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A Mixture of Canonical Vine Copula-GARCH Approach for Modeling Dependence of European Electricity Markets

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Abstract : This paper employed a mixed Canonical Vine Copula-GARCH approach for modeling the dependence structures of European electricity markets. The electricity spot prices are taken from French, German, Spanish, Dutch, and British markets. The empirical result shows that pairwise positive dependence between markets is represented in Tree 1, in which there is positive spillover effect between the French and the other four markets. Moreover, the French, German, and Dutch markets have strong symmetric tail dependence, which suggests one market (one of the French, German, or Dutch markets) experiencing spikes or drops, conditional on the event that the other two markets are also experiencing spikes or drops. Additionally, we also found that when adding the condition under one or more markets, the relationships of some pairs still had dependence, while some other pairs became independent.

Keywords : electricity spot price; canonical vine copula; GARCH model; dependence

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1 Introduction

The liberalization process of the European electricity markets experienced over the last twenty years. Our goal in this paper is to measure dependence or co-movement between European electricity markets from a new perspective by using mixture C-vine copula based GARCH model. The copula based GARCH model is a popular model to research dependence or (co-movement) in many fields, while the vine copula began to be applied recently, and so, very few literature reviews are available on vine copula as regards its applications. It was Joe [1] who first proposed the pair-copula structure, and Bedford and Cooke [2,3] and Kurowicka and Cooke [4] who further developed it. Aas et al. [5] first proposed that one pair-copula construction can be mixed with bivariate copulas for any copula family. Additionally, they developed the graphical representation for the C-vine and D-vine copulas and the standard maximum likelihood estimation (MLE). Mendes et al. [7] used the D-vine model with four different bivariate copula families on a six-dimensional global portfolio in order to show how pair-copulas could be applied on a daily basis for constructing efficient frontiers, besides discussing its use for portfolio management. Brechmann and Czado [14] developed a regular vine copula based factor model for asset returns, which is the so-called Regular Vine Market Sector (RVMS) model and employed it to analyze the Euro Stoxx 50 index. This paper shows that the RVMS model provides good fits of the data and accurate VaR forecasts. Kim et al. [8] proposed a mixing of D-vine copulas for revealing complex and hidden dependence structures in multivariate data. Low et al. [6] used canonical vine copulas in the context of modern portfolio management. They found that asymmetric copula models can be used to forecast returns for portfolios ranging in assets from 3 to 12, and that they have better implementation benefits than the traditional models.

This paper seeks to measure the dependence between various European electricity markets by employing the mixed canonical vine copula based GARCH model. We pursue to analyze the spot prices of five major European electricity markets, which are France, Germany, Spain, the Netherlands, and the UK. Additionally, the procedure for measuring the dependence between the different European electricity markets is divided into two stages. We first specify the marginal model for each electricity price, followed by the joint model for the dependence. The AR-Skew-t-GARCH models are used for the marginal models of the electricity prices, which capture the most important features of price (e.g., heteroscedasticity and volatility clustering). For the joint distribution, we use the mixed canonical vine copula model to get the dependence structure. The following C-vine copulas with different dependence structures are considered: Gaussian and Frank BB8 copulas, which are symmetric with zero tail dependence; Student-t copula, which is symmetric with non-zero tail dependence; Gumbel, Joe, and BB6 copulas, which allow for asymmetric with upper tail dependence and zero lower tail dependence; Clayton copulas, which allow for asymmetric with lower tail dependence and zero upper tail dependence; and BB8, which is asymmetric with non-zero tail dependence.

This paper is organized as follows. Section 2 describes the Canonical Vine Copula-GARCH model used in the paper. Section 3 discusses the data and their properties, and Section 4 presents the empirical results with inference. The last section provides some concluding remarks.

2 Canonical Vine Copula Based GARCH Model

2.1 Copula function

A copula is a multivariate cumulative distribution function (CDF) with uniform marginal distribution that captures the dependence structure between random variables. Let $X_1, X_2, ., X_n$ be random variables with the marginal distribution $F_1, F_2, , F_n$, and let their joint distribution be H. Then, there exists a copula $C : [0, 1]^n \to [0, 1]$ that satisfies the following:

$$F(x) = H(x_1, x_2, \ldots, x_n) = C(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)), \tag{2.1}$$

and vice versa

$$C(u_1, \ldots, u_n) = F\left(F_1^{-1}(u_1), \ldots, F_n^{-1}(u_n)\right), \tag{2.2}$$

where $u_1, \ldots, u_n \in (0, 1)$and $F_1^{-1}(u_1), \ldots, F_n^{-1}(u_n)$ are the inverse distribution functions of the marginal. According to Sklar's Theorem (Sklar, [11]), we know that a joint distribution H can be separated into n univariate marginal distributions $F_1, F_2, , F_n$ and a copula C that could be used to measure the dependence structure between the variables $X_1, X_2, ., X_n$.

he corresponding density is

$$\begin{aligned} f(x) =& \frac{\partial^n F(x)}{\partial x_1 \ldots \partial x_n} = c(F_1(x_1), F_2(x_2), \ldots, F_n(x_n)) \prod_{i=1}^{n} f_i(x_i) \\ =& c(u_1, \ldots, u_n) \prod_{i=1}^{n} f_i(x_i), \end{aligned} \tag{2.3}$$

Moreover, copulas have another feature which is that the tail dependence of the variables measures the extreme co-movements between the variables. We express the parameters of the upper and lower tail dependences in terms of the copula between X_1 and X_2, with the marginal distribution functions F_1 and F_2 as

$$\begin{aligned} \lambda_U &= lim_{u \to 1} Pr\left[X_2 \geq F_2^{-1}(x_2) \middle| X_1 \geq F_1^{-1}(x_1)\right] \\ \lambda_L &= lim_{u \to 0} Pr\left[X_2 < F_2^{-1}(x_2) \middle| X_1 < F_1^{-1}(x_1)\right] \end{aligned} \tag{2.4}$$

where the values of both λ_U and λ_L lie between θ and 1. We call the tail dependence between X_1 and X_2 symmetric if the upper tail dependence parameter, λ_U, is equal to the lower tail dependence, λ_L; otherwise, it is asymmetric.

2.2 Pair-copula construction

Employing copula constructions to build flexible multivariate distributions is a very useful method in which the modeling principle is based on the decomposition of a multivariate density into a cascade of bivariate copulas (Emmanouil and Nikos, [10]). By employing the general form of a conditional marginal density, given in Equation (2.5), a d-dimensional multivariate density, given in Equation (2.3), can be represented by a product of pair-copulas in an iterative manner under suitable regularity conditions (Aas et al., [5]):

$$f\left(x\,|\upsilon\right) = c_{x,\upsilon_{-j}|\upsilon_{-j}}\left(F\left(x\,|\upsilon_{-j}\right), F\left(x\,|\upsilon_{-j}\right)\right) f\left(x\,|\upsilon_{-j}\right), \tag{2.5}$$

where υ is an n-dimensional vector, υ_j is any one component arbitrarily chosen from υ, and υ_{-j} is the vector excluding υ_j.

2.3 Canonical vines (C-vines)

This paper employs C-vines, which is a special case of the pair-copula. For the n-dimensional C-vine, the pairs at level 1 are i, for $i = 1, \ldots, n$, and for level $l(2 \le l < n)$, the (conditional) pairs are $l, i|1, \ldots, l-1$, for $i = l+1, \ldots, n$. That is, for the C-vine copula, the conditional copulas are specified for variables i and l, given those indexed as 1 to $l-1$ (Nikoloulopoulos, Joe, and Li, [13]). The density of n-dimensional C-vines is defined as follows (Aas et al., [5]):

$$f\left(x\right) = \prod_{k=1}^{n} f_k\left(x_k\right) \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{i,i+j|1:j-1}\left(F\left(x_i\,|x_1, \ldots, x_{j-1}\right), F\left(x_{i+j}\,|x_1, \ldots, x_{j-1}\right)\right) \tag{2.6}$$

where i runs over the edges in each tree and index j denotes the tree/level.

Figure 1 shows the specification of C-vines with five variables and four trees/levels. It consists of four trees, $T_j, j = 1, \ldots, n-1$. Additionally, Tree T_j has $n+1-j$ nodes and $n-j$ edges. The density of a five-dimensional canonical vine structure, which is plotted in Figure 1, can be written as follows:

$$\begin{aligned} f = & f_1 f_2 f_3 f_4 f_5 \times c_{12}\left(F_1, F_2\right) c_{12}\left(F_1, F_2\right) c_{13}\left(F_1, F_3\right) c_{14}\left(F_1, F_4\right) c_{15}\left(F_1, F_5\right) \\ & \times c_{23|1}\left(F_{2|1}, F_{3|1}\right) c_{24|1}\left(F_{2|1}, F_{4|1}\right) c_{25|1}\left(F_{2|1}, F_{5|1}\right) \\ & \times c_{34|12}\left(F_{3|12}, F_{4|12}\right) c_{35|12}\left(F_{3|12}, F_{5|12}\right) \\ & \times c_{45|123}\left(F_{4|123}, F_{5|123}\right) \end{aligned} \tag{2.7}$$

From Equation 2.6, we know that there are as many as $(n!)/2$ possible different C-vine structures. Hence the whole decomposition has $n(n-1)/2$ pair-copula families to be chosen from. Therefore, the crucial step is to select the C-vine decomposition and an appropriate copula family for each edge in the C-vine model by using the selection rules. These will be described in Section 2.5.

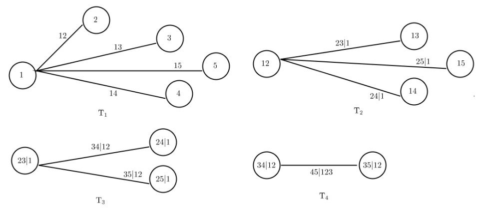

Figure 1: Tree decomposition, showing the five-dimensional canonical vine structure with four trees/levels

2.4 Inference for a C-vine copula model

This study uses two estimation procedures to estimate the C-vine parameters, which are maximum (log) likelihood estimation (MLE) and sequential estimation (SE). The log-likelihood of C-vine with parameter θ^{MLE} is defined as follows:

$$\ell\left(x, \theta^{MLE}\right) = \sum_{i=1}^{n-1} \sum_{j=1}^{n-j} \sum_{t=1}^{T} log\left[c_{i,i+j|1,\ldots,i-1}\left(F_{i|1,\ldots,i-1}, F_{i+j|1,\ldots,i-1}\left|\theta_{i,i+j|1,\ldots,i-1}\right.\right)\right] \tag{2.8}$$

where $F_{j|i_1,\ldots,i_m} = F\left(x_{t,j}\left|x_{t,i_1},\ldots,x_{t,i_m}\right.\right)$ and the marginal distribution are uniform.

We suppose that the i.i.d. data $u_t = (u_{1,t},\ldots,u_{n,t})^t$ for $t = 1,\ldots,T$ are available. According to the discussion given in Czado et al. [18], the steps for sequential estimation (SE) are described thus: (1) first, estimate the parameters $\hat{\theta}^{SE}_{1,j+1}$ of the unconditional bivariate copulas of Tree 1 using the data $(u_{1,t}, u_{j+1,t})$ for $j = 1,\ldots,n-1$; (2) given the estimated parameter vector $\hat{\theta}^{SE}$ in Tree 1, we compute the transformed variables for the second tree using h-functions and estimate the parameters $\hat{\theta}^{SE}_{1,j+1}$ $(j = 1,\ldots,n-2)$ of the conditional bivariate copulas of Tree 2 according to $c_{2,j+2|1}$ [2], which can be defined as

$$\hat{v}_{2|1,t} = F\left(u_{2,t}\left|u_{1,t};\hat{\theta}^{SE}_{1,1}\right.\right) = h\left(u_{2,t}\left|u_{1,t};\hat{\theta}^{SE}_{1,1}\right.\right)$$

$$\hat{v}_{j+2|1,t} = F\left(u_{j+2,t}\left|u_{1,t};\hat{\theta}^{SE}_{1,1}\right.\right) = h\left(u_{j+2,t}\left|u_{1,t};\hat{\theta}^{SE}_{1,1}\right.\right)$$

and (3) by following the same identification procedure, we can sequentially estimate the parameters of the bivariate copula for each nested set of trees in the C-vine structure until all the parameters of the bivariate copula are estimated.

[2] Aas et al. [5] define h-functions as $h(x,v;\theta) = h_\theta = \frac{\partial C_\theta(F_x(x),F_v(v))}{\partial F_v(v)} = \frac{\partial C_\theta(x,v)}{\partial v}$, where $x, v \sim U[0,1]$.

2.5 C-vine decomposition and copula selection

In general, there are $n!/2$ possible different C-vine structures. Hence the whole decomposition has $n(n-1)/2$ pair-copula families to be chosen from. Therefore, the crucial step is to select the C-vine decomposition and an appropriate copula family for each edge in the C-vine model by using the selection rules. The empirical selection rule for C-vine decomposition as discussed by Czado et al. [18] is followed in this paper. The empirical selection rule works thus: (1) estimate all the possible parameters of Kendall's $\tau_{(i.j)}$, and note them as $\tau_(i.j)$; (2) find the variable i^* that maximizes

$$\hat{\tau}_{i,sum} = \sum_{j=1}^{n} |\hat{\tau}_{i.j}| \tag{2.9}$$

over $i = 1, \ldots, n$. We select the most dependence with the other variables, i^*, and reorder these variables to be the first variable for Tree 1. As we form the $n-1$ transformed variables in the sequential estimation procedure. Follow the same identification procedure and recorder the $2\text{th}, \ldots, n-1\text{th}$ variable for Tree 1.

$$\hat{v}_{j+2|1,t} = h(u_{1,t}; \theta^{SE}_{1,j+1)j=0,\ldots,n-2,t=1,\ldots,T} \tag{2.10}$$

We estimate the parameters of all the pairwise Kendall's τ based on $n-1$ variables of size T and select those pilot variables i^{**} of ree 2 which maximize (Equation 2.5). We continue this procedure until all the pilot variables for each tree are determined.

Besides the steps described above, we should select an appropriate copula family for each edge in the C-vine model. In the later application, we consider ten kinds of copula families, namely, Gaussian, Student-t, Clayton, Gumbel, Frank, Joe, BB1, BB6, BB7, and BB8 copulas. The summary of the properties of these ten copulas and their statistical inference are reported in Table 1. We use AIC and BIC as well as goodness-of-fit tests to select an appropriate bivariate copula-family for every pair of variables. The AIC and BIC criteria correct the log-likelihood of a copula for the number of parameters (Dimann et al., [12]). Manner[7] and Brechmann ([9], Section 5.4) have conducted investigations on this previously, and shown that AIC and BIC are quite reliable criteria. We compute the AIC and BIC values for each family, and choose the copula family with the smallest AIC and BIC values. Additionally, we also do the independence test in the selection procedure. If this test indicates independence between variables, we need to take further steps and choose the independence copula.

2.6 Model for marginal distribution

We assume that the return of the electricity price r_t follows an AR (k)-GARCH (1,1) model with the standardized residual e_t satisfying the skewed-t distribution such that

$$\begin{aligned} r_t &= c_0 + \sum_{i=1}^{k} \phi_i r_{t-i} + e_t, \\ e_t &= \sqrt{h_t} z_t, z_t \sim SkT\left(z_t \middle| \xi, \lambda \right), \\ h_t &= \omega + \alpha {e^2}_{t-1} + \beta h_{t-1} \end{aligned} \tag{2.11}$$

where r_t is the daily return, e_t is the random error, h_t is the conditional variance of volatility of e_t at time t, and α and β are associated with the degree of innovation and volatility spillover effect from the previous period. z_t is the standardized residual assumed to be distributed as the following skewed-t distribution (Fernandez and Steel, [17]) in order to capture the possibly asymmetric and heavy-tailed characteristic of electricity price with λ DoF:

$$f(z_t) = \begin{cases} \frac{2}{\xi + \frac{1}{\xi}} f\left(\xi z_t\right), z_t < 0 \\ \frac{2}{\xi + \frac{1}{z}} f\left(\frac{z_t}{\xi}\right), z_t \geq 0 \end{cases}, \tag{2.12}$$

where ξ is the skewness parameter.

3 Data Description

This study focuses on the dependence between electricity prices by analyzing the spot electricity prices of five European electricity markets, namely, the French, German, Spanish, Dutch, and British markets. All the spot prices for the base period have been obtained from DataStream. The dataset covers the period from July 1, 2004, to June 30, 2013. DataStream does not provide weekend price data. None of the price series contains weekend price data; therefore, there are 2,437 daily price observations for each series in the base period.

Table 1: Bivariate Copula Families

Copula Name	Function $c(u,v)$	Parameter Range Tail	Dependence (lower, upper)
Gaussian	$\Phi_\rho\left(\Phi^{-1}(u), \Phi^{-1}(v)\right)$	$-1<\rho<1$	$(0,0)$
Student-t	$t_{\rho,n}\left[t_n^{-1}(u), t_n^{-1}(v)\right]$	$-1<\rho<1, n>1$	$\left(2t_{v+1}\left(-\sqrt{\frac{(v+1)(1-\rho)}{1+\rho}}\right), 2t_{v+1}\left(-\sqrt{\frac{(v+1)(1-\rho)}{1+\rho}}\right)\right)$
Clayton	$(u^{-\theta}+v^{-\theta}-1)^{-1/\theta}$	$\theta>0$	$(2^{-1/\theta},0)$
Gumbel	$\exp\left\{-\left((-lnu)^\theta+(-lnv)^\theta\right)^{1/\theta}\right\}$	$\theta\geq 1$	$(0,2-2^{1/\theta})$
Frank	$-\frac{1}{\tau}\ln\left(1+\frac{(\exp(-\theta u)-1)(\exp(-\theta v)-1)}{\exp(-\theta)-1}\right)$	$\theta\in$ R	$(0,0)$
Joe	$1-[(1-u)^\theta+(1-v)^\theta-(1-u)^\tau(1-v)^\theta]^{1/\theta}$	$\theta>1$	$(0,2-2^{1/\theta})$
BB1	$\left\{1+\left[\left(u^{-\theta}-1\right)^\delta+(v^{-\theta}-1)^\delta\right]^{1/\delta}\right\}^{-1/\theta}$	$\theta>0,\delta\geq 1$	$(2^{-1/(\theta\delta)},2-2^{1/\delta})$
BB6	$1-\left\{\exp\left[-(\log(1-(1-u)^\theta)\right]^\delta + \left[-\log(1-((1-v)^\theta)^\delta\right]^{1/\delta}\right\}^{1/\theta}$	$\theta\geq 0,\delta\geq 1$	$(0,2-2^{-1/(\theta\delta)})$
BB7	$1-\left\{1-\left[\left(1-(1-u)^\theta\right)^{-\delta}+\left(1-(1-v)^\theta\right)^{-\delta}-1\right]^{-1/\delta}\right\}^{1/\theta}$	$\theta\geq 1,\delta>0$	$(2^{-1/\delta},2-2^{1/\theta})$
BB8	$\frac{1}{\delta}\left\{1-\left[1-(1-\delta)^\theta\right]^{-1}\left[1-(1-\delta u)^\theta\right]\left[1-(1-\delta v)^\theta\right]\right\}$	$\theta\geq 1, 0<\delta\leq 1$	$(0,2-2^{1/\theta})$

Note: u and v are the CDFs of the standardized residuals from the marginal models, with the values of both u and v lying in the range $[0,1]$; ϕ_ρ is the bivariate standard normal distribution function with the correlation ρ; $\phi^{(}-1)$ is the inverse of the univariate standard normal distribution;$t_{\rho,n}$ is the bivariate Student-t distribution with n DoF and correlation ρ; $t_n^{(}-1)$ is the inverse of the standard univariate Student-t distribution; the correlation coefficient ρ is the Pearson's linear correlation.

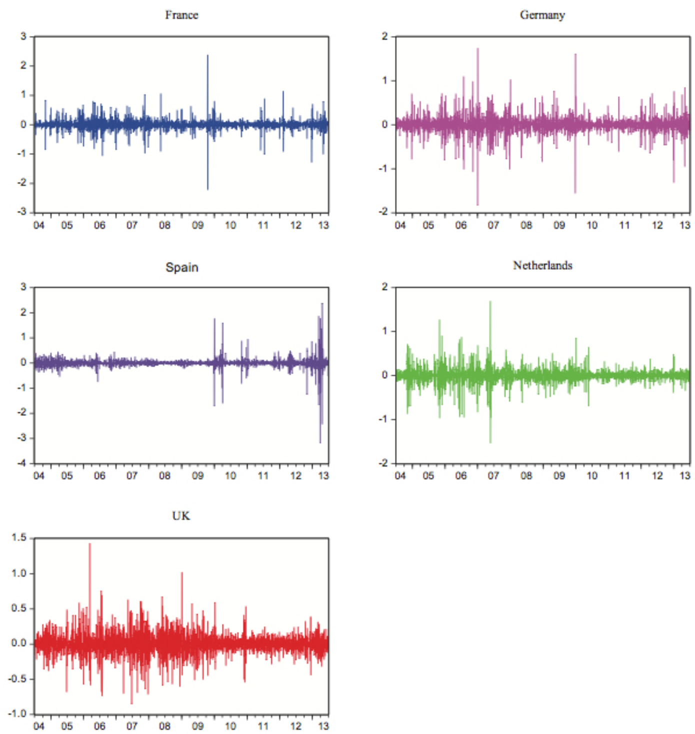

Figure 2: The return of the baseload price for each series

This study, however, does not analyze the level of price directly, but, instead, focuses on the returns of price. Returns are computed as the log-differences in the daily prices, as $r_t = \ln(p_t/p_{t-1})$, where p_t and p_{t-1} are the current and one-period lagged daily electricity prices. These plots clearly show extreme price spikes and volatility clustered for each return. Table 2 presents the descriptive statistics of the daily price returns. The mean of each return is almost close to zero and slightly positive for all the series considered in this paper. Skewness is slightly negative for the German, Spanish, and Dutch returns, and slightly positive for the French and British returns. All the returns are greater than 3 in kurtosis, which denotes a fat-tailed distribution. All the skewness of returns values are not equal to zero, which indicates an asymmetric distribution. The null hypotheses of the normality distribution for all the returns are rejected at the 1% significance level, which shows that normal distribution cannot approximate all the returns. Since the returns are found to have extreme price spikes and are volatility clustered, the unit root test for stationarity is essential. The augmented DickeyFuller (ADF) and the KwiatkowskiPhillipsSchmidtShin (KPSS) unit root tests are used to test the null hypothesis non-stationarity (or unit root) of returns. The ADF and KPSS tests have the opposite null hypotheses: the former has the unit root process as the null hypothesis, whereas the latter has the stationary process as the null hypothesis. The ADF statistics reject the null hypothesis at the 1% significant level of returns. On the other hand, the KPSS statistics do not reject the null hypothesis at the 1% significant level of returns. Therefore, it can be concluded that all the returns are stationary.

Table 2: Descriptive Statistics of Returns from 1 July 2004 to 28 June 2013

	France	Germany	Spain	The Netherlands	UK
Mean	0.0000	0.0001	0.0000	0.0002	0.0004
Maximum	2.3841	1.7356	2.3857	1.6892	1.4286
Minimum	-2.2139	-1.8304	-3.1828	-1.5402	-0.8529
Std. Dev.	0.1869	0.1995	0.2003	0.1685	0.1596
Skewness	0.0544	-0.1577	-1.2271	-0.0012	0.3313
Kurtosis	29.0096	16.1410	67.8199	17.1020	9.1699
JB	66157.0000	16897.0400	411471.8000	19447.4400	3765.5840
Probability	0.0000	0.0000	0.0000	0.0000	0.0000
ADF	-32.1533**	-20.5245**	-18.5637**	-20.2878**	-29.9868**
KPSS	0.0497	0.0676	0.0514	0.0481	0.0479
Observations	2347	2347	2347	2347	2347

4 Empirical Results

4.1 Results for marginal model

First, estimate the AR (k)-Skew-t-GJRGARCH (1,1) models for each return series. p and q are set from zero to a maximum of six lags, and the insignificant (with significant level 5%) autoregressive terms are deleted. AIC and BIC statistics are adopted to select the most suitable models. The estimated values of the marginal models are presented in Table 3. It can be noted that the ARCH parameter α and the GARCH parameter β are statistically significant, implying that conditional heteroskedasticity effects exist for all the return series. Moreover, the return series for the German, Spanish, and British electricity price markets exhibit significant leverage effect, which is drawn by the element η. The p-values of ARCH(10) are more than 10%, which indicates the successful removal of all ARCH-effects from the residual series. The skewness parameter ξ and the degrees of freedom λ of the skewed-t distribution for each return series are significant. Additionally, the degrees of freedom λ of the skewed-t distribution are small, ranging from 2 to 5. These results indicate that the error terms are not normal distribution and that the skewed-t distribution works reasonably well for all the return series.

4.2 Goodness-of-fit for marginal distribution

After filtering the original return series with the appropriate AR-GARCH models, we transformed non-parametrically the resulting standardized residual series to copula data u_{it} using the ECDF. Each copula data u_{it}t should be uniform $(0, 1)$;

Table 3: Estimated Coefficients for Marginal Model

	France	Germany	Spain	The Netherlands	UK
C_0	-0.0003	-0.0006	-0.0015*	0	0.0014*
	-0.006	-0.0004	-0.0007	-0.0004	-0.0006
ϕ_1	0.6434**	0.5576***	0.3255**	0.6383**	0.3523**
	-0.0247	-0.0253	-0.061	-0.023	-0.0279
ϕ_2	0.1063**	0.0678**	-0.0748*	-	0.0486*
	-0.0238	-0.0231	-0.031		-0.0235
ϕ_3	0.0831**	0.0677**	-	0.0877**	0.0449*
	-0.0202	-0.0213		-0.02198	-0.0225
ϕ_4	-	0.0727**	-	-	0.0453*
		-0.0207			-0.0206
ϕ_5	-	0.0330*	-	0.0392*	-
		-0.0175		-0.019	
	-0.9631**	-0.9683**	-0.6403**	-0.9641**	-0.8817**
	-0.0112	-0.0063	-0.06	-0.0085	-0.0194
ω	0.0031**	0.0058**	0.0006**	0.0014*	0.0008**
	-0.0009	-0.0011	-0.0001	-0.0006	-0.0001
α	0.2836**	0.3643**	0.1665**	0.3014**	0.2482**
	-0.0632	-0.0682	-0.0326	-0.0774	-0.0394
β	0.6483**	0.4788**	0.7046**	0.7139**	0.8145**
	-0.0584	-0.053	-0.0289	-0.0763	-0.0222
η	0.1412	0.2222*	0.2785**	-0.0334	-0.1614**
	-0.093	-0.0028	-0.0613	-0.0544	-0.04111
ξ	0.9232	0.8374**	0.8316**	0.9602**	1.3747**
	-0.0268	-0.0243	-0.0232	-0.0259	-0.0393
λ	2.9194	3.3211**	4.3213**	3.347688	4.6055**
	-0.1997	-0.2492	-0.3626	-0.2409	-0.4765
LL	1562.997	1341.973	2255.174	1734.633	1720.303
AIC	-1.3225	-1.1325	-1.9132	-1.4671	-1.4557
BIC	-1.2955	-1.1006	-1.8887	-1.4352	-1.4263
ARCH(10)	0.9947	0.9323	0.8238	0.6752	0.8541

Notes: The table shows the estimates and their standard errors (in parentheses) for the parameters of the marginal distribution model defined in Equation (3) and Equation (4). ** and * denote rejection of the null hypothesis at the 1% and 5% significance levels, respectively. ARCH(10) is the Engel's LM test for the ARCH effect in the residuals up to the 10th order. The total number of observations for each series is 2,347.

otherwise, the copula model could be mis-specified. By following the procedures discussed in Patton [15] and Reboredo [16], this paper used two steps to test u_{it}. First, a LjungBox (LB) test was used to examine the serial correlation under the null hypothesis of serial independence. To do so, $(u_{it} - \bar{u}_i)^k$ is regressed on the first 10 lags of the variables. Second, the KolmogorowSmirnov (KS) test is used to test the null hypothesis that the u_{it} are uniform (0,1). Table 4 reports the p-values of these tests. At the 5% significant level, all the null hypotheses are rejected, which implies that the marginal distribution model can demonstrate quite capably that these are not mis-specified. Hence, it can be safely concluded that the copula model can correctly capture the dependence between the six electricity markets.

Table 4: Goodness-of-fit Test for Marginal Distributions

	First moment LB test	Second moment LB test	Third moment LB test	Fourth moment LB test	KS test
France	0.1364	0.2603	0.1572	0.1673	1.0000
Germany	0.1904	0.0500	0.2314	0.1852	0.9991
Spain	0.1935	0.2027	0.7339	0.3195	0.9991
The Netherlands	0.2527	0.2186	0.8718	0.2272	0.9762
UK	0.8071	0.0592	0.9351	0.0563	1.0000

Notes: This table reports the p-values from the LjungBox (LB) tests for the serial independence of the first four moments of the variables$u_{i,t}$. We regress $(u_{i,t} - \bar{u}_i)^k$ on the first 10 lags of the variables for$k = 1, 2, 3, 4$. In addition, we present the p-values of the KolmogorowSmirnov (KS) test for the adequacy of the distribution model.

Table 5: Empirical Kendall's Matrix and Sum of Absolute Entries of Each Row for European Electricity Copula Data

	France	Germany	Spain	The Netherlands	UK	$\hat{\tau}_{\text{sum}}$
France	1.0000	0.4110	0.1410	0.4970	0.1940	2.2430
Germany	0.4110	1.0000	0.0961	0.4430	0.1700	2.1201
Spain	0.1410	0.0961	1.0000	0.0795	0.0310	1.3476
The Netherlands	0.4970	0.4430	0.0795	1.0000	0.1900	2.2095
UK	0.1940	0.1700	0.0310	0.1900	1.0000	1.5850

4.3 Selection and estimation of vine models

Figure 2 displays the scatter plots of the transformed standardized residual series, and it highlights the positive dependence within the transformed standardized residual series. This paper used the C-vine structure selection criterion described by Czado et al. [18] to select an appropriate C-vine copula model for the European electricity copula data. Table 5 describes the empirical Kendall's correlation matrix of the copula data and the sum of their values, denoted by $\hat{\tau}_{\text{sum}}$ and defined in the equation. Table 4 demonstrates that the France series has the maximum sum

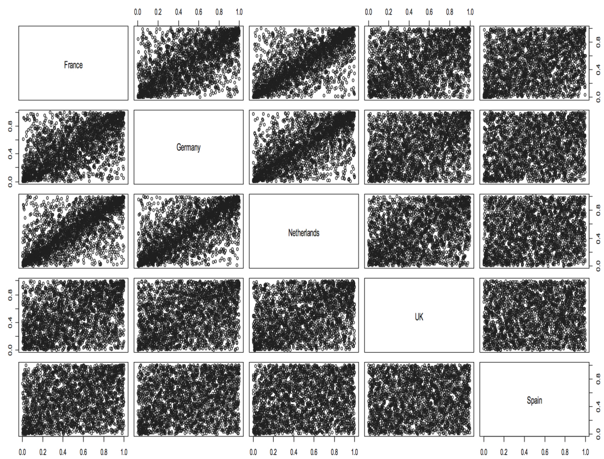

Figure 3: The scatter plots of the European electricity prices with standard normal margins

of the absolute values, $\hat{\tau}_{\text{sum}}$, and hence it is placed as the pilot variable in level 1 of the vine structure. Table 5 describes the empirical Kendall's matrix of the series, conditioned on France, and denoted by i^*, and the sum of the absolute entries of each row. In Table 6, we observe that the Germany series has the maximum sum of the absolute values, $\hat{\tau}_{\text{sum}}$, and hence it is placed as the pilot variable in level 2 of the vine structure. Following the same identification procedure, the permutation of the electricity copula data for the mixed C-vine structure is specified as (France, Germany, the Netherlands, UK, Spain) = (1, 2, 3, 4, 5).

Table 6: Goodness-of-fit Test for Marginal Distributions

	Germany$\|i^*$	Spain$\|i^*$	The Netherlands$\|i^*$	UK$\|i^*$	$\hat{\tau}_{\text{sum}}$
Germany$\|i^*$	1.0000	0.3815	0.1137	0.0201	1.5153
Spain$\|i^*$	0.2492	1.0000	0.0777	-0.0107	1.3375
The Netherlands$\|i^*$	0.0725	0.0777	1.0000	-0.0100	1.1602
UK$\|i^*$	0.0128	-0.0107	-0.0100	1.0000	1.0335

The resulting C-vine copula model and the sequential and maximum likelihood estimates are presented in Table 7. Regarding the copula selection, we used 10 different copula types to select the 10 in the total number of different pair-copulas in the C-vine model, and the selection of the appropriate copula types was based on the AIC and BIC criteria. The copula type with the smallest AIC/BIC value is chosen. Additionally, we also carried out the independence test for the C-vine model. The independence copula is selected for pair-copulas that cannot reject the null hypothesis of independence. From Table 7, it is evident that the sequential estimates are extremely close to the maximum likelihood estimates for all the estimated models, which is an indication that the employment of sequential estimation as the preferred optimization method is appropriate.

From the sequential selection, we note that the dependence between the French market and the other markets is the strongest and that the dependence between the Spanish market and the other markets is the weakest. From Table 7, we also understand that the French market co-movements are affected, to a large extent, by the neighboring German market. Between the French, German, and Dutch markets, especially, exist extreme co-movement. However, the relationship between the various markets becomes smaller, and it even becomes independent, when the conditions increase. For example, the relationship between Germany and Spain is independent under the condition of France. Additionally, the Dutch and Spanish markets do not have co-movement upon the condition of the French and German markets. Furthermore, the British and Spanish markets are independent upon the condition of the French, German, and Dutch markets. The Dutch and British markets are still dependent upon the condition of the German and French markets.

Table 7: Estimated Parameters of C-vine Copula

Level	Block	Family	Parameter	$\theta^{\hat{\text{SE}}}$	std. error	$\theta^{\hat{\text{MLE}}}$	λ_L	λ_U
1	$C_{1,2}$	Student-t	$\hat{\rho}$	0.5955	0.0139	0.6026	0.2786	0.2786
			$\hat{\nu}$	4.7606	0.5527	4.7668		
	$C_{1,3}$	Student-t	$\hat{\rho}$	0.7085	0.0117	0.7144	0.4823	0.4823
			$\hat{\nu}$	2.5390	0.1890	2.6530		
	$C_{1,4}$	Frank	$\hat{\theta}$	1.8124	0.1281	1.8190	0.0000	0.0000
	$C_{1,5}$	Student-t	$\hat{\rho}$	0.2269	0.0206	0.2282	0.0117	0.0117
			$\hat{\nu}$	12.4931	3.9000	12.4929		
2	$C_{2,4\|1}$	Student-t	$\hat{\rho}$	0.3815	0.019	0.3796	0.1173	0.1173
			$\hat{\nu}$	6.0683	0.7755	6.0714		
	$C_{2,3\|1}$	Student-t	$\hat{\rho}$	0.1173	0.0212	0.1205	0.0012	0.0012
			$\hat{\nu}$	17.5849	6.3332	17.5851		
	$C_{2,5\|1}$	Independence	$\hat{\rho}$	0.0000	0.0000	0.0000	0.0000	0.0000
3	$C_{3,4\|1,2}$	BB8	$\hat{\theta}$	1.1890	0.1380	1.1886	0.0000	0.0000
			$\hat{\delta}$	0.8700	0.1511	0.8695		
	$C_{3,5\|1,2}$	Independence	$\hat{\rho}$	0.0000	0.0000	0.0000	0.0000	0.0000
4	$C_{4,5\|1,2,3}$	Independence	$\hat{\rho}$	0.0000	0.0000	0.0000	0.0000	0.0000

Note: (France, Germany, Netherlands, UK, Spain) = (1, 2, 3, 4, 5).

5 Conclusion

This paper employed a combination canonical copula based GARCH model for modeling the dependence structures of European electricity markets. The five

European power markets are France, Germany, Spain, the Netherlands, and UK. The data consist of daily spot electricity prices for the base period, which is from 1 July 2004 to 28 June 2013. We used the mixed canonical copula based GARCH model approach and provided ten kinds of bivariate copula models for the selection.

Conditional dependence is used to measure spillover effects, that is, how the volatility of one market affects another market, conditional on the event. The conditional tail dependence is applied to investigate a particular market experiencing spikes and drops, conditional on the event that another market is also experiencing spikes and drops. The empirical results show that there is positive spillover effect between the French and the other four markets. Second, strong symmetric tail dependence exists between France, Germany, and the Netherlands. This suggests one market (one from among the French, German, and Dutch markets) experiencing spikes or drops, conditional on the event that the other two markets are also experiencing spikes or drops. When adding the condition of under one or more markets, relationships of some pairs are still found to have dependence, while correlations of some pairs are observed to become independent. For example, the relationship between Germany and Spain has independence under the condition of France. Additionally, the Dutch and Spanish markets do not have co-movement upon the condition of the French and German markets. The Dutch and British markets are still found to have dependence upon the condition of the German and French markets.

References

[1] Joe, H., 1996, Families of m-variate distributions with given margins and m(m1)/2 bivariate dependence parameters, in L. Rschendorf, B. Schweizer, and M. D. Taylor, eds., Distributions with fixed marginals and related topics, 120141 (Hayward: Institute of Mathematical Statistics, Hayward, CA).

[2] Bedford, T., and R. M. Cooke, 2001, Probability density decomposition for conditionally dependent random variables modeled by vines, Annals of Mathematics and Artificial Intelligence 32, 245268.

[3] Bedford, T., and R. M. Cooke, 2002, Vines a new graphical model for dependent random variables, The Annals of Statistics 30, 10311068.

[4] Kurowicka, D., and R. M. Cooke, 2006, Uncertainty Analysis with High Dimensional Dependence Modelling, first edition (Wiley).

[5] Aas, K., Czado, C., Frigessi, A., Bakken, H., 2009. Pair-copula constructions of multiple dependence. Insurance: Mathematics and Economics, 44(2), 182-198.

[6] Low, R.K.Y., Alcock, J., Robert Faff, R., Brailsford, T., 2013. Canonical vine copulas in the context of modern portfolio management: Are they worth it? Journal of Banking & Finance, 37, 3085-3099.

[7] Mendes, V.d.M.B., Mendes, S. M., Leal, P.C.R., 2010. Pair-copulas modeling in finance, Financial Markets and Portfolio Management 24, 193213.

[8] Kim, D., Kim, J.M., Liao, S.M., Yoon-Sung Jung, Y.s., 2013. Mixture of D-vine copulas for modeling dependence. Computational Statistics and Data Analysis, 64, 1-19.

[9] Brechmann, E.C., Schepsmeier, U., 2013. Modeling Dependence with C- and D-Vine Copulas: The R Package CDVine. Journal of Statistical Software, 52(3).

[10] Emmanouil, K.N., Nikos, N., 2012. Extreme Value Theory and mixed Canonical vine Copulas on modelling energy price risks. Working paper.

[11] Sklar, A., 1959, Fonctions de répartition à n dimensions et leurs marges, Publications del'Institut de Statistique de L'Université de Paris, 8, 229231.

[12] Diβmann, J., Brechmann, E.C., Czado, C., Kurowicka, D., 2013. Selecting and estimating regular vine copulae and application to financial returns. Computational Statistics and Data Analysis, 59, 52-69.

[13] Nikoloulopoulos, A.K., Joe, H., Li, H., 2012. Vine copulas with asymmetric tail dependence and applications to financial return data. Computational Statistics and Data Analysis, 56, 3659-3673.

[14] Brechmann, E. C., Czado, C., 2011. Risk management with high-dimensional vine copulas: An analysis of the euro stoxx 50, Working paper, Technische Universitt Mnchen, http://mediatum.ub.tum.de/doc/1079276/1079276.pdf.

[15] Fernandez, C., Steel, M.F.J., 1998. On Bayesian modeling of fat tails and skewness. Journal of the American Statistical Association 93, 359371.

[16] Patton, A., 2006. Modeling asymmetric exchange rate dependence. International economic review 47(2), 527-556.

[17] Reboredo, J., 2011. How do crude oil prices co-move? A copula approach. Energy Economics 33, 948-955.

[18] Czado, C., U. Schepsmeier, and A. Min, 2011, Maximum likelihood estimation of mixed C-vines with application to exchange rates, To appear in Statistical Modelling.

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