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Dependence Structure between World Crude Oil Prices: Evidence from NYMEX, ICE, and DME Markets

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Abstract : This paper examines the dependence structure between world crude oil prices using the D-vine copula based GARCH model to analyze three random variables, namely, Light crude futures 1-Pos (NYMEX), Brent crude futures 1-Pos (ICE), and Oman crude futures 1-Pos (DME). We find that NYMEX–ICE, NYMEX–DME, and ICE–DME have relatively strong dependence. In addition, we find the evidences for asymmetric tail dependence in each pair with the values of upper tail and lower tail dependences of three pair-copulas as being quite close to each other. Therefore, our findings support the "one great pool" hypothesis. Moreover, the results from the D-vine copula model indicate that the ICE is an important variable that governs the interactions in the dependence structure between the NYMEX and the DME. In other words, the change in the oil price of the ICE will impact quite significantly the prices of the NYMEX and the DME.

Keywords : futures crude oil prices; empirical Kendall's tau; distance measure; vine copula

2010 Mathematics Subject Classification : 91G70; 62P20 (2010 MSC)

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1 Introduction

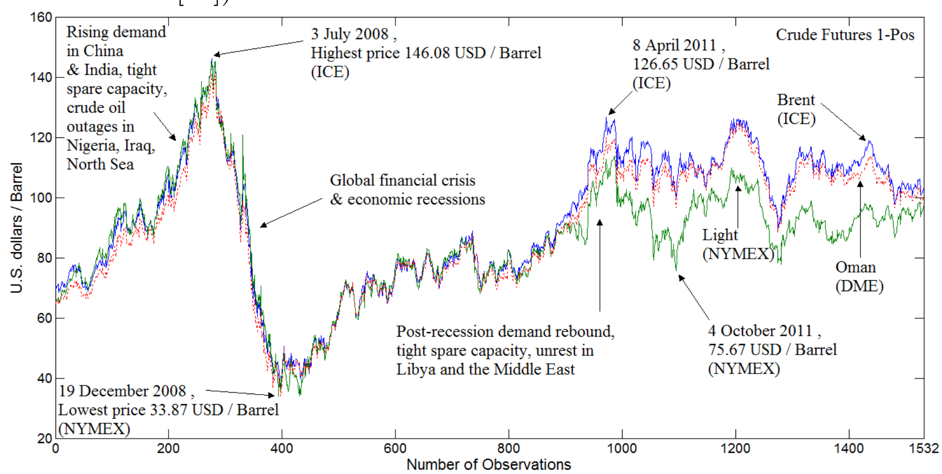
The observation "world oil market, like the world ocean, is one great pool" was proposed by Adelman [1, 2]. This assumption implies that the crude oil markets in each region are linked together or have integration. Moreover, Adelman [3] said that the transportation of oil between nations was relatively easy; oil exporters tend to seek the markets that make it more profitable for them and, thus, cause those oil markets to become "a single world market". As for the point of view of Adelman [1, 2], there were different empirical studies that supported the "one great pool" assumption. The crude oil benchmark prices in the international market (e.g., Brent, West Texas Intermediate [WTI], Dubai, Oman, and Maya) were used for the studies and several types of econometric models were utilized to analyze the data. Starting with Hammoudeh et al. [4], they used the threshold cointegration method to study the relationship between pairs of crude oil benchmark prices. They found out that there was a long-run equilibrium relationship between different crude oil benchmark prices. Reboredo [5] used copula based GARCH model to study the dependence structure between the crude oil benchmark prices in international crude oil markets. It was found that in times of crude oil market stress, the crude oil price in each market tends to have co-movement with the same intensity. AlMadi and Zhang [6] used vector error correction model (VECM) and Granger causality tests. The empirical results showed that the four crude oil benchmarks prices were found to be cointegrated; in addition, the following facts were identified: WTI significantly leads Brent, Dubai, and Oman; Brent significantly leads Dubai and Oman; and Oman moderately leads Dubai. Therefore, we can definitely say that if a market has supply and demand shocks/price shock, then it has an impact on other regional markets.

For analyzing the relationship of crude oil prices between the markets, most of the studies in the previous works (see [4, 5, 6]) used the bivariate model. In fact, the random variables that were used in those studies were also related to other variables. For hypothesis testing, in the context of world crude oil, the market is globalized or regionalized? Clearly, it is a multivariate model that we need in order to analyze the relationship between several markets (where there are more than two random variables) or to analyze the multivariate joint probability in higher dimensions. As a result, we are convinced that this model would be more appropriate than the bivariate model because the multivariate model can take all the variables to be considered into account. In order to fill the gap of bivariate model, this paper proposes the vine copula model [7, 8] to study the dependence structure between the prices of crude oil in three continents, namely, North America, Europe, and Asia, which are likely to share significant relationship, as is evident from Figure 1.

The vine copula model is a flexible tool to analyze the dependence structure in a multivariate setting. It allows us to define the relationship structure between the variables by using expert knowledge or concordance of data, or both, and it can describe the relationship between the variables through the graphical model, or through what are called pair-copulas, as shown in Figure 2.

Figure 1 displays the graph of crude oil futures prices and presents the major events which affected the prices during the period from 1 June 2007 to 28 June 2013. In this study, we used the crude oil futures prices which represented the

Figure 1: The crude oil futures prices of the NYMEX, ICE, and DME (sources: Ecowin database, Hamilton [9], OECD [10], Australian Institute of Petroleum [11])



crude oil prices of each continent: Light crude futures 1-Pos of the New York Mercantile Exchange (NYMEX), for North America; Brent crude futures 1-Pos of the Intercontinental Exchange (ICE), for Europe; and Oman crude futures 1-Pos of the Dubai Mercantile Exchange (DME), for Asia. The daily closing prices during the period from 26 December 2008 to 28 June 2013 were used for the analysis. We used the data of this period because it is a period in which the oil prices have rebounded from being the lowest after the shocks from the global financial crisis.

The vine copula model is used to analyze the dependence structure between three random variables of the crude oil futures prices. More specifically, we want to examine the following: (1) the order of the relationship of the three crude oil markets through an appropriate vine tree structure and (2) the particular market that is a key variable that governs the interactions within these three markets. The research results from this study will provide more understanding regarding the relationships between crude oil markets and their dependence structure, which will be useful for policy makers in that they will be able to monitor the changes in the crude oil prices for risk prevention, in the energy security context, and risk management, for investment in the commodity market; furthermore, the results will be useful for an improved understanding of the "one great pool" hypothesis.

The remainder of this work is organized as follows: part two is the methodology, and part three consists of the data and the empirical findings. Finally, part four makes up the conclusions.

2 Methodology

The objective of the study is to analyze the relationships between three crude oil prices: the NYMEX, ICE, and DME. First, we have to give the definitions of the variables, which are as follows: the NYMEX is the Light crude futures 1-Pos price, the ICE is the Brent crude futures 1-Pos price, and the DME is the Oman crude futures 1-Pos price. The ARMA-GARCH model is used to find out the "marginal distributions" for the copula model since this model has been widely used for modeling the volatility of the time series data in the financial field. The residuals (ϵ_t) from the appropriate marginal models of the three data series will be standardized. The standardized residuals (z_t) will then be transformed using the empirical distribution function and, thereafter, we obtain the marginals. These marginals are then used as inputs to the copula data $(F_1(x_1), F_2(x_2), F_3(x_3))$. Next, the vine copula model is used to analyze the dependence structure; also used are two approaches for specifying the structure of the D-vine model: (1) the empirical Kendall's tau, which is rank correlation, and (2) the distance measure that is based on the idea of information-theoretic entropy.

2.1 Marginal distribution model

Different models are appropriate for different time series data. Therefore, we adopt ARMA(p,q)-GARCH(1,1) model [12] with skewed student-T distribution residual (SkT) for the marginal distribution of the log-difference $\ln \dfrac{P_t}{P_{t-1}}$ of the crude oil future prices 1-Pos (y_t): the NYMEX, ICE, and DME.

2.1.1 ARMA(p,q)-GARCH(1,1)

$$y_t = a_0 + \sum_{i=1}^{p} a_i y_{t-i} + \sum_{i=1}^{q} b_i \varepsilon_{t-i} + \varepsilon_t \tag{2.1}$$

$$\varepsilon_t = z_t \sqrt{h_t}, z_t \sim SkT(\nu, \lambda) \tag{2.2}$$

$$h_t = \omega_t + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \tag{2.3}$$

In equation (2.1) is presented the ARMA(p,q) process, where y_{t-i} is an autoregressive term of y_t and ε_t is an error term. Equation (2.2) then defines this residual as the product between the conditional variance h_t and a random variable z_t. The residual ε_t will be standardized by $\varepsilon_t/\sqrt{h_t}$ to be a standardized residual z_t. The z_t is assumed to follow the skewed student-T (SkT) distribution with the shape parameter ν and the skewness parameter λ. Equation (2.3) presents the GARCH(1,1) process, where $\omega_t > 0, \alpha \geq 0, \beta \geq 0$ are sufficient to ensure that the conditional variance $h_t > 0$. The $\alpha\varepsilon_{t-1}^2$ represents the ARCH term and α refers to the short-run persistence of shocks, while βh_{t-1} represents the GARCH term and β refers to the contribution of shocks to the long-run persistence $(\alpha+\beta)$. The second moment condition is $\alpha + \beta < 1$.

In this study, the R-package fGarch by Wuertz and Chalabi [13] is used to estimate the parameters of the ARMA(p,q)-GARCH(1,1) model.

2.2 Copula functions

The fundamental theorem of copula is the Sklar's theorem, which was proposed by Sklar [14].

Let F be an n-dimensional distribution function with marginal distributions $F_1, ..., F_n$. Then there exists a copula C for all $x = (x_1, ..., x_n)^{'} \in [-\infty, \infty]^n$, such that

$$F(x) = C(F_1(x_1), ..., F_n(x_n)) \tag{2.4}$$

If $F_1, ..., F_n$ are continuous, then C is unique. Conversely, if C is a copula and $F_1, ..., F_n$ are distribution functions, then the above function $F(x)$ in equation (2.4) is a joint distribution function with marginal distribution $F_1, ..., F_n$. C can be interpreted as the distribution function of an n-dimensional random variable on $[0, 1]^n$ with uniform margins [7].

We used the various copula families contained in the R-package CDVines to measure the dependence of the pair-copula, including Gaussian, Student's T, Clayton, Gumbel, Frank, Joe, BB1, Rotated Clayton 180°, Rotated Gumbel 180°, Rotated Joe 180°, Rotated BB1 180°.

2.3 Vine copula modeling

Modeling dependencies in high dimension by the standard multivariate copula is inflexible because they do not allow for different dependency structures between pairs of variables [15]. Vine copulas can cross over this restriction; vine copulas are a flexible tool for illustrating the multivariate copulas through graphical models. The multivariate copulas are constructed from a cascade of bivariate copulas (called pair-copulas), as a result of which we are able to select bivariate copulas from a wide range of families.

This study used D-vine copula modeling to analyze the dependence between the crude oil futures prices of the NYMEX, ICE, and DME. The modeling of the D-vine copula is as follows: first an appropriate D-vine tree structure has to be specified; next, adequate copula families have to be selected and estimated [7].

2.3.1 Structure of D-vine

We let the structure of D-vine be given by the data itself. To construct a D-vine structure, we need to select the order of the variables in the first tree, as the first step. There are many approaches to ordering the sequences of variables, such as the empirical Kendall's tau, the Spearman's rho, the distance measure [15], and the degree of freedom parameters of the Student's T copula [16]. This paper used the empirical Kendall's tau and the distance measure, and compared the results from these two approaches.

Empirical Kendall's tau

The Kendall's rank correlation, or the empirical Kendall's tau ($\overline{\tau}_n$), as in equation (2.5), is used to measure the degree of dependence in each pair of the transformed standardized residuals of the data set. A high value of $\overline{\tau}_n$ means that there is high dependency between the two variables. The strongest dependencies, in terms of absolute empirical values of pairwise Kendall's tau, are used as the first pair in the first, and is subsequently followed by the next. The selection of the D-vine structure is based on the one that maximizes the sum of the corresponding absolute value of $\overline{\tau}_n$ in the first tree.

$$\overline{\tau}_n = \frac{P_n - Q_n}{(\frac{n}{2})} = \frac{4}{n(n-1)} P_n - 1 \tag{2.5}$$

where P_n and Q_n are the number of concordant and discordant pairs, respectively. The two pairs, (X_i, Y_i) and (X_j, Y_j), can be said to be concordant when $(X_i - X_j)(Y_i - Y_j) > 0$, and discordant when $(X_i - X_j)(Y_i - Y_j) < 0$ [17].

Distance Measure

There are many approaches to measuring the distance between probability distributions or data set. This study used the approach to distance measure which is closely related to divergence measures based on the idea of information-theoretic entropy first presented by Shannon [18]. This divergence measure is symmetric and is referred to as the non-directional divergence measure. It qualifies as distance measure [19]. The formula can be written as given in equation (2.6).

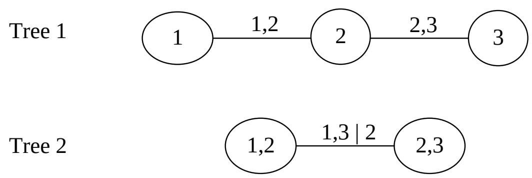
$$I(f_1, f_2) = K(f_1, f_2) = \int (f_1 - f_2) \log \frac{f_1}{f_2} dy \tag{2.6}$$

where $I(f_1, f_2)$ is the distance measure between the probability functions f_1 and f_2 of the standardized residual. A low value of $I(f_1, f_2)$ means that there is high association, or high affinity between f_1 and f_2. For ordering variables, the lowest $I(f_1, f_2)$ is used as the first pair in the first tree, and is subsequently followed by the next. The selection of the D-vine structure is based on the one that minimizes the sum of the corresponding absolute value of $I(f_1, f_2)$ in the first tree.

D-vine tree

Thereafter, we order the sequences of the variables in the first tree by the empirical Kendall's tau and the distance measure. We can construct the D-vine structure for three variables as shown in Figure 2.

Figure 2: The pair-copulas of three-dimensional D-vine trees



2.3.2 Density function of D-vine

We present the three dimensions case in our study. Let $X = (X_1, X_2, X_3) \sim F$ with corresponding densities f_1, f_2, f_3. Then (see [8]):

$$f(x_1, x_2, x_3) = f(x_1) \cdot f(x_2) \cdot f(x_3) \cdot c_{1,2}(F_1(x_1), F_2(x_2)) \cdot c_{2,3}(F_2(x_2), F_3(x_3)) \\ \cdot c_{1,3|2}(F_{1|2}(x_1 \mid x_2), F_{3|2}(x_3 \mid x_2)) \tag{2.7}$$

where $c_{1,2}, c_{2,3}$, and $c_{1,3|2}$ denote the densities of bivariate copulas $C_{1,2}, C_{2,3}$, and $C_{1,3|2}$, respectively. $F_{1|2}$ and $F_{3|2}$ are the marginal conditional distributions that can be derived from formula (2.8).

The vine copulas involve marginal conditional distributions. The general form of a conditional distribution function is $F(x \mid v)$, given by

$$F(x \mid v) = \frac{\partial C_{x,v_j|v_{-j}}(F(x \mid v_{-j}), F(v_j \mid v_{-j}))}{\partial F(v_j \mid v_{-j})} \tag{2.8}$$

where v denotes all the conditional variables and $C_{x,v_j|v_{-j}}$ is a bivariate copula distribution function. When v is univariate, the marginal condition distribution, for example, $F_{1|2}$ can be presented as

$$F_{1|2}(x_1 \mid x_2) = \frac{\partial C_{12}(F_1(x_1), F_2(x_2))}{\partial F_2(x_2)} \tag{2.9}$$

2.4 D-vine copula estimation

In the R-package CDVines, the maximum likelihood was used to estimate the parameters of the copulas. The log-likelihood of the D-vine copula with three dimensions in equation (2.7) can be written as

$$\sum_{t=1}^{T} \log[c_{1,2}(F_1(x_{1,t}), F_2(x_{2,t})) \cdot c_{2,3}(F_2(x_{2,t}), F_3(x_{3,t})) \\ \cdot c_{1,3|2}(F_{1|2}(x_{1,t} \mid x_{2,t}), F_{3|2}(x_{3,t} \mid x_{2,t}))] \tag{2.10}$$

3 Data and Empirical Results

This study used the oil prices from three major markets, the NYMEX, ICE, and DME, to analyze the dependence structure. The observations regarding the three data series were obtained from the EcoWin database during the period from 26 December 2008 to 28 June 2013.

We used the crude futures 1-Pos of daily closing prices and each data series was transformed into the log-difference ($\ln P_t - \ln P_{t-1}$), before it was used for analysis using the GARCH model and the vine copula.

Table 1 presents the descriptive statistics of the log-difference of three crude futures 1-Pos: the NYMEX, ICE, and DME. All of the three data series have a positive average growth rate, exhibiting positive skewness. If there is positive skewness, it means that the market has an upward trend, or that there is substantial probability of a big positive return. The kurtosis of these data is greater than 3. Hence, this kurtosis can be said to be super Gaussian and leptokurtic. This means that the growth rates of the empirical data have a typically spiky probability distribution function with heavy tails. The null hypotheses of normality of the Jarque-Bera tests are rejected in all the data series. The Augmented Dickey-Fuller test shows that these data series are stationary at p-value less than 0.01.

Table 1: Data Descriptive Statistics for Log-difference of Crude Oil Futures Price 1-Pos

	NYMEX	ICE	DME
Mean	0.001	0.001	0.001
Median	0.001	0.001	0.001
Maximum	0.133	0.127	0.134
Minimum	-0.131	-0.097	-0.091
Std. Dev.	0.023	0.020	0.019
Skewness	0.188	0.006	0.088
Kurtosis	8.276	7.141	7.649
p-value of Jarque-Bera	(0.01)	(0.01)	(0.01)
p-value of ADF test	(< 2.2e-16)	(< 2.2e-16)	(< 2.2e-16)
No. of Observations	1135	1135	1135

Table 2 presents the appropriate marginal models for the log-difference of three crude futures 1-Pos data: The ARMA(3,2)-GARCH(1,1) with skewed student-T residual for the NYMEX data and the ARMA(1,1)-GARCH(1,1) with skewed student-T residual for the ICE and DME data. The models are selected by using the AIC criterion. For the NYMEX, the $\alpha + \beta$ is 0.986, for the ICE and the DME, the $\alpha + \beta$ are 0.992 and 0.994, respectively; this implies that their volatilities have a long-run persistence. For the short-run effect of the unexpected factors, we considered the event from the α parameters of the NYMEX, the ICE and the DME. The results showed that they have the values 0.073, 0.056 and 0.049, and

Table 2: Results of ARMA(p,q)-GARCH(1,1) with Skewed Student-T Residual for Log-difference of Crude Oil Futures Price 1-Pos

	NYMEX	SE (p-value)	ICE	SE (p-value)	DME	SE (p-value)
mu	8.06e-06	4.27e-07 (<2e-16***)	7.207e-05	8.82e-05 (0.414)	6.638e-05	8.22e-05 (0.419)
ar1	2.42e-01	2.40e-05 (<2e-16***)	8.135e-01	9.69e-02 (<2e-16***)	8.170e-01	9.37e-02 (<2e-16***)
ar2	6.96e-01	2.48e-05 (<2e-16***)	-	-	-	-
ar3	9.91e-03	2.56e-05 (<2e-16***)	-	-	-	-
ma1	-3.07e-01	3.18e-05 (<2e-16***)	-8.356e-01	9.20e-02 (<2e-16***)	-8.436e-01	8.90e-02 (<2e-16***)
ma2	-7.14e-01	3.19e-05 (<2e-16***)	-	-	-	-
ω	5.64e-06	3.56e-06 (0.113)	2.96e-06	2.07e-06 (0.152)	2.54e-06	1.84e-06 (0.168)
α	7.27e-02	2.32e-02 (0.002**)	5.64e-02	1.81e-02 (0.002**)	4.87e-02	1.69e-02 (0.004**)
β	9.14e-01	2.75e-02 (<2e-16***)	9.36e-01	2.04e-02 (<2e-16***)	9.46e-01	1.85e-02 (<2e-16***)
ν (shape)	8.13	1.69e+00 (1.47e-06***)	6.79	1.34e+00 (3.66e-07***)	4.77	6.94e-01 (6.61e-12***)
λ (skewness)	8.55e-01	4.19e-02 (<2e-16***)	9.00e-01	3.74e-02 (<2e-16***)	9.06e-01	3.68e-02 (<2e-16***)
LL	2,888.76		2,986.98		3,042.67	

Note: Significant codes: 0 "***"; 0.001 "**"; 0.01 "*" 0.05.

that this has a small impact on volatility.

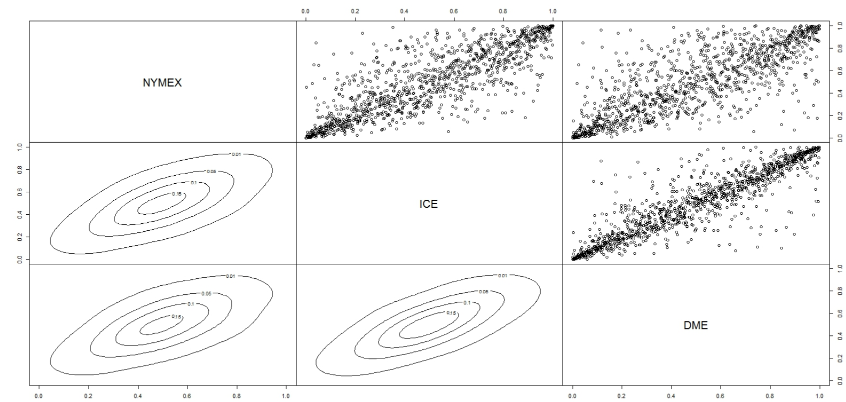
Next, we transformed the standardized residuals from the ARMA-GARCH model into the uniform [0,1] by using the empirical distribution function $F_n(x) = \dfrac{1}{n+1}\sum_{i=1}^{n} 1(X_i \leq x)$, where $X_i \leq x$ is the order statistics and 1 is the indicator function. The transformed data were used in the Kolmogorov-Smirnov (K-S) test for uniformity [0,1] and the Box-Ljung test for serial correlation. The results showed that these marginal distributions are uniform and i.i.d. so our marginal distributions were not misspecified and can be used for the copula model.

Figure 3 illustrates the scatter plots of the three bivariate margins, NYMEX–ICE, NYMEX–DME, and ICE–DME. The data show the clustering in both the upper and the lower tail dependences. The pair-copula of ICE–DME shows stronger dependence in both the upper and the lower tails, compared to the other pairs.

In order to better understand the relationship between these pair copulas, the results of the dependence structure were analyzed by using the bivariate copula model, as presented in Table 3.

The analysis is performed by taking into consideration the results of the AIC, the BIC, and the goodness-of-fit tests of the Cramér-von Mises (CvM) and the

Figure 3: The scatter plots of NYMEX–ICE, ICE–DME, and NYMEX–DME



Kolmogorov-Smirnov (K-S) tests in Table 4. As for the first pair-copula, NYMEX–ICE, the Rotated BB1 180° copula is appropriate to explain the dependence structure of this pair-copula. The Kendall's tau correlation (τ) is 0.646, and the lower (T^L) and upper tail (T^U) dependences are 0.680 and 0.758.

As far as the second pair-copula, NYMEX–DME, is concerned, the Rotated BB1 180° copula is appropriate for explaining the dependence structure of this pair-copula. The Kendall's tau correlation is 0.594, and the lower and upper tail dependences are 0.644 and 0.737.

As for the last pair-copula, ICE–DME, the BB1 copula is appropriate to explain the dependence structure of this pair-copula. The Kendall's tau correlation is 0.741, and the lower and upper tail dependences are 0.767 and 0.688.

The results demonstrate that the NYMEX, ICE, and DME have relatively strong dependence. Hence, we can safely infer that these crude oil futures prices move closely together, especially the ICE and the DME.

Next, we used the D-vine copula model to analyze the dependence structure between the crude oil futures prices and especially to examine which oil market is a key variable that governs the interactions within these three markets.

3.1 D-vine Structure

The empirical Kendall's tau $\overline{\tau}_n$ and the distance measure $I(f_1, f_2)$ were used to select the order of the variables in the first tree. Table 5 shows the empirical Kendall's tau matrix, which was computed from the transformed standardized residuals of the NYMEX, ICE, and DME. A high value of $\overline{\tau}_n$ means that there is "high dependency". The strongest dependencies in terms of absolute empirical values of $\overline{\tau}_n$ are used as the first pair in the first tree, which is subsequently followed by the next. The selection of the D-vine structure is based on the one

Table 3: Bivariate Copula Analysis of NYMEX–ICE, NYMEX–DME, and ICE–DME

Pair copula	Copula family	Parameter	SE (p-value)	τ	T^L	T^U	AIC	BIC
NYMEX –ICE	Rotated BB1 180°	$\theta = 0.264$	0.062 (0.000)	0.646	0.680	0.758	-1,478.90	-1,468.80
		$\delta = 2.496$	0.090 (0.000)					
NYMEX –DME	Rotated BB1 180°	$\theta = 0.165$	0.058 (0.002)	0.594	0.644	0.737	-1,205.20	-1,195.10
		$\delta = 2.276$	0.079 (0.000)					
ICE –DME	BB1	$\theta = 1.023$	0.104 (0.000)	0.741	0.767	0.688	-2,074.50	-2,064.40
		$\delta = 2.552$	0.111 (0.000)					

Note: For the CvM and K-S tests, the critical value $\alpha = 5\%$. If p-value > 0.05, it means that the dependence structure of the data series is appropriate for the chosen family of copulas.

Table 4: P-value of CvM and K-S tests of Goodness-of-fit test based Kendall's process

	NYMEX–ICE, Rotated BB1 180°	NYMEX–DME, Rotated BB1 180°	ICE–DME, BB1
P-value of CvM	0.64	0.73	0.09
P-value of KS	0.39	0.77	0.16

Table 5: Empirical Kendall's tau Matrix

	NYMEX	ICE	DME
NYMEX	1	0.643	0.597
ICE	0.643	1	0.752
DME	0.597	0.752	1

that maximizes the sum of the corresponding absolute value of $\overline{\tau}_n$ in the first tree, as can be understood from Figure 4.

Table 6 presents the distance measure matrix, which was computed from the skewed student-T distribution of the standardized residuals of the NYMEX, ICE, and DME. A low value of $I(f_1, f_2)$ means that there is "high association", or "high affinity" between f_1 and f_2. The lowest $I(f_1, f_2)$ is used as the first pair in the first tree, which is subsequently followed by the next. The selection of the D-vine structure is based on the one that minimizes the sum of the corresponding absolute

Figure 4: The order of the variables in the first tree of the D-vine structure by the empirical Kendall's tau

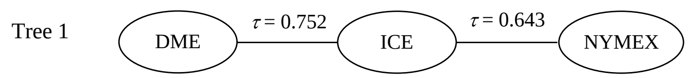
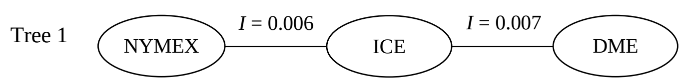


Table 6: Symmetric Distance Measure Matrix

	NYMEX	ICE	DME
NYMEX	0	0.006	0.021
ICE	0.006	0	0.007
DME	0.021	0.007	0

Figure 5: The order of the variables in the first tree of the D-vine structure by the symmetric distance measure



value of $I(f_1, f_2)$ in the first tree, as can be seen from Figure 5.

The findings as displayed in Figure 4 and Figure 5 demonstrate that for the three variables of the crude oil prices chosen for this study, the order of variables in the first tree of D-vine by the empirical Kendall's tau approach is different from that by the distance measure approach, in reversed direction. In addition, the NYMEX and the DME were linked by ICE in both the structures.

3.2 Results of D-vine models

As the next step, the appropriate D-vine tree structures were specified by the empirical Kendall's tau (Figure 4) and the distance measure (Figure 5). Then, adequate copula families were selected and estimated. For the estimation of the copula parameters, we followed the process as presented in Aas et al. [8] and Brechmann and Schepsmeier [7]. First, we estimated the parameters of the three copulas involved by a sequential procedure that involved only the bivariate estimation for each individual pair-copula. Next, we used the parameters from the sequential estimation as the starting value to maximize the full log-likelihood procedure, or what is called a joint MLE estimation. Thereafter, the copula parameters of each pair can be obtained from the joint MLE estimation.

Figure 6 presents the results of Model 1: D-vine model by the empirical

Figure 6: The Model 1: D-vine model by the empirical Kendall's tau sequencing

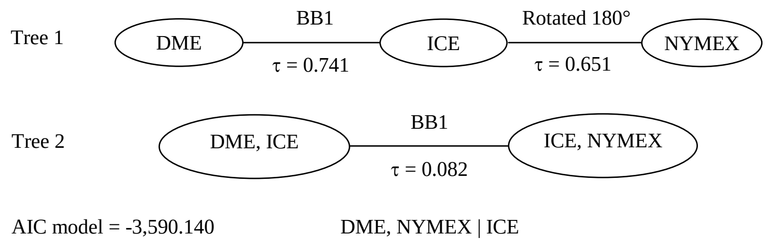
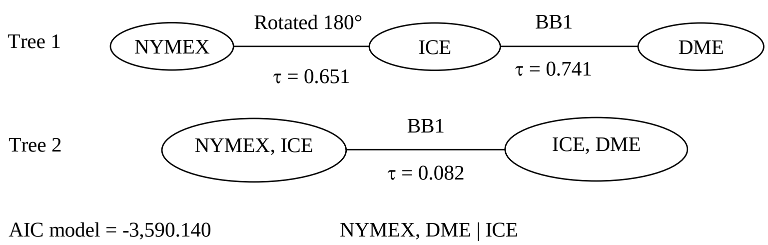


Figure 7: The Model 2: D-vine model by the distance measure sequencing



Kendall's tau sequencing. Each pair-copula consists of the copula families and their Kendall's tau correlations that are transformed from the copula parameters by a joint MLE estimation.

Figure 7 presents the results of Model 2: D-vine model by the distance measure sequencing. When we compare these results with the results from the empirical Kendall's tau, it can be seen that this three dimensional model provides the same estimates of the parameters of interest and the value of AIC.

In addition, we also fitted Model 3 and Model 4, the D-vine models with different orders of the variables to determine the better appropriate structure of the D-vine model for our data, and the results from the joint MLE estimation are shown in Figure 8 and Figure 9.

By taking into consideration the Akaike Information Criterion (AIC) value of each model, we found that Model 1 and Model 2, the D-vine models by the empirical Kendall's tau and by the distance measure sequencing, provide better fit than Model 3 and Model 4.

Figure 8: The Model 3: D-vine model with different orders of the variables: the DME, NYMEX, and ICE

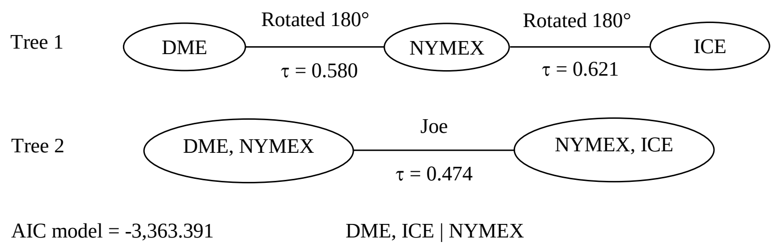
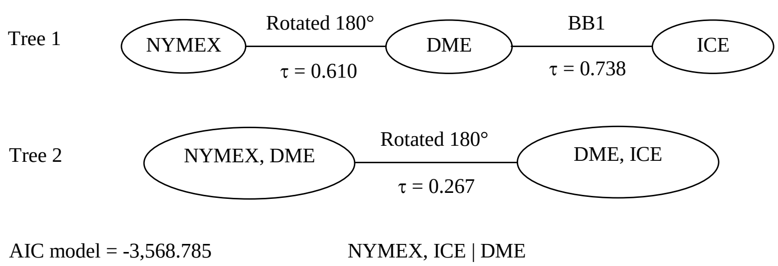


Figure 9: The Model 4: D-vine model with different orders of the variables: the NYMEX, DME, and ICE



Model 1 and Model 2 provide the same estimates of the parameters of interest, which is that the DME and the NYMEX are linked by the ICE, as demonstrated in Figure 6 and Figure 7. For this reason, we will explain the results of only one model.

Model 2: D-vine copula model, which is modeled by the distance measure, reveals that there exists a positive dependence for each pair-copula, which estimated by a joint MLE. The first pair is the NYMEX–ICE, for which the rotated BB1 180° copula is the best fit, with two copula parameters, 0.267 and 2.530, a Kendall's tau correlation of 0.651, and the lower and upper tail dependences of 0.685 and 0.760, respectively.

The second pair is the ICE–DME, and the BB1 copula is chosen to explain the dependence structure of this pair-copula with two copula parameters, 1.083 and 2.505, a Kendall's tau correlation of 0.741, and the lower and upper tail dependences of 0.775 and 0.681, respectively.

The last conditional pair-copula of NYMEX–DME given ICE in Tree 2 provides that the BB1 copula is its best fit with two copula parameters, 0.069 and 1.053, a Kendall's tau correlation of 0.082, and the lower and upper tail dependences of 0.000 and 0.069, respectively. This Kendall's tau correlation by the conditional pair-copula of NYMEX–DME given ICE is less than those that are obtained by using the bivariate copula analysis of NYMEX–DME, which is 0.594, as presented in Table 6. This implies that the ICE has an influence on the dependence between the NYMEX and the DME, and that it is an important variable that governs the interactions within these variables.

4 Conclusions

This study used the GARCH model and the D-vine copula model to analyze the relationships between three random variables which we used to represent the crude oil prices of three different continents: Light crude futures 1-Pos of the New York Mercantile Exchange (NYMEX) for North America, Brent crude futures 1-Pos of the Intercontinental Exchange (ICE) for Europe, and Oman crude futures 1-Pos of the Dubai Mercantile Exchange (DME) for Asia. The daily closing prices during the period from 26 December 2008 to 28 June 2013 of three crude futures 1-Pos were used to conduct the analysis.

We found that the log-difference of the crude futures 1-Pos of the NYMEX data was appropriate with the ARMA(3,2)-GARCH(1,1) with skewed student-T residual. As for the log-differences of the crude futures 1-Pos of the ICE and the DME data, they were appropriate with the ARMA(1,1)-GARCH(1,1) with skewed student-T residuals. Moreover, it was observed that the three data series had long-run persistence.

The results from the bivariate copula analysis of and comparison between the crude futures 1-Pos of these three markets revealed that the relationships between the NYMEX–ICE pair, the NYMEX–DME pair, and the ICE–DME pair had comovement. These findings correspond to the findings obtained in a previous study conducted by Reboredo [5]. In addition, we discovered evidences of asymmetric tail dependence in each pair. The NYMEX–ICE and NYMEX–DME pairs showed that the upper tail dependence was greater than the lower tail dependence, with the rotated BB1 copula families. As for the ICE–DME pair, it presented that the lower tail dependence was greater than the upper tail dependence, with the BB1 copula family. However, the values of the upper tail and the lower tail dependences of the three pair-copulas were quite close to each other. Furthermore, these findings support the "one great pool" hypothesis propounded in Adelman [1, 2], which, again, corresponds to the research studies of Hammoudeh et al. [4], Reboredo [5], and AlMadi and Zhang [6].

As far as specifying the D-vine structures in the cases of the three variables are concerned, we found that the D-vine models by the empirical Kendall's tau and the distance measure provided the best fit by giving better AIC values. In addition, these two models with the three dimensional copula provided the same estimates of the parameters of interest as well as the AIC values. The results of the bivariate and the D-vine copula models indicated that the ICE had an influence on the dependence between the crude oil prices of the NYMEX and the DME, and that it was an important variable that governs the interactions within these crude oil markets.

From the findings, it can be concluded that the crude oil prices of North America, Europe, and Asia in the case of crude futures 1-Pos have relatively strong dependence, and that regardless of whether it is an upward or a downward trend, their prices tend to move together. This finding is useful for decision planning of energy security in many countries in each of these regions. In addition, the evidence of the upper and the lower tail dependences between these three markets can be useful in risk management for investment in the commodity market. This information can tell us about the probability of joint occurrence of extreme events in crude oil prices.

Moreover, the best and the most advantageous finding of this study is it has given us the knowledge that among the three crude oil markets, the ICE is a crude oil market that has much influence. In other words, the change in oil prices in the ICE will impact quite significantly the oil prices in the NYMEX and the DME, in the same direction. Therefore, the price of the crude futures 1-Pos of the ICE is the appropriate information, or should be used as the indicator for monitoring the change in the oil prices of the NYMEX and the DME.

Regarding further studies in this field, we recommend that they include more of the related variables that represent the oil price movements in other crude oil markets also in the different regions of the world for a better understanding of the "one great pool" hypothesis.

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