_2$-s^*g open set and F is $\tau_1\tau_2$-s^*g closed set in X.

(b) $(\tau_1, \tau_2)^*$-s^*g locally closed* if $A = G \cap F$ where G is $\tau_1\tau_2$-s^*g open set and F is τ_2-closed in X.

(c) $(\tau_1, \tau_2)^*$-s^*g locally closed** if $A = G \cap F$ where G is τ_1-open and F is $\tau_1\tau_2$-s^*g closed in X.

Remark 3.2.

(a) *The class of all* $(\tau_1, \tau_2)^*$-s^*g *locally closed sets in* (X, τ_1, τ_2) *is denoted by* $(\tau_1, \tau_2)^*$-$S^*GLC(X, \tau_1, \tau_2)$.

(b) *The class of all* $(\tau_1, \tau_2)^*$-s^*g *locally closed* sets in* (X, τ_1, τ_2) *is denoted by* $(\tau_1, \tau_2)^*$-$S^*GLC^*(X, \tau_1, \tau_2)$.

(c) *The class of all* $(\tau_1, \tau_2)^*$-s^*g *locally closed** sets in* (X, τ_1, τ_2) *is denoted by* $(\tau_1, \tau_2)^*$-$S^*GLC^{**}(X, \tau_1, \tau_2)$.

Example 3.3. *Let* $X = \{a, b, c\}, \tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{b, c\}\}$. *Then,* $\tau_1\tau_2$-s^*g *open sets in* (X, τ_1, τ_2) *are* $\phi, X, \{a\}, \{b, c\}$ *and* $\tau_1\tau_2$-s^*g *closed sets in* (X, τ_1, τ_2) *are* $X, \phi, \{a\}, \{b, c\}$. *Then*

(a) $(\tau_1, \tau_2)^*$-s^*g *locally closed sets in* (X, τ_1, τ_2) *are* $\phi, X, \{a\}, \{b, c\}$.

(b) $(\tau_1, \tau_2)^*$-s^*g *locally closed* sets in* (X, τ_1, τ_2) *are* $\phi, X, \{a\}, \{b, c\}$.

(c) $(\tau_1, \tau_2)^*$-s^*g *locally closed** sets in* (X, τ_1, τ_2) *are* $\phi, X, \{a\}, \{b, c\}$.

Definition 3.4. A subset A of a bitopological space (X, τ_1, τ_2) is said to be

(a) $(\tau_1, \tau_2)^*$-g locally closed set if $A = G \cap F$ where G is $\tau_1\tau_2$-g open set and F is $\tau_1\tau_2$-g closed set in X.

(b) $(\tau_1, \tau_2)^*$-sg locally closed set if $A = G \cap F$ where G is $\tau_1\tau_2$-sg open set and F is $\tau_1\tau_2$-sg closed set in X.

(c) $(\tau_1, \tau_2)^*$-gs locally closed set if $A = G \cap F$ where G is $\tau_1\tau_2$-gs open set and F is $\tau_1\tau_2$-gs closed set in X.

Example 3.5. Let $X = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a, b\}\}, \tau_2 = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$. Then all the subsets of X are $(\tau_1, \tau_2)^*$-g locally closed, $(\tau_1, \tau_2)^*$-sg locally closed and $(\tau_1, \tau_2)^*$-gs locally closed in X.

Theorem 3.6. *In any bitopological space* (X, τ_1, τ_2),

(i) $A \in (\tau_1, \tau_2)^*$-$S^*GLC^*(X, \tau_1, \tau_2) \Rightarrow A \in (\tau_1, \tau_2)^*$-$S^*GLC(X, \tau_1, \tau_2)$.

(ii) $A \in (\tau_1, \tau_2)^*$-$S^*GLC^{**}(X, \tau_1, \tau_2) \Rightarrow A \in (\tau_1, \tau_2)^*$-$S^*GLC(X, \tau_1, \tau_2)$.

(iii) $A \in \tau_1\tau_2$-$S^*GC(X, \tau_1, \tau_2) \Rightarrow A \in (\tau_1, \tau_2)^*$-$S^*GLC(X, \tau_1, \tau_2)$.

(iv) $A \in \tau_1\tau_2$-$S^*GO(X, \tau_1, \tau_2) \Rightarrow A \in (\tau_1, \tau_2)^*$-$S^*GLC(X, \tau_1, \tau_2)$.

Proof. (i) Since A is $(\tau_1, \tau_2)^*$-s^*g locally closed* subset in (X, τ_1, τ_2), we have $A = G \cap F$ where G is $\tau_1\tau_2$-s^*g open set and F is τ_2-closed in X. Since every τ_2-closed set is $\tau_1\tau_2$-s^*g closed in (X, τ_1, τ_2), $A = G \cap F$ where G is $\tau_1\tau_2$-s^*g open and F is $\tau_1\tau_2$-s^*g closed in (X, τ_1, τ_2). Therefore, $A \in (\tau_1, \tau_2)^*$-$S^*GLC(X, \tau_1, \tau_2)$.

(ii) Since A is $(\tau_1, \tau_2)^*$-s^*g locally closed** subset in (X, τ_1, τ_2), we have $A = G \cap F$ where G is τ_1-open and F is $\tau_1\tau_2$-s^*g closed in (X, τ_1, τ_2). Since every

τ_1-open set is $\tau_1\tau_2$-s^*g open in $(X, \tau_1, \tau_2), A = G \cap F$ where G is $\tau_1\tau_2$-s^*g open and F is $\tau_1\tau_2$-s^*g closed in (X, τ_1, τ_2). Therefore, $A \in \tau_1\tau_2$-$S^*GLC(X, \tau_1, \tau_2)$.

(iii) Since $A = A \cap X$ and A is $\tau_1\tau_2$-s^*g closed and X is $\tau_1\tau_2$-s^*g open in (X, τ_1, τ_2), we have $A \in (\tau_1, \tau_2)^*$-$S^*GLC(X, \tau_1, \tau_2)$.

(iv) Since $A = A \cap X$ and A is $\tau_1\tau_2$-s^*g open and X is $\tau_1\tau_2$-s^*g closed in (X, τ_1, τ_2), we have $A \in (\tau_1, \tau_2)^*$-$S^*GLC(X, \tau_1, \tau_2)$. □

Remark 3.7. *The converses of (i), (ii), (iii) and (iv) of the above theorem are not true in general as can be seen from the following examples.*

Example 3.8. *In Example 3.5* $\{b, c, d\}$ *is* $(\tau_1, \tau_2)^*$-s^*g *locally closed in* (X, τ_1, τ_2), *but not* $(\tau_1, \tau_2)^*$-s^*g *locally closed* in* (X, τ_1, τ_2).

Example 3.9. *In Example 3.3,* $\{b\}$ *is* $(\tau_1, \tau_2)^*$-s^*g *locally closed in* (X, τ_1, τ_2), *but not* $(\tau_1, \tau_2)^*$-s^*g *locally closed** in* (X, τ_1, τ_2).

Example 3.10. *In Example 3.5,* $\{b, c, d\}$ *is* $(\tau_1, \tau_2)^*$-s^*g *locally closed in* (X, τ_1, τ_2), *but not* $\tau_1\tau_2$-s^*g *open in* (X, τ_1, τ_2) *and* $\{a\}$ *is* $(\tau_1, \tau_2)^*$-s^*g *locally closed in* (X, τ_1, τ_2), *but not* $\tau_1\tau_2$-s^*g *closed in* (X, τ_1, τ_2).

Recall that a bitopological space (X, τ_1, τ_2) is a pairwise door space [23] if every subset of (X, τ_1, τ_2) is either τ_i-open or τ_j-closed, for $i, j = 1, 2$ and $i \neq j$.

Theorem 3.11. *If* (X, τ_1, τ_2) *is pairwise door space, then every subset of* X *is both* $(\tau_1, \tau_2)^*$-s^*g *locally closed and* $(\tau_2, \tau_1)^*$-s^*g *locally closed.*

Proof. Since (X, τ_1, τ_2) is pairwise door space, every subset of (X, τ_1, τ_2) is either τ_1-open or τ_2-closed and τ_2-open or τ_1-closed. Since every τ_1-open (resp. τ_2-closed) subset of (X, τ_1, τ_2) is $\tau_1\tau_2$-s^*g open (resp. $\tau_1\tau_2$-s^*g closed), we have every subset of (X, τ_1, τ_2) is either $\tau_1\tau_2$-s^*g open or $\tau_1\tau_2$-s^*g closed. Since every $\tau_1\tau_2$-s^*g open and $\tau_1\tau_2$-s^*g closed subset of (X, τ_1, τ_2) is $(\tau_1, \tau_2)^*$-s^*g locally closed, we have every subset of X is $(\tau_1, \tau_2)^*$-s^*g locally closed. Similarly we can prove that every subset of X is $(\tau_2, \tau_1)^*$-s^*g locally closed. □

Theorem 3.12. *For a subset* A *of a bitopological space* (X, τ_1, τ_2), *the following are equivalent.*

(a) $A \in (\tau_1, \tau_2)^*$-$S^*GLC^*(X, \tau_1, \tau_2)$.

(b) $A = G \cap [\tau_2\text{-}cl\,(A)]$ *for some* $\tau_1\tau_2$-s^*g *open set* G.

(c) $A \cup \{X - [\tau_2\text{-}cl\,(A)]\}$ *is* $\tau_1\tau_2$-s^*g *open.*

(d) $[\tau_2\text{-}cl\,(A)] - A$ *is* $\tau_1\tau_2$-s^*g *closed.*

Proof. $(a) \Rightarrow (b)$: Since A is $(\tau_1, \tau_2)^*$-s^*g locally closed* set in (X, τ_1, τ_2), we have $A = G \cap F$ where G is $\tau_1\tau_2$-s^*g open set and F is τ_2-closed in X. Since $A \subseteq \tau_2$-cl (A) and $A \subseteq G$, we have

$$A \subseteq G \cap [\tau_2\text{-cl } (A)] \tag{1}$$

Since $A \subseteq F$ and F is τ_2-closed in X, we have τ_2-cl $(A) \subseteq F$. Therefore, $G \cap [\tau_2$-cl $(A)] \subseteq G \cap F = A$. Hence

$$G \cap [\tau_2\text{-cl } (A)] \subseteq A \tag{2}$$

From(1) and (2), we have $A = G \cap [\tau_2$-cl $(A)]$ for some $\tau_1\tau_2$-s^*g open set G in (X, τ_1, τ_2).

$(b) \Rightarrow (a)$: Suppose that $A = G \cap [\tau_2$-cl $(A)]$ for some $\tau_1\tau_2$-s^*g open set G in (X, τ_1, τ_2). Since τ_2-cl (A) is τ_2-closed in (X, τ_1, τ_2) and G is $\tau_1\tau_2$-s^*g closed in (X, τ_1, τ_2), we have $A \in (\tau_1, \tau_2)^*$-$S^*GLC^*(X, \tau_1, \tau_2)$

$(b) \Rightarrow (c)$: Since $A = G \cap [\tau_2$-cl $(A)]$ for some $\tau_1\tau_2$-s^*g open set G in (X, τ_1, τ_2), we have $A \cup \{X - [\tau_2\text{-cl } (A)]\} = \{G \cap [\tau_2\text{-cl } (A)]\} \cup \{X - [\tau_2\text{-cl } (A)]\} = G$. Therefore, $A \cup \{X - [\tau_2\text{-cl } (A)]\}$ is $\tau_1\tau_2$-s^*g open.

$(c) \Rightarrow (b)$: Suppose that $A \cup \{X - [\tau_2\text{-cl } (A)]\}$ is $\tau_1\tau_2$-s^*g open in (X, τ_1, τ_2). Let $G = A \cup \{X - [\tau_2\text{-cl } (A)]\}$. Then G is $\tau_1\tau_2$-s^*g open in (X, τ_1, τ_2). Now, $G \cap [\tau_2\text{-}cl(A)] = [A \cup \{X - [\tau_2\text{-}cl(A)]\}] \cap [\tau_2\text{-}cl(A)] = \{[A \cup [\tau_2\text{-}cl(A)]^C\} \cap [\tau_2\text{-}cl(A)] = \{A \cap [\tau_2\text{-}cl(A)]\} \cup \{[\tau_2\text{-}cl(A)]^C \cap [\tau_2\text{-}cl(A)]\} = A \cup \phi = A$. Therefore, $A = G \cap [\tau_2$-cl $(A)]$ for some $\tau_1\tau_2$-s^*g open set G in (X, τ_1, τ_2).

$(c) \Rightarrow (d)$: Suppose that $A \cup \{X - [\tau_2\text{-cl } (A)]\}$ is $\tau_1\tau_2$-s^*g open in (X, τ_1, τ_2). Let $G = A \cup \{X - [\tau_2\text{-cl } (A)]\}$. Since G is $\tau_1\tau_2$-s^*g open in (X, τ_1, τ_2), we have $X - G$ is $\tau_1\tau_2$-s^*g closed in (X, τ_1, τ_2). Now, $X - G = X - [A \cup \{X - [\tau_2\text{-}cl(A)]\}]$ $= (X - A) \cap \{X - [\tau_2\text{-}cl(A)]\} = (X - A) \cap [\tau_2\text{-}cl(A)] = \tau_2\text{-}cl(A) - A$. Therefore, τ_2-cl $(A) - A$ is $\tau_1\tau_2$-s^*g closed in (X, τ_1, τ_2).

$(d) \Rightarrow (c)$: Suppose that τ_2-cl $(A) - A$ is $\tau_1\tau_2$-s^*g closed in (X, τ_1, τ_2). Let $F = \tau_2$-cl $(A) - A$. Then F is $\tau_1\tau_2$-s^*g closed in (X, τ_1, τ_2) implies that $X - F$ is $\tau_1\tau_2$-s^*g open in (X, τ_1, τ_2). Now, $X - F = X - \{[\tau_2\text{-}cl(A)] - A\} = X \cap \{[\tau_2\text{-}cl(A)] - A\}^C$ $= X \cap \{[\tau_2\text{-}cl(A)] \cap A^C\}^C = X \cap \{[\tau_2\text{-}cl(A)]^C \cup (A^C)^C\} = X \cap \{[\tau_2\text{-}cl(A)]^C \cup A\}$ $= \{X \cap [\tau_2\text{-}cl(A)]^C\} \cup \{X \cap A\} = [\tau_2\text{-}cl(A)]^C \cup A = \{X - [\tau_2\text{-}cl(A)]\} \cup A$. Hence $A \cup \{X - [\tau_2\text{-cl } (A)]\}$ is $\tau_1\tau_2$-s^*g open in (X, τ_1, τ_2). □

Theorem 3.13. *In a bitopological space (X, τ_1, τ_2), the following are equivalent.*

(a) $A - [\tau_1\text{-}int\ (A)]$ is τ_2-s^*gopen in (X, τ_1, τ_2).

(b) $[\tau_1\text{-}\ int\ (A)] \cup [X - A]$ is $\tau_1\tau_2$-s^*g closed in (X, τ_1, τ_2).

(c) $G \cup [\tau_1\text{-}int\ (A)] = A$ for some $\tau_1\tau_2$-s^*g open set G in (X, τ_1, τ_2).

Proof. (a) $\Rightarrow$ (b) : Now, $X - \{A - [\tau_1\text{-}int(A)]\} = X \cap \{A - [\tau_1\text{-}int(A)]\}^C = X \cap [A \cap \{\tau_1\text{-}int(A)\}^C]^C = X \cap \{A^C \cup [\{\tau_1\text{-}int(A)\}^C]^C\} = X \cap \{A^C \cup [\tau_1\text{-}int(A)]\} = \{A^C \cup [\tau_1\text{-}int(A)]\} = [\tau_1\text{-}int(A)] \cup [X - A]$. Since $A - [\tau_1\text{-int }(A)]$ is $\tau_1\tau_2\text{-}s^*g$ open, we have $X - \{A - [\tau_1\text{-int }(A)]\} = [\tau_1 - int(A)] \cup [X - A]$ is $\tau_1\tau_2\text{-}s^*g$ closed in (X, τ_1, τ_2).

(b) $\Rightarrow$ (a) : Suppose that $[\tau_1\text{-int }(A)] \cup [X - A]$ is $\tau_1\tau_2\text{-}s^*g$ closed in (X, τ_1, τ_2). Since $[\tau_1\text{-int }(A)] \cup [X - A]$ is $\tau_1\tau_2\text{-}s^*g$ closed, we have $X - \{[\tau_1\text{-int }(A)] \cup [X - A]\}$ is $\tau_1\tau_2\text{-}s^*g$ open. Now, $X - \{[\tau_1\text{-}int(A)] \cup [X - A]\} = X \cap \{[\tau_1\text{-}int(A)] \cup [X - A]\}^C = X \cap \{[\tau_1\text{-}int(A)] \cup A^C\}^C = X \cap \{[\tau_1\text{-}int(A)]^C \cap (A^C)^C\} = X \cap \{[\tau_1\text{-}int(A)]^C \cap A\} = A \cap [\tau_1\text{-}int(A)]^C = A - [\tau_1\text{-}int(A)]$. Therefore, $A - [\tau_1\text{-int }(A)]$ is $\tau_1\tau_2\text{-}s^*g$ open in (X, τ_1, τ_2).

(b) $\Rightarrow$ (c) : Suppose that $[\tau_1\text{-int }(A)] \cup [X - A]$ is $\tau_1\tau_2\text{-}s^*g$ closed. Let $U = [\tau_1\text{-int }(A)] \cup [X - A]$. Then U is $\tau_1\tau_2\text{-}s^*g$ closed. Then U^C is $\tau_1\tau_2\text{-}s^*g$ open. Now, $U^C \cup [\tau_1\text{-}int(A)] = \{[\tau_1\text{-}int(A)] \cup [X - A]\}^C \cup [\tau_1\text{-}int(A)] = \{[\tau_1\text{-}int(A)]^C \cap (A^C)^C\} \cup [\tau_1\text{-}int(A)] = \{[\tau_1\text{-}int(A)]^C \cap A\} \cup [\tau_1\text{-}int(A)] = \{[\tau_1\text{-}int(A)]^C \cup [\tau_1\text{-}int(A)]\} \cap \{A \cup [\tau_1\text{-}int(A)]\} = X \cap A = A$. Take $G = U^C$. Then $A = G \cup [\tau_1\text{-int }(A)] = A$ for some $\tau_1\tau_2\text{-}s^*g$ open set in (X, τ_1, τ_2).

(c) $\Rightarrow$ (b) : Suppose that $A = G \cup [\tau_1\text{-int }(A)] = A$ for some $\tau_1\tau_2\text{-}s^*g$ open set G in (X, τ_1, τ_2). Now, $[\tau_1\text{-}int(A)] \cup [X - A] = \tau_1\text{-}int(A) \cup A^C = [\tau_1\text{-}int(A)] \cup \{G \cup [\tau_1\text{-}int(A)]\}^C = [\tau_1\text{-}int(A)] \cup \{G^C \cap [\tau_1\text{-}int(A)]^C\} = \{[\tau_1\text{-}int(A)] \cup G^C\} \cap \{[\tau_1\text{-}int(A)] \cup [\tau_1\text{-}int(A)]^C\} = \{[\tau_1\text{-}int(A)] \cup G^C\} \cap X\} = \{[\tau_1\text{-}int(A)] \cup G^C\} = X - G$. Since G is $\tau_1\tau_2\text{-}s^*g$ open in (X, τ_1, τ_2), we have $X - G$ is $\tau_1\tau_2\text{-}s^*g$ closed in (X, τ_1, τ_2). Therefore, $[\tau_1\text{-int }(A)] \cup [X - A]$ is $\tau_1\tau_2\text{-}s^*g$ closed in (X, τ_1, τ_2). □

Remark 3.14. *Even A and B are not $(\tau_1, \tau_2)^*$-s^*g locally closed sets in (X, τ_1, τ_2), $A \cup B$ is $(\tau_1, \tau_2)^*$-s^*g locally closed in general as can be seen from the following example.*

Example 3.15. *In Example 3.3, $A = \{b\}, B = \{c\}$ are not $(\tau_1, \tau_2)^*$-s^*g locally closed sets in (X, τ_1, τ_2), but $A \cup B = \{b, c\}$ is $(\tau_1, \tau_2)^*$-s^*g locally closed set in (X, τ_1, τ_2).*

Remark 3.16. *Since every $(\tau_1, \tau_2)^*$-s^*g locally closed set is the intersection of a $\tau_1\tau_2$-s^*g open set and $\tau_1\tau_2$-s^*g closed set, we can conclude the following.*

Theorem 3.17. *A subset A of (X, τ_1, τ_2) is $(\tau_1, \tau_2)^*$-s^*g locally closed if and only if A^C is the union of a $\tau_1\tau_2$-s^*g open set and $\tau_1\tau_2$-s^*g closed set.*

Remark 3.18. *Every τ_1-open set {resp. τ_2-closed set} is $\tau_1\tau_2$-s^*g open {resp. $\tau_1\tau_2$-s^*g closed}. Accordingly, we conclude the following.*

Theorem 3.19.

(a) *Every τ_1-open set is $(\tau_1, \tau_2)^*$-s^*g locally closed and every τ_2-closed set is $(\tau_1, \tau_2)^*$-s^*g locally closed.*

(b) *Every $\tau_1\tau_2$-locally closed set is $(\tau_1,\tau_2)^*$-s^*g locally closed, $(\tau_1,\tau_2)^*$-s^*g locally closed* and $(\tau_1,\tau_2)^*$-s^*g locally closed**.*

Remark 3.20. *But the converses of the assertions of above theorem are not true in general as can be seen in the following examples.*

Example 3.21.

(a) *In Example 3.3, $\{b,c\}$ is $(\tau_1,\tau_2)^*$-s^*g locally closed set in (X,τ_1,τ_2), but $\{b,c\}$ is not τ_2-closed in (X,τ_1,τ_2).*

(b) *In Example 3.3, $\{b,c\}$ is $(\tau_1,\tau_2)^*$-s^*g locally closed set in (X,τ_1,τ_2), but $\{b,c\}$ is not τ_1 - open in (X,τ_1,τ_2).*

(c) *In Example 3.5, $\{b\}$ is a $(\tau_1,\tau_2)^*$-s^*g locally closed set, $(\tau_1,\tau_2)^*$-s^*g locally closed* set and $(\tau_1,\tau_2)^*$-s^*g locally closed** set but not $\tau_1\tau_2$-locally closed in (X,τ_1,τ_2).*

Remark 3.22. *Since every $\tau_1\tau_2$-s^*g closed set is $\tau_1\tau_2$-g closed, $\tau_1\tau_2$-sg closed and $\tau_1\tau_2$-gs closed, we conclude the following.*

Theorem 3.23.

(a) *Every $(\tau_1,\tau_2)^*$-s^*g locally closed is $(\tau_1,\tau_2)^*$-g locally closed.*

(b) *Every $(\tau_1,\tau_2)^*$-s^*g locally closed is $(\tau_1,\tau_2)^*$-sg locally closed.*

(c) *Every $(\tau_1,\tau_2)^*$-s^*g locally closed is $(\tau_1,\tau_2)^*$-gs locally closed.*

Remark 3.24. *But none of the assertions of the above theorem are reversible in general as can be seen in the following example.*

Example 3.25. *Let $X=\{a,b,c\}$, $\tau_1=\{\phi,X,\{a\}\}$, $\tau_2=\{\phi,X,\{a\},\{a,c\}\}$. Then $\{a,b\}$ is a $(\tau_1,\tau_2)^*$-g locally closed set, $(\tau_1,\tau_2)^*$-sg locally closed set and $(\tau_1,\tau_2)^*$-gs locally closed set, but not $(\tau_1,\tau_2)^*$-s^*g locally closed in (X,τ_1,τ_2).*

From the above results we conclude the following.

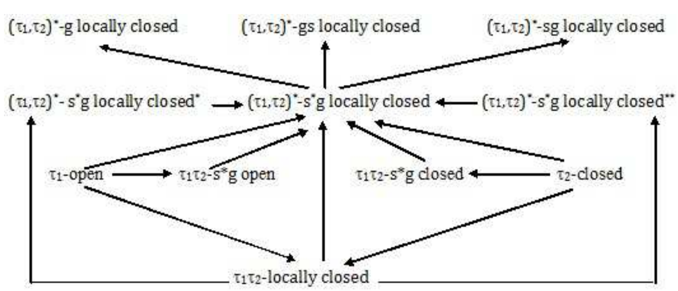

Fig 1. Relationship between several locally closed sets

Since the finite intersection of τ_1-open sets is τ_1-open and the intersection of two $\tau_1\tau_2$-s^*g closed sets is $\tau_1\tau_2$-s^*g closed, we immediately get the following theorem.

Theorem 3.26. *In any bitopological space (X, τ_1, τ_2), intersection of two $(\tau_1, \tau_2)^*$-s^*g locally closed** sets is $(\tau_1, \tau_2)^*$-s^*g locally closed**.*

In this sequel our next result exhibits the intersection of a $(\tau_1, \tau_2)^*$-s^*g locally closed set and a τ_2-closed set in a bitopological space.

Theorem 3.27. *If $A \in (\tau_1, \tau_2)^*$-$S^*GLC(X, \tau_1, \tau_2)$ and B is τ_2-closed in X, then $A \cap B \in (\tau_1, \tau_2)^*$-$S^*GLC(X, \tau_1, \tau_2)$.*

Proof. It is obvious since every τ_2-closed set is $\tau_1\tau_2$-s^*g closed and the intersection of two $\tau_1\tau_2$-s^*g closed sets is $\tau_1\tau_2$-s^*g closed. □

Our next result is an immediate consequence of the above theorem.

Theorem 3.28. *If $A \in (\tau_1, \tau_2)^*$-$S^*GLC(X, \tau_1, \tau_2)$ and B is $\tau_1\tau_2$-s^*g closed in X, then $A \cap B \in (\tau_1, \tau_2)^*$-$S^*GLC(X, \tau_1, \tau_2)$.*

Remark 3.29. *The complement of a $(\tau_1, \tau_2)^*$-s^*g locally closed set in (X, τ_1, τ_2) is not $(\tau_1, \tau_2)^*$-s^*g locally closed in general and hence the finite union of $(\tau_1, \tau_2)^*$-s^*g locally closed sets need not be $(\tau_1, \tau_2)^*$-s^*g locally closed in (X, τ_1, τ_2). The next examples show the claim.*

Example 3.30. *In Example 3.5, $\{a\}$ is a $(\tau_1, \tau_2)^*$-s^*g locally closed set,but its complement $\{b, c\}$ is not $(\tau_1, \tau_2)^*$-s^*g locally closed in (X, τ_1, τ_2).*

Example 3.31. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}, \{b, c\}\}$, $\tau_2 = \{\phi, X, \{b\}, \{a, c\}\}$. Then, $A = \{b\}$, $B = \{c\}$ are $(\tau_1, \tau_2)^*$-s^*g locally closed sets, but $A \cup B = \{b, c\}$ is not $(\tau_1, \tau_2)^*$-s^*g locally closed in (X, τ_1, τ_2).*

Theorem 3.32. *In a bitopological space (X, τ_1, τ_2), the following are equivalent.*

(a) A is $(\tau_1, \tau_2)^*$-s^*g locally closed if and only if A^C is $(\tau_1, \tau_2)^*$-s^*g locally closed.

(b) $(\tau_1, \tau_2)^*$-s^*g locally closed sets are closed under finite union.

Proof. $(a) \Rightarrow (b)$: Suppose that A is $(\tau_1, \tau_2)^*$-s^*g locally closed if and only if A^C is $(\tau_1, \tau_2)^*$-s^*g locally closed. Let A, B be $(\tau_1, \tau_2)^*$-s^*g locally closed. Then by our assumption, A^C, B^C are $(\tau_1, \tau_2)^*$-s^*g locally closed. Consequently, $(A \cup B)^C = A^C \cap B^C$ is $(\tau_1, \tau_2)^*$-s^*g locally closed. Therefore, $A \cup B$ is $(\tau_1, \tau_2)^*$-s^*g locally closed.

$(b) \Rightarrow (a)$: Suppose that $(\tau_1, \tau_2)^*$-s^*g locally closed sets are closed under finite union. Let A be $(\tau_1, \tau_2)^*$-s^*g locally closed. Then $A = G \cap F$ where G is $\tau_1\tau_2$-s^*g open and F is $\tau_1\tau_2$-s^*g closed in X. Since G^C is $\tau_1\tau_2$-s^*g closed and F^C is $\tau_1\tau_2$-s^*g open in X and every $\tau_1\tau_2$-s^*g open is $(\tau_1, \tau_2)^*$-s^*g locally closed and $\tau_1\tau_2$-s^*g closed set is $(\tau_1, \tau_2)^*$-s^*g locally closed, we have A^C is $(\tau_1, \tau_2)^*$-s^*g locally closed by our assumption. Similarly, we can prove if A^C is $(\tau_1, \tau_2)^*$-s^*g locally closed then A is $(\tau_1, \tau_2)^*$-s^*g locally closed. □

Conclusion

Thus we have studied properties of $(\tau_1, \tau_2)^*$-s^*g locally closed sets in the context of bitopological spaces. Borges [24] showed that locally closed sets play an important role in the context of simple extensions and some results of Engelking [25] indicate that locally closed subsets are of some interest in the setting of local compactness, Čech-Stone compactifications, or Čech complete spaces. Further research can be undertaken in this direction.

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