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Pasting Lemmas for Some Continuous Functions

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Abstract : One of the most important concepts in topological space is the pasting lemma for continuous functions. It plays an important role in algebraic topology. In the recent years pasting lemmas for some stronger and weaker forms of continuous functions such as g-continuous functions, gp-continuous functions, gpr-continuous functions, g^*b-continuous functions have been introduced by several mathematicians. In this sequel, the pasting lemmas for s^*g-continuous functions, rg^*-continuous functions and g^*r-continuous functions have been introduced in this paper.

Keywords : s^*g-continuous functions; rg^*-continuous functions; g^*r-continuous functions.
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1 Introduction and Preliminaries

Levine introduced generalized closed sets [1] in topological spaces in 1970. According to him, a set A is generalized closed {see also [2]} if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open. After the work of Levine on generalized closed sets as the natural extension of closed sets, several types of generalized closed sets such as s^*g-closed sets [3], rg^*-closed sets [4], g^*r-closed sets [4], regular generalized closed sets [5, 6], generalized preclosed sets [7], generalized pre regular closed [8], g^*b-closed [9] etc have been introduced in topological spaces. Consequently,

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the new types of continuous mappings and irresolute maps, namely g-continuous [10], s^*g-continuous [11], rg-continuous [12], rg^*-continuous [12], g^*r-continuous [12], gp-continuous [13], gpr-continuous [14], g^*b-continuous [15], gc-irresolute [10], s^*g-irresolute [11], rg-irresolute [12], rg^*-irresolute [12], g^*r-irresolute [12], gp-irresolute [13], gpr-irresolute [14], g^*b-irresolute [15]and so on are also studied. Moreover, the pasting lemmas for continuous functions are established over last two decades. Anitha et al. [16] established the pasting lemma for rg-continuous, gc- irresolute, and gp-continuous functions. In this sequel, the pasting lemmas for s^*g-continuous functions, rg^*-continuous functions and g^*r-continuous functions have been introduced in this paper.

Throughout the paper, (X, τ) or X denote the topological space on which no separation axiom is assumed unless explicitly stated. Let A be a subset of X. Then the closure and interior of A are the intersection of all closed sets containing A and union of all open sets contained in A respectively and they are denoted by $cl(A)$ and $int(A)$. A set A is called semi open [17], {resp. regular open [18], preopen [19], b-open [20]} if $A \subseteq cl[int(A)]$ { resp. $A = int[cl(A)]$, $A \subseteq int[cl(A)]$ and $A \subseteq cl[int(A)] \cup int[cl(A)]$}. The complements of semi open, regular open, pre open, b-open sets are called semi closed, regular closed, pre closed sets, b-closed respectively. The regular closure, regular interior, preclosure, preinterior, b-closure and b-interior of a set A are defined in similar way of closure and interior of a set A. Moreover, A is called s^*g-closed [3] {resp. rg-closed [5], rg^*-closed [4], g^*r-closed [4], gp-closed [7], gpr-closed and g^*b-closed [9]} if $cl(A) \subseteq U$ {resp. $cl(A) \subseteq U$, $rcl(A) \subseteq U$, $rcl(A) \subseteq U$, $pcl(A) \subseteq U$, $pcl(A) \subseteq U$ and $bcl(A) \subseteq U$} whenever $A \subseteq U$ and U is semi open {resp. regular open, regular open, open, open, regular open and g-open}. The complements of semi open, {resp. regular open, preopen, s^*g-closed, rg-closed, rg^*-closed, g^*r-closed, gp-closed, gpr-closed, g^*b-closed} are called semi closed, {resp. regular closed, preclosed, s^*g-open, rg-open, rg^*-open, g^*r-open, gp-open, gpr-open and g^*b-open}.

A function $f : (X, \tau) \to (Y, \sigma)$ is called continuous, {resp. g-continuous, rg-continuous, gp-continuous, gpr-continuous, s^*g-continuous, rg^*-continuous, g^*r-continuous, g^*b-continuous} if $f^{-1}(V)$ is closed {resp. g-closed, rg-closed, gp-closed, gpr-closed, s^*g-closed, rg^*-closed, g^*r-closed, g^*b-closed} for every closed set V in Y. A function $f : (X, \tau) \to (Y, \sigma)$ is called irresolute, {resp. gc-irresolute, rg-irresolute, gp-irresolute, gpr-irresolute, s^*g-irresolute, rg^*-irresolute, g^*r-irresolute, g^*b-irresolute} if $f^{-1}(V)$ is semi closed {resp. g-closed, rg-closed, gp-closed, gpr-closed, s^*g-closed, rg^*-closed, g^*r-closed, g^*b-closed} for every semi closed {resp. g-closed, rg-closed, gp-closed, gpr-closed, s^*g-closed, rg^*-closed, g^*r-closed, g^*b-closed} set V in Y. A collection $\{A_\alpha : \alpha \in I\}$ of subsets of a space X is locally finite if every point of X has a neighborhood that intersects only finitely many members of $\{A_\alpha : \alpha \in I\}$.

2 Main Results

Theorem 2.1. *Let X, Y be topological spaces, $V \subseteq Y$ and $f : X \to Y$. Then f is s^*g-continuous if and only if V is closed in Y implies that $f^{-1}(V)$ is s^*g-closed in X.*

Proof. **Necessity:** Let $f : X \to Y$ be s^*g-continuous. Let $V \subseteq Y$ be closed. Then $V^C \subseteq Y$ is open in Y. Since $f : X \to Y$ is s^*g-continuous, $f^{-1}(V)$ is s^*g-open in X. Hence $f^{-1}(V^C) = [f^{-1}(V)]^C$ is s^*g-closed in X.

Sufficiency: Suppose that V is closed in Y implies that $f^{-1}(V)$ is s^*g-closed in X. Let A be open in Y. Then A^C is closed in Y. By our assumption, $f^{-1}(A^C) = [f^{-1}(A)]^C$ is s^*g-closed in X. Consequently, $f^{-1}(A)$ is s^*g-open in X. It completes the proof. □

Theorem 2.2. *Pasting lemma for s^*g-continuous functions.*
Let $X = A \cup B$ where A and B are both open and s^*g-closed in X. Let $f : A \to Y$ and $g : B \to Y$ be s^*g-continuous (s^*g-irresolute). If $f(x) = g(x)$ for every $x \in A \cap B$. The f and g combine to give a s^*g-continuous (s^*g-irresolute) function $h : X \to Y$, defined by setting $h(x) = f(x)$ if $x \in A$ and $h(x) = g(x)$ if $x \in B$.

Proof. Let U be closed (s^*g-closed) in Y. Then, $h^{-1}(U) = f^{-1}(U) \cup g^{-1}(U)$. Since $f : A \to Y$ is s^*g-continuous (s^*g-irresolute), $f^{-1}(U)$ is s^*g-closed in A. Since $g : B \to Y$ is s^*g-continuous (s^*g-irresolute), $g^{-1}(U)$ is s^*g-closed in B. Since A and B are both open and s^*g-closed in X, $f^{-1}(U)$ and $g^{-1}(U)$ are s^*g-closed in X. Since the union of two s^*g-closed sets is s^*g-closed, we have $h^{-1}(U) = f^{-1}(U) \cup g^{-1}(U)$ is s^*g-closed in X. It completes the proof □

Recall that arbitrary union of $cl(A_i), i \in I$ is contained in closure of arbitrary union of subsets A_i in any topological space. The equality holds if the collection $\{A_i, i \in I\}$ is locally finite.

Theorem 2.3 ([21]). *The arbitrary union of s^*g-closed sets $A_i, i \in I$ in a topological space (X, τ) is s^*g-closed if the family $\{A_i, i \in I\}$ is locally finite.*

Theorem 2.4. *Pasting lemma for s^*g-continuous functions.*
Let $X = \bigcup A_\alpha$ where $\{A_\alpha, \alpha \in I\}$ is locally finite and A_α is both open and s^*g-closed in X for each $\alpha \in I$. Let $f_\alpha : A_\alpha \to Y$ be s^*g-continuous (s^*g-irresolute) for each $\alpha \in I$ such that $f_\alpha(x) = f_\beta(x)$ for every $x \in A_\alpha \cap A_\beta$. The f_α, $\alpha \in I$ combine to give a s^*g-continuous (s^*g-irresolute) function $h : X \to Y$, defined by setting $h(x) = f_\alpha(x)$ for $x \in A_\alpha$.

Proof. Let U be closed (s^*g-closed)in Y. Then, $h^{-1}(U) = \bigcup f_\alpha^{-1}(U)$. Since $f_\alpha : A_\alpha \to Y$ is s^*g-continuous (s^*g-irresolute), $f_\alpha^{-1}(U)$ is s^*g-closed in A_α. Since A_α is both open and s^*g-closed in X for each $\alpha \in I$, $f_\alpha^{-1}(U)$ is s^*g-closed in X. Since $\{A_\alpha, \alpha \in I\}$ is locally finite, we have $h^{-1}(U) = \bigcup f_\alpha^{-1}(U)$ is s^*g-closed in X. It completes the proof □

Theorem 2.5. *Pasting lemma for rg^*-continuous functions.*
Let $X = A \cup B$ where A and B are both open and rg^*-closed in X. Let $f : A \to Y$ and $g : B \to Y$ be rg^*-continuous (rg^*-irresolute). If $f(x) = g(x)$ for every $x \in A \cap B$. The f and g combine to give a rg^*-continuous (rg^*-irresolute) function $h : X \to Y$, defined by setting $h(x) = f(x)$ if $x \in A$ and $h(x) = g(x)$ if $x \in B$.

Proof. Let U be closed (rg^*-closed)in Y. Then, $h^{-1}(U) = f^{-1}(U) \cup g^{-1}(U)$. Since $f : A \to Y$ is rg^*-continuous (rg^*-irresolute), $f^{-1}(U)$ is rg^*-closed in A. Since $g : B \to Y$ is rg^*-continuous (rg^*-irresolute), $g^{-1}(U)$ is rg^*-closed in B. Since A and B are both open and rg^*-closed in X, $f^{-1}(U)$ and $g^{-1}(U)$ are rg^*-closed in X. Since the union of two rg^*-closed sets is rg^*-closed, we have $h^{-1}(U) = f^{-1}(U) \cup g^{-1}(U)$ is rg^*-closed in X. It completes the proof □

Theorem 2.6. *Pasting lemma for g^*r-continuous functions.*
Let $X = A \cup B$ where A and B are both regular open and g^*r-closed in X. Let $f : A \to Y$ and $g : B \to Y$ be g^*r-continuous (g^*r-irresolute). If $f(x) = g(x)$ for every $x \in A \cap B$. The f and g combine to give a g^*r-continuous (g^*r-irresolute) function $h : X \to Y$, defined by setting $h(x) = f(x)$ if $x \in A$ and $h(x) = g(x)$ if $x \in B$.

Proof. Let U be closed (g^*r-closed) in Y. Then, $h^{-1}(U) = f^{-1}(U) \cup g^{-1}(U)$. Since $f : A \to Y$ is g^*r-continuous (g^*r-irresolute), $f^{-1}(U)$ is g^*r-closed in A. Since $g : B \to Y$ is g^*r-continuous (g^*r-irresolute), $g^{-1}(U)$ is g^*r-closed in B. Since A and B are both regular open and g^*r-closed in X, $f^{-1}(U)$ and $g^{-1}(U)$ are g^*r-closed in X. Since the union of two g^*r-closed sets is g^*r-closed, we have $h^{-1}(U) = f^{-1}(U) \cup g^{-1}(U)$ is g^*r-closed in X. It completes the proof □

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References

[1] N. Levine, Generalized closed sets topology, Rendiconti del Cirwlo Matematico di Palermo, Serie II, 19 (1970) 89–96.

[2] K. Kannan, On Levine's generalized closed sets: a survey, Research Journal of Applied Sciences, Engineering and Technology 4 (11) (2012) 1612–1615.

[3] K. Chandrasekhara Rao, K. Joseph, Semi star generalized closed sets, Bulletin of Pure and Applied Sciences 19E (2) (2000) 281–290.

[4] K. Chandrasekhara Rao, N. Palaniappan, Regular generalized star closed sets, Bulletin of Pure and Applied Sciences 19E (2) (2000) 291–306.

[5] K. Chandrasekhara Rao, N. Palaniappan, Regular generalized closed sets, Kyungpook Math. J. 33 (2) (1993) 211–219.

[6] N. Palaniappan, Regular generalized closed sets, Acta Ciencia Indica 26 M (2000) 67–71.

[7] I.A. Rani, Studies on Generalizations of Generalized Closed Sets and Maps in Topological Spaces, Ph.D. Thesis, Bharathiar University, Coimbatore, 1997.

[8] Y. Gnanambal, On generalized pre regular closed sets in topological spaces, Indian J. Pure Appl. Math. 28 (3) (1997) 351–360.

[9] D. Vidhya, R. Parimelazhagan, g^*b-closed sets in topological spaces, Int. J. Contemp. Math. Sciences 7 (2012) 1305–1312.

[10] K. Balachandran, P. Sundaram, H. Maki, On generalized continuous maps in topological spaces, Mem. Fac. Sci. Kochi Univ. (Math.) 12 (1991) 5–13.

[11] K. Chandrasekhara Rao, K. Joseph, Semi star generalized homeomorphisms in topological spaces, Applied Sci. Periodical J. Engineering Applied Sciences 3 (2007) 176–187.

[12] N. Palaniappan, Studies on Regular Generalized Closed Sets and Maps in Topological Spaces, Ph.D. Thesis, Alagappa University, Karikudi, 1995.

[13] I.A. Rani, K. Balachandran, J. Dontchev, Some characterizations of gp-irresolute and gp-continuous maps between topological spaces, Mem. Fac. Sci. Kochi Univ. Ser.A. Math. 20 (1999) 93–104.

[14] Y. Gnanambal, K. Balachandran, On gpr-continuous functions in topological spaces, Indian J. Pure Appl. Math. 30 (1999) 581–593.

[15] D. Vidhya, R. Parimelazhagan, g^*b-continuous maps and pasting lemma in topological spaces, Int. J. Math. Analysis 6 (47) (2012) 2307–2315.

[16] M. Anitha, R. Selvi, P. Thangavelu, Pasting lemmas for g-continuous functions, Missouri Journal of Mathematical Sciences 21 (1) (2009) 28–33.

[17] N. Levine, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly 70 (1963) 36–41.

[18] M.H. Stone, Applications of the theory of boolean rings to the general topology, Trans. A. M. S. 41 (1937) 375–481.

[19] A.S. Mashhour, M.E.A.E. Monsef, S.N.E. Deeb, On pre- continuous and weak pre-continuous mappings, Proc. Math. Phys. Soc. Egyp. 53 (1982) 47–53.

[20] D. Andrijevic, On b-open sets, Mat. Vesink 48 (1996) 59–64.

[21] K. Kannan, D. Narasimhan, K. Chandrasekhara Rao, On semi star generalized closed sets in bitopological spaces, Bol. Soc. Paranaense de Mat. (3s.) 28 (1) (2010) 29–40.

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