Thai Journal of Mathematics
Volume 12 (2014) Number 1 : 71–80
http://thaijmath.in.cmu.ac.th
ISSN 1686-0209

(ψ, α, β)-Weak Contraction in Partially Ordered G-Metric Spaces

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Abstract : In this paper we have generalized the weak contraction principle to coincidence point and common fixed point results in partially ordered G-metric spaces. We illustrate our results with the help of an example.

Keywords : partially ordered set; G-metric space; coincidence point; fixed point; g-non-decreasing mapping; weak contraction; control functions.
2010 Mathematics Subject Classification : 54H25; 47H10.

1 Introduction

Alber and Guerre-Delabriere in [1] suggested a generalization of the Banach contraction mapping principle by introducing the concept of weak contraction in Hilbert spaces. Rhoades [2] had shown that the result which Alber et al. had proved in Hilbert spaces [1] is also valid in complete metric spaces. Weakly contractive mappings and mappings satisfying other weak contractive inequalities have been discussed in several works, some of which are noted in [3, 4]. Khan et al. [5] introduced the use of a control function in metric fixed point problems. This function was referred to as 'altering distance function' by the authors of [5]. This function and its extensions have been used in several problems of fixed point theory, some of which are noted in [6, 7]. In recent times, fixed point

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theory has developed rapidly in partially ordered metric spaces, that is, in metric spaces endowed with a partial ordering [8, 9]. Using the control functions the weak contraction principle has been generalized in metric spaces [10] and in partially ordered metric spaces in [11]. In [12], the weak contraction principle has been generalized by using three functions. Compatibility of two mappings introduced by Jungck [13] is an important concept in the context of common fixed point problems in metric spaces. This concept has been weakened to compatibilities of type A, type B, type C and finally to weak compatibility [14, 15]. Note that a lot of authors have proved various fixed-point and coupled point results for one or two self-mappings in the setting of metric, cone metric, ordered metric or G-metric spaces, see for instance [16–23].

Now we give preliminaries and basic definitions which are used throughout the paper.

2 Preliminaries

Definition 2.1 ([24])**.** Let X be a non-empty set, $G : X \times X \times X \longrightarrow \mathbb{R}^+$ be a function satisfying the following properties:

(G1) $G(x, y, z) = 0$ if $x = y = z$.

(G2) $0 < G(x, x, y)$ for all $x, y \in X$ with $x \neq y$.

(G3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $y \neq z$.

(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \cdots$. (symmetry in all three variables).

(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$ (rectangle inequality).

Then the function G is called a *generalized metric*, or, more specially, a *G-metric* on X, and the pair (X, G) is called a *G-metric space.*

Definition 2.2 ([24])**.** Let (X, G) be a G-metric space, and let (x_n) be a sequence of points of X. We say that (x_n) is *G-convergent* to $x \in X$ if $\lim_{n,m\longrightarrow+\infty} G(x, x_n, x_m) = 0$, that is, for any $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x, x_n, x_m) < \varepsilon$, for all $n, m \geq N$. We call x the limit of the sequence and write $x_n \longrightarrow x$ or $\lim_{n\longrightarrow+\infty} x_n = x$.

Proposition 2.3 ([24])**.** *Let (X, G) be a G-metric space. The following are equivalent:*

(1) (x_n) *is G-convergent to* x.

(2) $G(x_n, x_n, x) \longrightarrow 0$ *as* $n \longrightarrow +\infty$.

(3) $G(x_n, x, x) \longrightarrow 0$ *as* $n \longrightarrow +\infty$.

(4) $G(x_n, x_m, x) \longrightarrow 0$ *as* $n, m \longrightarrow +\infty$.

Definition 2.4 ([24]). Let (X, G) be a G-metric space. A sequence (x_n) is called a *G-Cauchy sequence* if, for any $\varepsilon > 0$, there is $N \in \mathbb{N}$ such that $G(x_n, x_m, x_l) < \varepsilon$ for all $m, n, l \geq N$, that is, $G(x_n, x_m, x_l) \longrightarrow 0$ as $n, m, l \longrightarrow +\infty$.

Proposition 2.5 ([25]). *Let (X, G) be a G-metric space. Then the following are equivalent*

(1) *the sequence (x_n) is G-Cauchy.*

(2) *for any $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \varepsilon$, for all $m, n \geq N$.*

Proposition 2.6 ([24]). *Let (X, G) be a G-metric space. A mapping $f : X \longrightarrow X$ is G-continuous at $x \in X$ if and only if it is G-sequentially continuous at x, that is, whenever (x_n) is G-convergent to x, $(f(x_n))$ is G-convergent to $f(x)$.*

Proposition 2.7 ([24]). *Let (X, G) be a G-metric space. Then, the function $G(x, y, z)$ is jointly continuous in all three of its variables.*

Definition 2.8 ([24]). A G-metric space (X, G) is called *G-complete* if every G-Cauchy sequence is G-convergent in (X, G).

Definition 2.9 ([14]). (Weakly Compatible Mappings) Two mappings $f, g : X \longrightarrow X$ are *weakly compatible* if they commute at their coincidence points, that is, if $ft = gt$ for some $t \in X$ implies that $fgt = gft$.

Lemma 2.10 ([15]). *If f and g are either compatible, or compatible of type A (resp. type B or type C), then f and g are weakly compatible.*

Definition 2.11 ([15]). (g-Non Decreasing Mapping) Suppose $(X, \preceq)$ is a partially ordered set and $f, g : X \longrightarrow X$ are mappings of X to itself. f is said to be *g-non-decreasing* if for $x, y \in X$, $gx \preceq gy$ implies $fx \preceq fy$.

3 Main Results

Theorem 3.1. *Let $(X, \preceq)$ be a partially ordered set and suppose that (X, G) is a G-complete metric space. Let $f, g : X \longrightarrow X$ be such that $f(X) \subseteq g(X)$, f is g-non-decreasing, $g(X)$ is closed and*

$$\psi(G(fx, fy, fz)) \leq \alpha(G(gx, gy, gz)) - \beta(G(gx, gy, gz)) \tag{3.1}$$

for all $x, y, z \in X$ such that $gx \preceq gy \preceq gz$, where $\psi, \alpha, \beta : [0, +\infty) \longrightarrow [0, +\infty)$ are such that, ψ is continuous and monotone non-decreasing, α is continuous, β is lower semi-continuous,

$$\psi(t) = 0 \text{ if and only if } t = 0,\ \alpha(0) = \beta(0) = 0 \tag{3.2}$$

and

$$\psi(t) - \alpha(t) + \beta(t) > 0 \text{ for all } t > 0. \tag{3.3}$$

Also, if any nondecreasing sequence (x_n) in X converges to z, then we assume

$$x_n \preceq z \text{ for all } \; n \geq 0. \tag{3.4}$$

If there exists $x_0 \in X$ such that $gx_0 \preceq fx_0$, then f and g have a coincidence point.

Proof. By the condition of the theorem there exists $x_0 \in X$ such that $gx_0 \preceq fx_0$. Since $f(X) \subseteq g(X)$, we can define $x_1 \in X$ such that $gx_1 = fx_0$, then $gx_0 \preceq fx_0 = gx_1$. Since f is g-non decreasing, we have $fx_0 \preceq fx_1$. In this way we construct the sequence (x_n) recursively as

$$fx_n = gx_{n+1} \text{ for all } \; n \geq 1 \tag{3.5}$$

for which

$$gx_0 \preceq fx_0 = gx_1 \preceq fx_1 = gx_2 \preceq fx_2 \preceq \cdots \preceq fx_{n-1} = gx_n \preceq fx_n = gx_{n+1} \preceq \cdots \tag{3.6}$$

If any two consecutive terms in the sequence (x_n) are equal, then the conclusion of the theorem follows. So we assume that

$$G(fx_{n-1}, fx_{n-1}, fx_n) \neq 0 \quad \text{for all} \quad n \geq 1. \tag{3.7}$$

Let, if possible, for some n

$$G(fx_{n-1}, fx_{n-1}, fx_n) < G(fx_n, fx_n, fx_{n+1}).$$

Substituting $x = y = x_n$ and $z = x_{n+1}$ in (3.1), using (3.5), (3.6) and the monotone property of ψ, we have

$$\begin{aligned} \psi\big(G(fx_{n-1}, fx_{n-1}, fx_n)\big) & \\ & < \psi\big(G(fx_n, fx_n, fx_{n+1})\big) \\ & \leq \alpha(G(gx_n, gx_n, gxn+1)) - \beta(G(gx_n, gx_n, gx_{n+1})) \\ & = \alpha(G(fx_{n-1}, fx_{n-1}, fx_n)) - \beta(G(fx_{n-1}, fx_{n-1}, fx_n)). \end{aligned} \tag{3.8}$$

By (3.3), we have that $G(fx_{n-1}, fx_{n-1}, fx_n) = 0$, which contradicts (3.7). Therefore, for all $n \geq 1$

$$G(fx_n, fx_n, fx_{n+1}) \leq G(fx_{n-1}, fx_{n-1}, fx_n).$$

It follows that the sequence $(G(fx_n, fx_n, fx_{n+1}))$ is a monotone decreasing sequence of non-negative real numbers and consequently there exists $r \geq 0$ such that

$$\lim_{n \longrightarrow +\infty} G(fx_n, fx_n, fx_{n+1}) = r. \tag{3.9}$$

Taking $n \longrightarrow +\infty$ in (3.8) and using the lower semi continuity of β and the continuities of ψ and α, we obtain $\psi(r) \leq \alpha(r) - \beta(r)$, which, by (3.3), implies that $r = 0$. Hence

$$\lim_{n \longrightarrow +\infty} G(fx_n, fx_n, fx_{n+1}) = 0. \tag{3.10}$$

Next we show that (fx_n) is a Cauchy sequence. If not, then there exists some $\varepsilon > 0$ for which we can find two sequences $(fx_{m(k)})$ and $(fx_{n(k)})$ of (fx_n), $n(k) > m(k) > k$, for all $k \geq 0$,

$$G(fx_{m(k)}, fx_{m(k)}, fx_{n(k)}) \geq \varepsilon \tag{3.11}$$

and

$$G(fx_{m(k)}, fx_{m(k)}, fx_{n(k)-1}) < \varepsilon. \tag{3.12}$$

By (3.12) and the rectangle inequality, we have for all $k \geq 0$,

$$\begin{aligned}\varepsilon \leq G(fx_{m(k)}, fx_{m(k)}, fx_{n(k)}) &\leq G(fx_{m(k)}, fx_{m(k)}, fx_{n(k)-1}) \\ &+G(fx_{n(k)-1}, fx_{n(k)-1}, fx_{n(k)}) < \varepsilon + G(fx_{n(k)-1}, fx_{n(k)-1}, fx_{n(k)}).\end{aligned}$$

Taking $k \longrightarrow +\infty$ in the above inequality and using (3.10) we obtain

$$\lim_{k \longrightarrow +\infty} G(fx_{m(k)}, fx_{m(k)}, fx_{n(k)}) = \varepsilon. \tag{3.13}$$

Also, by rectangle inequality and using that $G(x, x, y) \leq 2G(x, y, y)$ for any $x, y \in X$, for all $k \geq 0$, we have

$$\begin{aligned}&G(fx_{m(k)-1}, fx_{m(k)-1}, fx_{n(k)-1}) \\ &\quad\leq G(fx_{m(k)-1}, fx_{m(k)-1}, fx_{m(k)}) + G(fx_{m(k)}, fx_{m(k)}, fx_{n(k)}) \\ &\qquad + G(fx_{n(k)}, fx_{n(k)}, fx_{n(k)-1}) \\ &\quad\leq G(fx_{m(k)-1}, fx_{m(k)-1}, fx_{m(k)}) + G(fx_{m(k)}, fx_{m(k)}, fx_{n(k)}) \\ &\qquad + 2G(fx_{n(k)-1}, fx_{n(k)-1}, fx_{n(k)})\end{aligned}$$

and

$$\begin{aligned}&G(fx_{m(k)}, fx_{m(k)}, fx_{n(k)}) \\ &\quad\leq G(fx_{m(k)}, fx_{m(k)}, fx_{m(k)-1}) + G(fx_{m(k)-1}, fx_{m(k)-1}, fx_{n(k)-1} \\ &\qquad + G(fx_{n(k)-1}, fx_{n(k)-1}, fx_{n(k)}) \\ &\quad\leq 2G(fx_{m(k)-1}, fx_{m(k)-1}, fx_{m(k)}) + G(fx_{m(k)-1}, fx_{m(k)-1}, fx_{n(k)-1}) \\ &\qquad + G(fx_{n(k)-1}, fx_{n(k)-1}, fx_{n(k)}).\end{aligned}$$

Taking limit as $k \longrightarrow +\infty$ in the above two inequalities and using (3.10) and (3.13) we have

$$\lim_{k \longrightarrow +\infty} G(fx_{m(k)-1}, fx_{m(k)-1} fx_{n(k)-1}) = \varepsilon. \tag{3.14}$$

Again, by (3.6), we have that the elements $gx_{m(k)}$ and $gx_{n(k)}$ are comparable. Putting $x = y = x_{n(k)}$ and $z = x_{m(k)}$ in (3.1), for all $k \geq 0$, by (3.5), we have

$$\begin{aligned}&\psi(G(fx_{m(k)}, fx_{m(k)}, fx_{n(k)})) \\ &\quad\leq \alpha(G(gx_{m(k)}, gx_{m(k)}, gx_{n(k)})) - \beta(G(gx_{m(k)}, gx_{m(k)}, gx_{n(k)})) \\ &\quad= \alpha(G(fx_{m(k)-1}, fx_{m(k)-1}, fx_{n(k)-1})) - \beta(G(fx_{m(k)-1}, fx_{m(k)-1}, fx_{n(k)-1})).\end{aligned}$$

Taking $k \longrightarrow +\infty$ in the above inequality, using (3.14), the continuities of ψ and α and the lower semi continuity of β, we obtain $\psi(\varepsilon) \leq \alpha(\varepsilon) - \beta(\varepsilon)$. Then, by (3.3), we have $\varepsilon = 0$, which is a contradiction. It then follows that (fx_n) is a Cauchy sequence and hence (fx_n) is convergent in the complete G-metric space (X, G). Since $g(X)$ is closed and by (3.5), $fx_n = gx_{n+1}$ for all $n \geq 0$, we have that there exists $z \in X$ for which

$$\lim_{n \longrightarrow +\infty} gx_n = \lim_{n \longrightarrow +\infty} fx_n = gz. \tag{3.15}$$

Now we prove that z is a coincidence point of f and g. From (3.6), we have (gx_n) is a non-decreasing sequence in X. By (3.15) and a condition of our theorem,

$$gx_n \preceq gz. \tag{3.16}$$

Putting $x = y = x_n$ in (3.1), by the virtue of (3.16), we get

$$\psi(G(fx_n, fx_n, fz)) \leq \alpha(G(gx_n, gx_n, gz)) - \beta(G(gx_n, gx_n, gz)).$$

Taking $n \longrightarrow +\infty$ in the above inequality, using (3.2) and (3.15), we have $G(gz, gz, fz) = 0$, that is,

$$fz = gz. \tag{3.17}$$

This completes the proof. □

Theorem 3.2. *If in Theorem 3.1 it is additionally assumed that*

$$gz \preceq ggz \tag{3.18}$$

where z is as in (3.4) and f and g are weakly compatible then f and g have a common fixed point in X.

Proof. Following the proof of the Theorem 3.1 we have (3.15), that is, a non-decreasing sequence (gx_n) converging to gz. Then by (3.18) we have $gz \preceq ggz$. Since f and g are weakly compatible, by (3.17), we have that $fgz = gfz$. We set

$$w = gz = fz. \tag{3.19}$$

Therefore, we have

$$gz \preceq ggz = gw. \tag{3.20}$$

Also

$$fw = fgz = gfz = gw. \tag{3.21}$$

If $z = w$, then z is a common fixed point. If $z \neq w$, then, by (3.1), we have

$$\psi(G(gz, gz, gw)) = \psi(G(fz, fz, fw)) \leq \alpha(G(gz, gz, gw)) - \beta(G(gz, gz, gw)).$$

From (3.3), $gz = gw$. Then, by (3.19) and (3.21), we have $w = gw = fw$. This completes the proof of Theorem 3.2. □

Remark 3.1. *Continuity of f is not required in Theorem 3.1. If we assume f to be continuous then (3.4) is no longer required for the theorem and can be omitted.*

Remark 3.2. *In view of Lemma 2.10, the result of Theorem 3.2 is valid if we assume f and g to be compatible, compatible of type A, type B or type C.*

4 Example

Let $X = [0, 1]$. We define a partial order $\preceq$ on X as $x \preceq y$ if and only if $x \geq y$ for all $x, y \in X$. Define $G : X \times X \times X \longrightarrow \mathbb{R}^+$ by

$$G(x, y, z) = |x - y| + |x - z| + |y - z|$$

for all $x, y, z \in X$. Then (X, G) is a complete G-metric space. Let $f, g : X \longrightarrow X$ be defined as, $fx = x - \frac{5}{6}x^2$ and $gx = x - \frac{1}{3}x^2$ for all $x \in [0, 1]$. Let ψ, α, β : $[0, +\infty) \longrightarrow [0, +\infty)$ be defined as $\psi(t) = t$, for $t \in [0, 1]$, $\alpha(t) = t$ for $t \in [0, 1]$ and $\beta(t) = \frac{t^2}{6}$ for $t \in [0, 1]$. Without loss of generality we assume that $x > y > z$ and verify the inequality (3.1). For all $x, y, z \in [0, 1]$ with $x > y > z$,

$$G(fx, fy, fz) = (x - y) - \frac{5}{6}(x^2 - y^2) + (x - z) - \frac{5}{6}(x^2 - z^2) + (y - z) - \frac{5}{6}(y^2 - z^2)$$

and

$$G(gx, gy, gz) = (x - y) - \frac{1}{3}(x^2 - y^2) + (x - z) - \frac{1}{3}(x^2 - z^2) + (y - z) - \frac{1}{3}(y^2 - z^2).$$

Now,

$$\begin{aligned}
&\alpha(G(gx, gy, gz)) - \beta(G(gx, gy, gz)) \\
&\quad = (x - y) - \frac{1}{3}(x^2 - y^2) + (x - z) - \frac{1}{3}(x^2 - z^2) + (y - z) - \frac{1}{3}(y^2 - z^2) \\
&\qquad - \frac{\Big[(x - y) - \frac{1}{3}(x^2 - y^2) + (x - z) - \frac{1}{3}(x^2 - z^2) + (y - z) - \frac{1}{3}(y^2 - z^2)\Big]^2}{6}.
\end{aligned}$$

Since $(x - y) - \frac{1}{3}(x^2 - y^2) \leq (x - y)$ and $x > y > z$, we have

$$\begin{aligned}
&\Big[(x - y) - \frac{1}{3}(x^2 - y^2) + (x - z) - \frac{1}{3}(x^2 - z^2) + (y - z) - \frac{1}{3}(y^2 - z^2)\Big]^2 \\
&\quad \leq \big((x - y) + (x - z) + (y - z)\big)^2 = (x - y)^2 + (x - z)^2 + (y - z)^2 \\
&\qquad + 2\Big((x - z)(y - z) + (x - y)(x - z) + (x - y)(y - z)\Big) \\
&\quad \leq (x^2 - y^2) + (x^2 - z^2) + (y^2 - z^2) + 2\big((x - z)^2 + (x - z)^2 + (x - z)^2\big) \\
&\quad \leq (x^2 - y^2) + 7(x^2 - z^2) + (y^2 - z^2).
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\alpha(G(gx,gy,gz)) - \beta(G(gx,gy,gz)) \\
&\quad \geq (x-y) - \frac{1}{3}(x^2-y^2) + (x-z) - \frac{1}{3}(x^2-z^2) + (y-z) - \frac{1}{3}(y^2-z^2) \\
&\qquad - \frac{(x^2-y^2)+7(x^2-z^2)+(y^2-z^2)}{6} \\
&\quad = (x-y) - \frac{1}{2}(x^2-y^2) + (x-z) - \frac{5}{6}(x^2-z^2) + (x-y) - \frac{1}{2}(y^2-z^2) \\
&\quad \geq (x-y) - \frac{5}{6}(x^2-y^2) + (x-z) - \frac{5}{6}(x^2-z^2) + (x-y) - \frac{5}{6}(y^2-z^2) \\
&\quad = \psi\big(G(fx,fy,fz)\big).
\end{aligned}$$

Therefore, the inequality (3.1) is satisfied. Then, with any choice of x_0 in $(0,1)$, all the conditions of Theorem 3.1 are satisfied. Also f and g are weakly compatible. Further g also satisfies (3.18). Hence Theorem 3.2 is also applicable to this example. Here $z=0$ is a coincidence point as well as common fixed point of f and g.

Acknowledgement : The authors would like to express their sincere thanks to the editor and reviewers for their valuable suggestions.

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(Received 10 April 2012)
(Accepted 30 September 2012)