Thai Journal of Mathematics
Volume 12 (2014) Number 1 : 25–32
http://thaijmath.in.cmu.ac.th
ISSN 1686-0209

A Note on Strongly Sum Difference Quotient Graphs

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Abstract : Recently, Adiga and Shivakumar Swamy [1] have introduced the concept of strongly sum difference quotient (SSDQ) graphs and shown that all graphs such as cycles, flowers and wheels are SSDQ graphs. They have also derived an explicit formula for $\alpha(n)$, the maximum number of edges in a SSDQ graphs of order n in terms of Eulers phi function. In this paper, we show that much studied families of graphs such as Mycielskian of the path P_n and the cycle C_n, $C_n \times P_n$, double triangular snake graphs and total graph of C_n are strongly sum difference quotient graphs.

Keywords : graph labeling; strongly sum difference quotient graphs; Mycielskian of the graph.
2010 Mathematics Subject Classification : 05C78; 05C22.

1 Introduction

A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. During the past few years, a lot of research work has been done on graph labeling [1–5] and several labeling techniques have been studied. Most of these interesting problems have been motivated by practical problems. In an interesting paper [4], Beineke and Hegde introduced the concept

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of strongly multiplicative graph and shown that all graphs like trees, wheels and grids are strongly multiplicative. Motivated by this, Adiga and Shivakumar Swamy [1] have introduced the concept of strongly sum difference quotient (SSDQ) graph and studied it in detail.

Through out this paper by a graph we mean a finite, undirected, connected graph without loops or multiple edges. By a labeling f of a graph $G(V, E)$ of order n we mean an injective mapping

$$f : V(G) \longrightarrow \{1, 2, \ldots, n\}.$$

Adiga and Shivakumar Swamy [1] have defined the sum difference quotient function

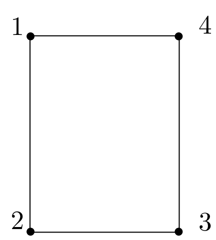
$$f_{sdq} : E(G) \longrightarrow Q$$

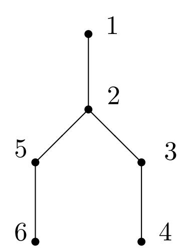
by

$$f_{sdq}(e) = \frac{|f(v) + f(w)|}{|f(v) - f(w)|}$$

if e join v and w.

A graph with n vertices is said to be strongly sum difference quotient (SSDQ) graph if its vertices can be labeled $1, 2, \ldots, n$, such that the sum difference quotient function f_{sdq} is injective, i.e., the values $f_{sdq}(e)$ on the edges are all distinct. For example, the following graphs are strongly sum difference quotient graphs:





The main purpose of this paper is to show that some families of graphs like Mycielskian of the path P_n and the cycle C_n, $C_n \times P_n$, double triangular snake graphs and total graph of C_n are strongly sum difference quotient graphs.

2 Some Classes of Strongly Sum Difference Quotient Graphs

In this section we show that Mycielskian graph of path P_n and cycle C_n, $C_n \times P_n$, double triangular snake graphs and total graph of C_n are strongly sum difference Quotient (SSDQ) graphs.

In search for triangle-free graphs with arbitrary large chromatic numbers, Mycielski [6] developed an interesting graph transformation as follows. For a graph

$G = (V, E)$, the Mycielskian of G is the graph $\mu(G)$ with vertex set $V \cup U \cup w$, where $U = \{u_1 : v_1 \in V\}$ and is disjoint from V, and edge set $E \cup \{v_1u_2 : v_1v_2 \in E\} \cup \{u_2w : u_2 \in U\}$. The vertex u_1 is called the twin of the vertex v_1 (and v_1 the twin of u_1) and the vertex w is the root of $\mu(G)$. In recent times, there has been an increasing interest in the study of Mycielskians, especially in the study their circular chromatic numbers (see, for instance, [7–10]).

Theorem 2.1. *For all $n \geq 2$, the Mycielskian graph $\mu(P_n)$ of the path P_n is a SSDQ graph.*

Proof. We label the vertices of $\mu(P_n)$ as shown in the following Figure 2.1.

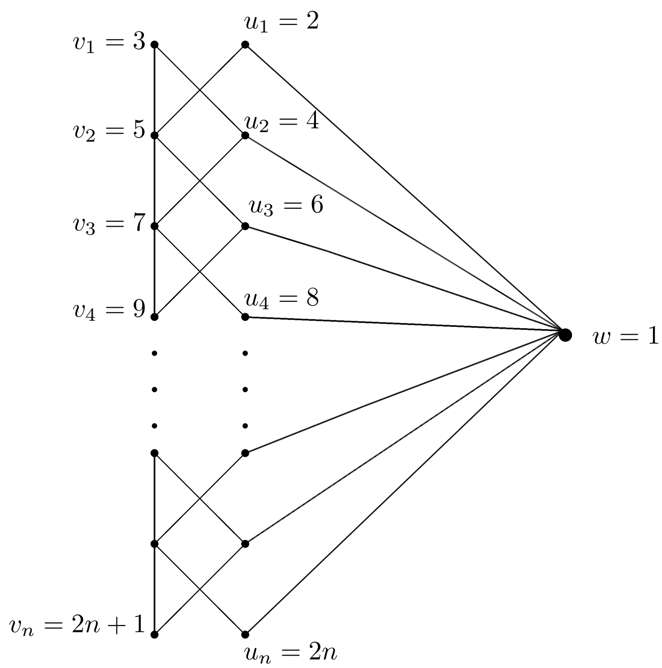


Figure 2.1

The values of the edges v_iv_{i+1} are

$$2i + 2, \qquad \text{for } i = 1, 2, \ldots, n-1, \tag{2.1}$$

all distinct. The values of the edges v_iu_{i+1} are

$$4i + 3, \qquad \text{for } i = 1, 2, \ldots, n-1, \tag{2.2}$$

which are strictly increasing. The values of the edges $v_{i+1}u_i$ are

$$\frac{4i + 3}{3}, \qquad \text{for } i = 1, 2, \ldots, n-1, \tag{2.3}$$

all distinct and the values of the edges u_iw are

$$\frac{2i+1}{2i-1}, \quad \text{for } i = 1, 2, \ldots, n-1, n, \tag{2.4}$$

which are decreasing and hence distinct. Now we show that the edge values in equation (2.1) and (2.2) are distinct. Otherwise

$$2i + 2 = 4j + 3 \quad \text{for some } i \text{ and } j.$$

$$\text{i.e., } 2i = 4j + 1,$$

which is not possible. Similarly, we can show that all the edge values of $\mu(P_n)$ are distinct. Hence, $\mu(P_n)$ is SSDQ graph. □

Theorem 2.2. *For all $n \geq 2$, the Mycielskian graph $\mu(C_n)$ of the cycle C_n is a SSDQ graph.*

Proof. We label the vertices of $\mu(C_n)$ as shown in the following Figure 2.2.

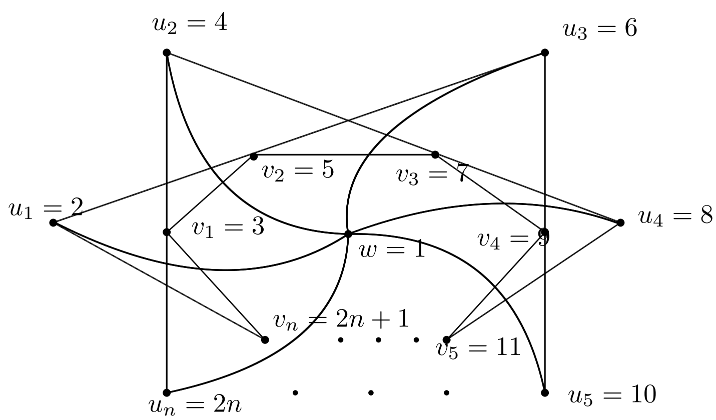


Figure 2.2

The edge value of v_nv_1 is $\frac{n+2}{n-1}$ and values of the edges v_iv_{i+1} are

$$2(i+1), \quad \text{for } i = 1, 2, \ldots, n-1, \tag{2.5}$$

which are strictly increasing and hence distinct and $\frac{n+2}{n-1} < 2(i+1)$. The values of the edges v_iu_{i+1} are

$$4i + 3, \quad \text{for } i = 1, 2, \ldots, n-1, \tag{2.6}$$

which are also strictly increasing. The edge value $\frac{2n+3}{2n-3}$ of v_1u_n and values of the edges $v_{i+1}u_i$ are

$$\frac{4i+3}{3}, \quad \text{for } i = 1, 2, \ldots, n-1, \tag{2.7}$$

are all distinct and $\frac{n+2}{n-1} < \frac{2n+3}{2n-3} < \frac{4k+3}{3}$ and the values of the edges $u_i w$

$$\frac{2i+1}{2i-1}, \qquad \text{for } i = 1, 2, \ldots, n-1, \tag{2.8}$$

are strictly decreasing and hence distinct. Note that all edge values in equations (2.5) to (2.8) are all distinct. Hence $\mu(C_n)$ is a SSDQ graph. □

Definition 2.1. If $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are two graphs then the cartesian product of G_1 and G_2 denoted by $G_1 \times G_2 = G(V, E)$ consists a vertex set $V = V_1 \times V_2$ and $(x_1, y_1), (x_2, y_2) \in V_1 \times V_2$ are adjacent if $x_1 x_2 \in E_1$ and $y_1 = y_2$ or $y_1 y_2 \in E_2$ and $x_1 = x_2$.

Theorem 2.3. *For all $n \geq 3$, $C_n \times P_2$ is a SSDQ graph.*

Proof. We label the vertices of $C_n \times P_2$ as shown in the following Figure 2.3.

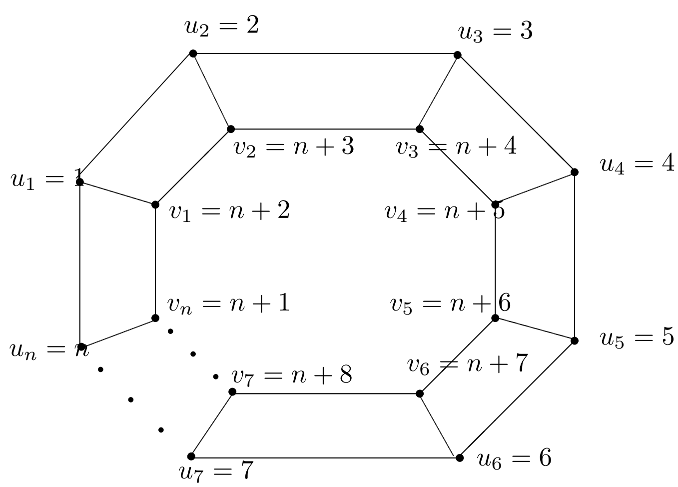


Figure 2.3

The edge value $\frac{n+1}{n-1}$ of $u_1 u_n$ and values of the edges $u_i u_{i+1}$ are

$$2i+1, \qquad \text{for } i = 1, 2, \ldots, n-1, \tag{2.9}$$

are distinct and $\frac{n+1}{n-1} < 2i+1$. The edge values $\frac{3n+1}{n-1}$ of $v_{n-1} v_n$, $2n+3$ of $v_n v_1$ and values of the edges $v_i v_{i+1}$ are

$$2n+3+2i, \qquad \text{for } i = 1, 2, \ldots, n-2, \tag{2.10}$$

all distinct and $\frac{n+1}{n-1} < \frac{3n+1}{n-1} < 2n+3 < 2n+2i+3$. The edge value $2n+1$ of $v_n u_n$ and the values of edges $v_i u_i$ are

$$\frac{n+1+2i}{n+1}, \qquad \text{for } i = 1, 2, \ldots, n-1, \tag{2.11}$$

which are increasing and hence distinct and $\frac{n+1+2i}{n+1} < 2n+1$ also $\frac{n+1}{n-1} < \frac{3n+1}{n-1} < 2n+1 < 2n+3$. Note that all edge values in equations (2.9) to (2.11) are all distinct. Hence $C_n \times P_2$ is SSDQ graph. □

Definition 2.2. A double triangular snake graph consists of two triangular snakes that have a common path. That is, a double triangular graph is obtained from a path $v_1, v_2, \ldots, v_n$ by joining v_i and v_{i+1} to a new vertex u_i for $i = 1, 2, \ldots, n-1$ and to a new vertex w_i for $i = 1, 2, \ldots, n-1$.

Theorem 2.4. *The double triangular snake graph is a SSDQ graph.*

Proof. We label the vertices v_i by $3n-1-i$ for $i = 1, 2, \ldots, n$ and rest of the vertices as shown in the following Figure 2.4.

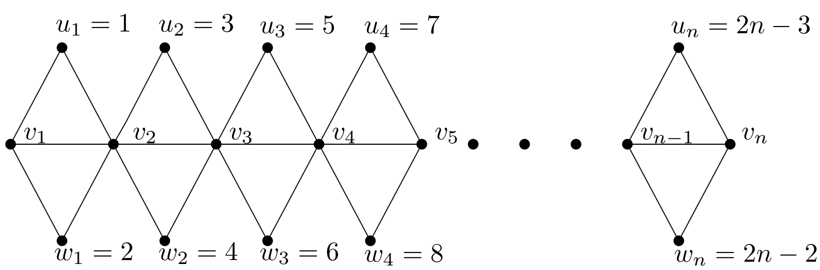


Figure 2.4

The values of the edges $v_i v_{i+1}$ are

$$6n-3-2i, \quad \text{for } i = 1, 2, \ldots, n-1, n, \tag{2.12}$$

which are strictly increasing and hence distinct. The values of the edges $v_i u_i$ are

$$\frac{3n-2+i}{3n-3i} \quad \text{for } i = 1, 2, \ldots, n-1, \tag{2.13}$$

all distinct. Values of the edges $v_{i+1} u_i$ are

$$\frac{3n+i-3}{3n-1-3i}, \quad \text{for } i = 1, 2, \ldots, n-1, \tag{2.14}$$

which are strictly increasing. The values of the edges $v_i w_i$ are

$$\frac{3n+i-1}{3n-1-3i}, \quad \text{for } i = 1, 2, \ldots, n-1, \tag{2.15}$$

all are distinct for all i and the values of the edges $v_{i+1} w_i$ are

$$\frac{3n+i-2}{3n-3i-2}, \quad \text{for } i = 1, 2, \ldots, n-1, \tag{2.16}$$

which are decreasing and hence distinct. Note that all edge values in equations (2.12) to (2.16) are all distinct. Hence, the double triangular snake graph is SSDQ graph. □

Definition 2.3. The total graph $T(G)$ of graph $G(V, E)$ has a point set $V(G) \cup E(G)$, and two points of $T(G)$ are adjacent whenever they are neighbors in G.

Theorem 2.5. *For all $n \geq 3$, the total graph $T(C_n)$ of C_n the cycle is a SSDQ Graph.*

Proof. We label the vertices v_i by $n+1+i$ for $i = 1, 2, \ldots, n-1$ and rest of vertices of $T(C_n)$ as shown in the following Figure 2.5.

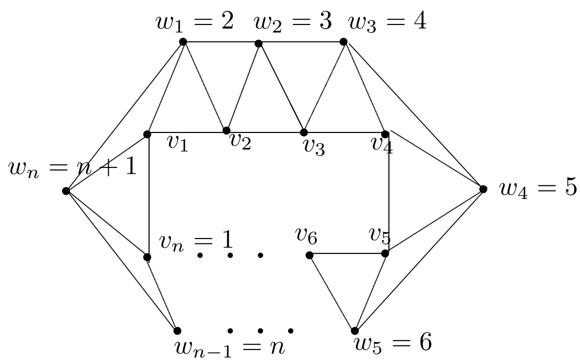


Figure 2.5

The edge value $\frac{n+3}{n-1}$ of $w_1 w_n$ and values of the edges $w_i w_{i+1}$ are

$$2i + 3, \qquad \text{for } i = 1, 2, \ldots, n-1, \tag{2.17}$$

all strictly increasing, hence distinct and $\frac{n+3}{n-1} < 2i+3$. The edge value $\frac{2n+1}{2n-1}$ of $v_{n-1}v_n$ and $\frac{n+3}{n+1}$ of $v_n v_1$ also values of the edges $v_i v_{i+1}$ are

$$2n + 3 + 2i, \qquad \text{for } i = 1, 2, \ldots, n-1, \tag{2.18}$$

all distinct and $\frac{2n+1}{2n-1} < \frac{n+3}{n+1} < 2n+3+2i$. The edge value $\frac{n+2}{n}$ of $w_n v_n$ and values of the edges $w_i v_i$ are

$$\frac{n + 2i + 2}{n}, \qquad \text{for } i = 1, 2, \ldots, n-1, \tag{2.19}$$

which all are distinct and $\frac{n+2}{n} < \frac{n+2+2i}{n}$ and the edge values $\frac{n+1}{n-1}$ of $w_{n-1}v_n$, $2n+3$ of $w_n v_1$ and the values of the edges $w_i v_{i+1}$

$$\frac{n + 3 + 2i}{n + 1} \qquad \text{for } i = 1, 2, \ldots, n-2, \tag{2.20}$$

are all distinct and $\frac{n+1}{n-1} < \frac{n+3+2i}{n+1} < 2n+3$. Note that all edge values in equations (2.17) to (2.20) are all distinct. Hence, $T(c_n)$ is a SSDQ graph. □

Acknowledgement : The authors are thankful to Prof. Chandrashekar Adiga for his encouragement and suggestions.

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(Received 4 June 2011)
(Accepted 8 September 2012)