Thai Journal of Mathematics (2014) 73–90
Special Issue on : Copula Mathematics and Econometrics
http://thaijmath.in.cmu.ac.th
Online ISSN 1686-0209

Investigation of the Dependence Structure Between Imports and Manufacturing Production Index of Thailand using Copula-Based GARCH Model

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Abstract : This paper aims at investigating the dependence structure between the imports and the manufacturing production index of Thailand using a copula-GARCH approach. We applied skewed student-t distribution to estimate all of the marginal distributions with ARMA(1,12)-GARCH(1,1), ARMA(1,2)-GARCH(1,1) to fit the manufacturing production index (MPI) and the imports of Thailand. The results of this paper suggest that the student-t copula is the most appropriate method to best fit the tail dependences in both static and time-varying copulas because the AIC and the BIC of this method are the lowest when compared with the candidates among the other types of copula.

Keywords : MPI; imports; ARMA-GARCH; copula; dependence structure; volatility mappings
2010 Mathematics Subject Classification : 62P20; 91B84 (2010 MSC)

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1 Introduction

In the past 20 years, both the World and the Thailand economy have enormously fluctuated. Thailand has been seriously affected by the volatility of the global economy and by the various economically impacted events. The Asian financial crisis, also known as Tom Yum Kung Crisis, rapidly spread across Southeast Asia and the East Asia region. The crisis started in Thailand in July 1997. The Thai government announced the floating of the Baht value after several efforts to support its fixed exchange rate pegging at the US dollar; however, this did not put an end to the problems. Thailand had accumulated a considerably big burden of foreign debt that made the country effectively bankrupt even before the actual collapse of its economy[2]. Later, Thailand had to ask for help from the International Monetary Fund (IMF), on August 11, 1997. This event caused panic throughout the region and also the World. Indonesia, South Korea, and Thailand were some of the most affected countries. Although the global economy started to recover in 2003, Thailand still faced other problems that continuously caused the fluctuation of its economy, such as the 2004 Indian Ocean tsunami, resulting in Thailand's economic retardation, and the Severe Acute Respiratory Syndrome (SARS), resulting in a significant decline in tourists coming to Thailand. With the reduction in income from tourists, Thailand's GDP slightly reduced. The Subprime mortgage crisis, or the United States Housing Bubble, was another major financial crisis dragging the global economy back into a downturn, again, in 2008. This time, many of the US financial institutions went bankrupt, while others had to face the problem of liquidity and had to find financial support for further operations. Next, Thailand faced a severe flooding during 2011. The World Bank estimated 1,425 billion Baht in economic damage and losses because of this flooding[3]. All the problems mentioned earlier clearly influenced the economic volatility and the economic growth of Thailand in different ways, including the effects on the GDP, the imports, the exports, the investment, the employment, and the government expenditure.

At present, we can determine the economic growth of any country by the calculation of the volume of the various national produce via the Gross Domestic Production (GDP), which means that if any country has high economic growth, the country's GDP will be good, as well. On the contrary, if any country has low economic growth, the country's GDP will also be less. However, recently, the alternative of the measurement of economic growth has changed. We can use an index called the manufacturing production index (MPI) which can represent the use of the GDP also. This index serves as an indicator for the volume of production as well as the direction of the manufacturing sector. The higher the MPI is, the greater the expansion of the economy.

[2] `http://www.euromoney.com/Article/1005746/When-the-world-started-to-melt.html.`

[3] `http://www.worldbank.org/en/news/2011/12/13/world-bank-supports-thailands-post-floods-recovery-effort,World Bank.13 December 2011.[online; cited January 2012].`

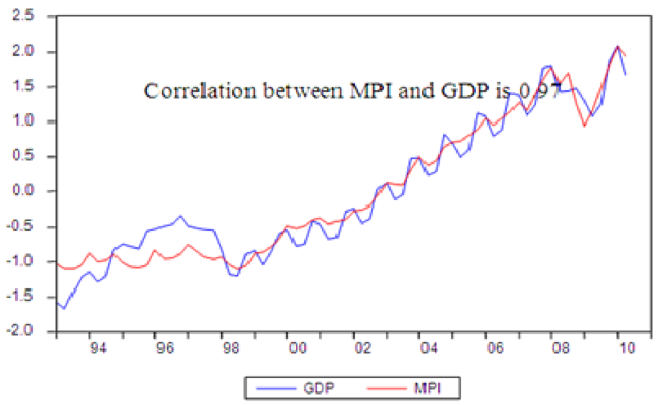

Figure 1: The correlation between the MPI and the Gross Domestic Production of Thailand.

Upon referring to Figure 1, it can be seen that the GDP and the MPI of Thailand are closely related since the industrial outputs make up a proportion of 40% of GDP, which activates the largest proportion of the test correlation between the MPI and the GDP as equal to 0.97, which is very high. Thus, MPI could be a good representative of GDP.

An increase in the MPI will have two impacts on the imports. Firstly, if the MPI increases, it implies that there must be more production in the industrial sector. In the manufacture of certain products, the manufacturers, however, have to import foreign materials known as intermediate materials to generate the final products. The import volume of the intermediate materials will increase the total volume of imports. Secondly, in terms of consumption, if the MPI increases, it means that the country has produced more and more products, and, therefore, the employment will expand and the public revenues will increase, resulting in increased consumption. Some of the products that consumers need might have to be imported from other countries. This will also increase the total volume of imports.

Thus, the purpose of this study was to investigate the relationship between the MPI and Thailand's imports, and how it affected Thailand's economic expansion as well as Thai income using a copula based on the GARCH models.

The generalized autoregressive condition heteroskedasticity (GARCH) model and its extended models are very popular econometric tools to study the volatility of time series; in addition, several studies have used copulas based on ARMA-GARCH to estimate marginal distribution. Wang [18] examined a copula-based GARCH dependence model of the Shanghai and Shenzhen stock markets, and they found that a copula is able to clearly describe the dependence of the marginal distributions and pass over the limitations of the marginal distributions. In addition, t-copula gives the best explanation for the dependence, not only for the two stock returns but also for the tail dependence. Patton [14] utilized the modeling asymmetric exchange rate dependence by testing the asymmetric dependence between Deutsche Mark of Germany and Yen of Japan with the US dollar, separate pre-Euro and post-Euro. The results showed that the Mark-US dollar and the Yen-US dollar exchange rates were more correlated when they were depreciating against the Dollar than when they were appreciating. Ning [13] applied the copulas to explain the dependence structure between the equity market and the foreign exchange market. The relationship between the equity market and the currencies of the five most developed countries - the United States, the United Kingdom, Germany, France, and Japan - was studied, and the significance in all the return pairs was observed; it was observed that the dependences were still significant but weaker after the launch of the Euro. Aloui [1] examined the conditional dependence structure between oil prices and US exchange rate by a copula-GARCH approach. It showed that all the oil-exchange rate pairs were strongly significant and symmetric. The depreciation of the US dollar was attributed to the increase in oil price. Wu [19] studied the economic value of the co-movement between oil price and exchange rate using copula-based GARCH models. The result of this paper demonstrated that crude oil and US dollar exhibit an asymmetric, or tail dependence, structure. More risk-averse investors were willing to pay higher fees to change from a static strategy to a dynamic strategy based on the copula-based GARCH models. Li [10] applied the GARCH-copula approach to estimate the value at risk for portfolios. It was found that a GJR model with a skewed student-t distribution was a better fit for the dependence structure than a GARCH model, and after estimating the parameters, the Monte Carlo simulations were implemented to estimate the Value at Risk. Chinnakum [5] determined the relationship between the GDP, imports, and exports in Asian countries using a copula-VAR analysis. The results showed that there was no casual relationship between export and GDP growth for Laos and Philippines, while the results for Singapore, Indonesia, and Vietnam approved only the ELG hypothesis, and not the GLE hypothesis.

The remainder of this article is organized as follows: section 2 presents preliminaries. Section 3 discusses main results. Section 4 shows conclusion.

2 Preliminaries

This paper applied the ARMA-GARCH and copula approach to simultaneously estimate the correlation between the dependence structures of imports and manufacturing production index of Thailand. The copula approach was first described by Sklar [16] and Nelsen [12], and it has been widely used to investigate dependence between variables. In addition, it has been extensively analyzed, applied, and solved in financial, econometrics, and statistics problems. The calculation methodology is begun by using the skewed student-t distribution with the ARMA-GARCH model to estimate the marginal distributions. We took the standard residuals into the uniform [0,1], and then used the copula function to link the marginal distributions simultaneously in order to analyze the dependence between the manufacturing production index (MPI) and the imports of Thailand.

2.1 Skewed student-t distribution

Fernandez and Steel [8] made use of a skewed student-t distribution which displays both flexible tails and possible skewness, each entirely controlled by separate scalar parameters. The formula of the skewed student-t distribution is presented as

$$P(b_i|v,\gamma) = \frac{2}{\gamma + \frac{1}{\gamma}} \left\{ f_v\left(\frac{b_i}{\gamma}\right) I_{[0,\infty)}(b_i) + f_v(\gamma b_i) I_{(-\infty,0)}(b_i) \right\} \tag{2.1}$$

where $f_{v(\cdot)}$ is unimodal and symmetric around 0,γ is the skewness parameter that is assigned,$\gamma \in (0,\infty)$, I indicates the indicator function, and v is the degree of freedom. Arnold and Groeneveld [2] put forward a proposal about the skewness measures, as follows:

$$SM(b|\gamma) = \frac{\gamma^2 - 1}{\gamma^2 + 1} \tag{2.2}$$

If $\lim_{\gamma\to 0} SM(b|\gamma) = -1$, it means that there exists extreme left skewness, if $\lim_{\gamma\to 0} SM(b|\gamma) = 1$, it means that there exists extreme right skewness, and if $\gamma = 1$, there exists no skewness.

2.2 ARMA-GARCH model

In this section we describe the GARCH model which was developed by Bollerslev [3]. It is an expansion of the ARCH progress generated by Engle [7] in 1982. We selected GARCH to investigate the volatility of the manufacturing production index (MPI) and the imports of Thailand. We proposed ARMA-GARCH which combines the autoregressive (AR) and the moving average (MA) terms to find the marginal distributions and the standard innovations that accept the skewed student-t distribution. The GARCH (p,q) approach includes the p lags of the conditional variance in the linear and ARCH (q) condition variance equation, which is shown in the following equations as

$$Y_t = c + \sum_{i=1}^{k} \lambda_i Y_{t-i} + \sum_{i=1}^{l} \varphi_i \epsilon_{t-i} + \epsilon_t \tag{2.3}$$

$$\epsilon_t = h_t \eta_t \tag{2.4}$$

$$h_t^2 = \omega + \sum_{i=1}^{p} \alpha_i \epsilon_{t-1}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}^2 \tag{2.5}$$

where h_t^2 is the conditional variance of ϵ_t and η_t are the standard residuals which are i.i.d. with mean 0 and variance 1. For $\sum_{i=1}^{k} \lambda_i < 1$, $\omega > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, and $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1$. According to Svetlozar [17], η_t is not always realistic. If the distribution of the historical innovation $\eta_{t-n}, ..., \eta_t$ is heavier tailed than the normal, one can adjust the model and allow for a heavy-tailed distribution, specifically, a skewed student-t distribution.

2.3 Copula models of conditional dependence structure

The copula model is adopted to explain the symmetric or asymmetric dependence structure and the co-movement between the variables. There are two copula families: elliptical copulas (such as the Gaussian copula and the student-t copula) and the Archimedean copulas (such as the Gumbel copula, the Frank copula, the Clayton copula, and the Rotated Clayton copula). All of them are widely used in financial time series data. We can show as many copulas as possible.

- **Gaussian copula**

The Gaussian Copula is presented in the following form:

$$C_g(u,v) = H(\phi^{-1}(u), \phi^{-1}(v)) \tag{2.6}$$

$$C_\rho(u,v) = \int_{-\infty}^{\phi^{-1}(u)} \int_{-\infty}^{\phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} exp\left[-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right] dxdy \tag{2.7}$$

where ρ is the Pearson correlation between u and v, and are accepted from uniform [0,1], the coefficient being restricted to the interval (-1,1) which are the standard residuals for marginal distributions. The Gaussian copula is symmetric, in addition to the fact that there is no tail dependence.

- **Student-t copula**

Like the Gaussian copula, the student-t copula is an elliptical copula but it can capture the extreme dependence which is symmetric extreme dependence. The student-t copula is defined as

$$C(u,v) = \int_{-\infty}^{t_v^{-1}(u)} \int_{-\infty}^{t_v^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} \left[1 + \frac{x^2 - 2\rho xy + y^2}{v(1-\rho^2)}\right]^{-\frac{v+2}{2}} dxdy \tag{2.8}$$

where $t_v^{-1}(u)$ indicates the inverse of the CDF of the (univariate) student-t distribution function.ρ is the Pearson correlation which is the same as the Gaussian copula and v is the degree of freedom. If v is large enough, the student-t copula results will be similar to those of the Gaussian copula.

- **Clayton copula**

The Clayton copula was presented by Clayton [6], and is shown as

$$C(u,v) = (u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}, \theta \in (0,\infty)$$

where $\theta > 0$ to satisfies the decreasing means, which is able to reflect the lower tail dependence.

- **Rotated-Clayton copula**

In order to generate the upper tail dependence, we can use a particular copula which is called the rotated-copula; it is defined as

$$C_\theta^R = \left[(1-u)^{-\theta} + (1-v)^{-\theta} - 1\right]^{-\frac{1}{\theta}} \tag{2.9}$$

If (u, v) has copula $C_\theta(u, v)$, then $(1-u, 1-v)$ is distributed according to the rotated copula $C_\theta^R(u, v)$. The rotated copula has dependence in the upper tail (see Jondeau [9]).

- **Frank copula**

The Frank copula can be presented as

$$C_\theta^F = -\frac{1}{\theta}\ln\left[\frac{1+(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right] \tag{2.10}$$

and

$$C_\theta^F(u, v) = \frac{\theta(1-e^{-\theta})e^{-\theta(u+v)}}{\left[(1-e^{-\theta})-(1-e^{-\theta u})(1-e^{-\theta v})\right]^2} \tag{2.11}$$

The above function is the only Archimedean copula that satisfies the functional equation $C(u, v) = \widehat{C}(u, v)$. Then the marginal distribution becomes radially symmetric; further, $\theta \in (-\infty, +\infty)\,\{\theta\}$ also holds true.

- **Gumbel copula**

The Gumbel copula is presented as follows:

$$C_\theta^G(u, v) = \exp\left\{-\left[(-\ln u)^\theta + (-\ln v)^\theta\right]^{\frac{1}{\theta}}\right\}, \theta \in [1, \infty) \tag{2.12}$$

The Gumbel copula is an asymmetric copula of the Archimedean family. This family allows for upper tail dependence. Extreme dependence measures are functions of the Gumbel copula (see Liu [11]).

- **Rotated Gumbel copula**

The rotated Gumbel family of copulas is shown as

$$C_\theta^R = u + v - 1 + C_\theta(1-u, 1-v) \tag{2.13}$$

2.4 Time-varying in conditional copulas

Patton [14, 15] assumes that the functional form of the copula remains fixed over the sample whereas the parameters vary according to some evolutional equation. Moreover, because it is very difficult to know what might influence it to change, he proposed the Gaussian and SJC copulas to estimate and investigate the upper and lower tail dependence parameters, following something akin to a restricted ARMA(p, q) process. According to Patton's paper, it was found that there existed an autoregressive term and a forcing variable. In our paper, we used the mean absolute difference between u_t and v_t over the previous 12 and 2 observations as a forcing variable; besides that, we suggested two methods of employing the time-varying copula. The following are the two time-varying copulas suggested by us:

- **Time-varying Gaussian copula**

The first copula considered is the time-varying Gaussian copula, of which the evolution equation can be given as

$$\rho_t = \tilde{A}\Big(\omega_p + \beta_{N1} \cdot \rho_{t-1} + ... + \beta_{Np} \cdot \rho_{t-p} + \alpha_N \cdot \frac{1}{q} \sum_{j=1}^{q} \Phi^{-1}(u_{t-j}) \cdot \Phi^{-1}(v_{t-j})\Big) \quad (2.14)$$

where $\tilde{A}(x)$ is the logistic transformation which is determined as $\tilde{A}(x) \equiv (1 - e^{-x})(1 + e^{-x})^{-1}$, ρ_t is the correlation coefficient, as well as $\rho_t \in (-1, 1)$ at all times.

- **Time-varying Student-t copula**

The second copula considered is the time-varying student-t copula, which is the last family to be presented in this paper:

$$\rho_t = \tilde{A}\Big(\omega_p + \beta_{T1} \cdot \rho_{t-1} + ... + \beta_{Tp} \cdot \rho_{t-p} + \alpha_T \cdot \frac{1}{q} \sum_{i=1}^{q} \Phi^{-1}(u_{t-i}; v) \cdot \Phi^{-1}(v_{t-i}; v)\Big) \quad (2.15)$$

There are two parameters, ρ, v, which are the Pearson correlation coefficient and the degree of freedom, respectively. Our supposition is that not only the degree of freedom but also the correlation can change with time.

2.5 Canonical Maximum Likelihood (CML) method

The copula parameters may be estimated without specifying the margins; on the other hand, another method consists of transforming the sample data into uniform $[0, 1]$ and then estimating the copula parameters (Cherubini [4]). This method may be described as follows. First estimate the marginal using the empirical distribution $G_i(x_{it})$ with $i = 1, \ldots, n$. Second,estimate using the Full Maximum Likelihood method the copula parameters.

$$\hat{\theta}_{CML} = \arg\max_{\theta} \sum_{t=1}^{T} \ln c(\hat{G_1}(x_{1t}), \ldots, \hat{G_n}(x_{nt}); \theta_{CML}) \quad (2.16)$$

After estimating using the ARMA-GARCH model, we obtained the marginal distribution and the standard residual from each dependence measure. Furthermore, we made standard residuals transform into uniform (u and v) and T = observation.

$$\hat{u}_t = \frac{1}{T+1}\sum_{t=1}^{T} 1_{\eta_{1t} \leq \eta_1} \tag{2.17}$$

$$\hat{v}_t = \frac{1}{T+1}\sum_{t=1}^{T} 1_{\eta_{2t} \leq \eta_2} \tag{2.18}$$

3 Main Results

Our paper used the monthly the manufacturing production index (MPI) and the import data of Thailand starting January 1995 to August 2012, volume of the observation being 212. The data sources were the Bank of Thailand as well as the office of the industrial economics of Thailand. To decrease the problem of the data becoming stationary, the monthly data was transformed into the log-difference of each data; thus,

$$Y_t = 100 * (log(g_t) - log(g_{t-1})) \tag{3.1}$$

where g_t are the manufacturing production index (MPI) and the imports of Thailand at period t, respectively.

Table 1: Data Description and Statistics of Manufacturing Production Index (MPI) and Imports of Thailand

Statistics	MPI	Imports of Thailand
Mean	0.004062	0.006380
Median	0.005974	0.000967
Maximum	0.336898	0.278586
Minimum	-0.406959	-0.278700
Std. Dev.	0.076736	0.101632
Skewness	-0.849486	0.168155
Kurtosis	8.437819	2.820490
Jarque-Bera	286.698100	1.283732
Probability	0.000000	0.526309
ARCH LM statistic	46.515556	9.873827
Observation	212	212

Source: Computation.

Table 1 shows the summary statistics of each log-difference of these data. This table presents all the mean of the log-differences that are small relative to the standard deviation of each series. Both the series also display excess kurtosis which is positive. This means that the distribution of the two series has larger, thinner tails than normal distribution and negative skewness, except that the skewness of the imports of Thailand is positive. Therefore, this indicates that the distribution for the imports of Thailand is slightly skewed to the right. Generally, the Jarque-Bera statistics are large, which indicates that the choice of skewed-t distribution is justified in our paper; moreover, the ARCH LM test for all of these specifies a reject of null hypothesis at the 0.05 level.

Table 2 presents the unit root test at I(0) of all the log-differences of the series. We used the ADF-test (Augment Dickey-Fuller test) and the PP-test (Phillips and Person test) for computing the stationary value of each log-difference; all of these are significant of unit root in every class of data.

Table 2: Unit Root Test at I(0) of Variables

Variables	Statistics	With intercept	Trend and intercept	None
MPI	ADF-Test	-3.576***	-3.703***	-3.278***
	PP-Test	-29.777***	-31.530***	-24.560***
Imports of Thailand	ADF-Test	-7.754***	-7.807***	-7.646***
	PP-Test	-23.407***	-23.498***	-23.018***

Note: Significant code: "***" 0.01

3.1 Results for ARMA-GARCH model

After testing for the stationary values, as the next step we should estimate the volatility of each variable: the ARMA(1,12)-GARCH(1,1) and ARMA(1,2)-GARCH
(1,1) are employed in computing the values of the log-difference of the manufacturing production index (MPI) and the imports of Thailand, respectively. In addition, we hypothesize that the margins are skewed-t distributions. The estimates of the marginal distributions are presented in Table 3 and Table 4. The manufacturing production index (MPI) margin is the parameter that is mostly significant expect at MA(4), ω, and α. In the case of the imports of Thailand margin, as given in Table 4, the parameter is mostly significant expect at AR(1), and α is insignificant. In both the marginal distributions, we found that the values of γ are strongly significant and less than 1, which indicates that the log-differences of the manufacturing production index (MPI) and the imports of Thailand are skewed to the left. The parameter β is significant, thereby demonstrating that the conditional volatility is very consistent over time.

Table 3: Results of Manufacturing Production Index (MPI) of Thailand Using ARMA(1,12)-GARCH(1,1) Model

	Parameter	Std.error	t-statistic	P-value
$AR(1)$	-0.275315	0.084871	-3.244	0.001179**
$MA(1)$	-0.175002	0.066866	-2.617	0.008865**
$MA(2)$	0.082858	0.037453	2.212	0.026944*
$MA(3)$	0.065875	0.039941	1.649	0.099086.
$MA(4)$	0.058361	0.037476	1.557	0.119401
$MA(5)$	-0.124688	0.038802	-3.213	0.001312**
$MA(6)$	0.108021	0.032222	3.352	0.000801***
$MA(7)$	-0.146454	0.033024	-4.435	9.22e-06***
$MA(8)$	0.090153	0.029039	3.105	0.001906**
$MA(9)$	-0.066958	0.030396	-2.203	0.027604*
$MA(10)$	0.184476	0.035470	5.201	1.98e-07***
$MA(11)$	-0.303083	0.042767	-7.087	1.37e-12***
$MA(12)$	0.705413	0.054909	12.847	$< 2e-16$***
ω	0.001651	0.001218	1.356	0.175061
α	1.000000	0.685584	1.459	0.144672
β	0.452486	0.175697	2.575	0.010013*
γ	0.866182	0.048669	17.797	$< 2e-16$***
v	2.271379	0.242620	9.362	$< 2e-16$***

Note: Signif. codes: 0 "***", 0.001 "**", 0.01 "*", 0.05 ".", 0.1 " ".
Log likelihood: 358.32

Table 4: Result of Imports of Thailand Using ARMA(1,2)-GARCH(1,1) Model

	Parameter	Std.error	t-statistic	P-value
$AR(1)$	0.095260	0.143717	0.663	0.507438
$MA(1)$	-0.778162	0.121611	-6.399	1.57e-10***
$MA(2)$	0.450380	0.082129	5.484	4.16e-08***
ω	0.002129	0.001268	1.680	0.092973.
α	0.117370	0.072817	1.612	0.106996
β	0.632952	0.179850	3.519	0.000433***
γ	0.832204	0.076320	10.904	$< 2e-16$***
v	5.459118	2.734692	1.996	0.045907*

Note: Signif. codes: 0 "***", 0.001 "**", 0.01 "*", 0.05 ".", 0.1 " ".
Log likelihood: 225.13

3.2 Results of Kolmogorov-Smirnov and Box-Ljung Tests

Before the copula estimation, we obtained all the parameters based on the ARMA-GARCH model; in addition to that, we can find the standard residuals from these models to ascertain whether the marginal distributions can form joint copula models. In case the marginal distributions are not valid, their probability values are not going to be i.i.d. uniform $[0, 1]$; then, the copula will not be mis-specified (see Ning [13]). For this part, we proposed the Box-Ljung test to evaluate and verify whether both the univariate distributions still suffered the autocorrelation or not; the Komogorov-Smirnov (K-S) test was also applied to investigate whether the uniform (u and v) from each univariate distribution have been transformed to appropriate i.i.d. uniform $[0, 1]$ or not. Table 5 presents the p-value from the Kolmogorov-Smirnov test for the fit of the distribution model, with all of them also passing the statistical test at the 0.05 level.

Table 5: Kolmogorov-Smirnov Uniform Distribution Test

K-S Test of Both Margins of Uniform	Statistics	P-value
Margins 1 MPI (u)	0.0283	0.9957
Margins 2 Imports of Thailand (v)	0.0047	1.0000

Source: Computation.

Table 6 shows the p-values from the Box-Ljung test of serial independence of the first four moments of the u and v; all the parameters pass the statistical test at the 0.10 level. This means that at each marginal distribution, there is no autocorrelation.

Table 6: Box-Ljung Test of Both Margins for Autocorrelation

Marginals	Level	X-squared	P-value
Marginal 1	First Moment	7.3625	0.1950
Manufacturing	Second Moment	4.0922	0.5362
Production	Third Moment	9.3309	0.9657
Index (v)	Fourth Moment	4.5521	0.4729
Marginal 2	First Moment	2.7361	0.7406
Imports	Second Moment	2.3744	0.7953
of	Third Moment	3.8456	0.5719
Thailand (v)	Fourth Moment	3.3344	0.6486

Source: Computation.

3.3 Results of static copula and goodness-of-fit tests

In this section, we present the various copulas, including the constant copula of the elliptical family copula as well as the Archimedean family copulas, and consider the goodness-of-fit copulas, the Akaikes Information Criterion (AIC) and the Bayesian Information Criterion (BIC), which are the statistical criteria used for the selection of the copula. Table 7 shows all of the parameters of the various static copulas which provide the parameters of correlation between the different marginal distributions, standard error, log-likelihood, the AIC, and the BIC. All of the parameters of the copulas are obviously significant (see the p-value in the bracket below the parameter value, which shows that all of them also pass the test at the 0.05 level). Moreover, the AIC and BIC values of the student-t copula display better explanatory ability than the other copulas because the AIC and BIC values of this copula are -35.2466 and -35.2307, respectively, which is the lowest value from among all the copulas. According to the Canonical Maximum Likelihood (CML) method, the constant student-t copula fits the best. The tail dependence of this copula is equal to 0.3925, which implies that the tail dependence is very high.

Although it is possible to choose the best copula for explaining the dependence structures, to confirm whether the copula we choose is the best fit, we use the AIC and the BIC as the goodness-of-fit tests in order to correctly model the dependence structure. We would need to determine the goodness of fit of the copula. We use the "hit test" in Patton [14] to test the goodness of fit of the copula. Table 8 shows the Cramer-von Mises test (CvM test) and the Kolmogorov-Smirnov test (K-S test). The main objective of the CvM and K-S tests is to verify the null hypothesis. If the p-value of each of the copulas is less than 0.05, it indicates a rejection of the null hypothesis and it demonstrates that the model is well specified. In addition, most of the CvM and K-S tests of the copulas pass at the 0.05 level except for the Rotated Clayton and Gumbel copulas; besides, the student-t copula is the best copula among the seven cases. It is important to note here that the student-t copula has more ability for observation in the tail than the Gaussian copula.

Table 7: Estimated Results for Parameter Copulas

Copula	Parameter	Std.error	LL	AIC	BIC
Gaussian	0.3913 (0.0061)	0.0076	-17.6191	-35.2288	-35.2130
Clayton	0.5292 (0.0000)	0.0076	-15.3944	-30.7793	-30.7635
Rotated Clayton	0.4345 (0.0000)	0.0071	-11.4578	-22.9062	-22.8903
Frank	2.3987 (0.0000)	0.0298	-15.4510	-30.8926	-30.8768
Gumbel	1.2779 (0.0000)	0.0081	-13.6564	-27.3033	-27.2875
Rotated Gumbel	1.3151 (0.0000)	0.0047	-17.4612	-34.9130	-34.8971
Student-t	0.3925 (0.0000)	0.0036	-17.6280	-35.2466	-35.2307
d.f	90.2691 (0.0079)				

Note: the element in the brackets correspond to the p-value.
Source: Computation.

Table 8: Goodness of Fit of Copulas

Copula	CvM	K-S	Stat CvM	Stat K-S
Gaussian	0.7	0.9	0.04993865	0.52893330
Clayton	0.5	0.4	0.09790849	0.74954510
Rotated Clayton	0.0	0.3	0.17320740	0.85300470
Frank	0.6	0.3	0.06514864	0.71041270
Gumbel	0.0	0.0	0.21037180	0.87382380
Rotated Gumbel	0.5	0.9	0.05682949	0.56937010
Student-t	0.8	1.0	0.05462471	0.51876520

Source: Computation.

3.4 Results for time-varying copulas

In this part, we explain the analysis of the time-varying copula which applied only two families of copulas, namely, the student-t copula and the Gaussian copula. However, the student-t copula could be considered as the best fit for dependence because the AIC and BIC values of this copula are the lowest in comparison with those of all the other copulas. To examine the goodness of fit of the student-t copula, we use the CvM and K-S tests, whose values for this copula confirm that they are still the highest in comparison with those of all the other copulas. Certainly, we still use the AIC and the BIC as the criteria for choosing the best time-varying copula. If the AIC and BIC values are the smallest, then it means that that particular time-varying copula is the best. Table 9 shows the parameters of these copulas, the standard error, ω,α,β, the AIC, and the BIC values. We found that these copulas are also significant. Moreover, the student-t copula is still the best fitted because of the lowest AIC and BIC values.

Table 9: Estimated Results for Time-varying Copulas

Copula	Parameter	ω	α	β	AIC	BIC
Gaussian	ρ	0.4518	-0.0471	1.0142	-35.2990	-35.2835
	Std. error	0.1276	0.0177	0.3152		
Student-t	ρ	0.4648	-0.0471	0.9800	-35.3161	-35.3003
	Std. error	0.1235	0.0160	0.3070		

Note: Computation.

The comparison between the correlation of the constant student-t copula and the correlation of the time-varying student-t copula is shown in Figure 2. We can see that the smallest value of kendall.tau is close to 0.35 and the highest value is approximately 0.415; this implies that the non-linear correlation always exists at a high level and that the Kendalls tau always maintains a value above 0.39.

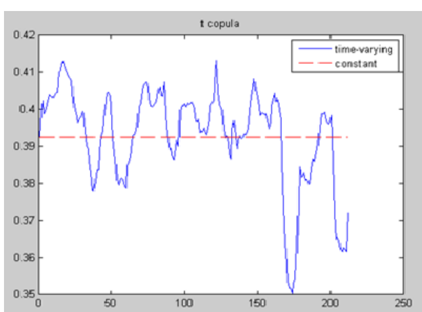

Figure 2: The comparison of the conditional correlations of the student-t copula.

4 Conclusion

This paper investigates the correlation of two dependence structures using copulas based on GARCH models with skewed student-t distribution. The ARMA-GARCH was employed for the estimation of the volatility of marginal distribution of the two dependences. After that, the Box-Ljung and Kolmogorov-Smirnov tests were applied to investigate whether it satisfied the hypothesis or not. We subsequently used various copulas to investigate the correlation between the manufacturing production index (MPI) and the imports of Thailand. This study confirms that the Canonical Maximum Likelihood method can be applied for the estimation approach using each of the copulas, and to study the correlation and co-movement of the economic growth model. In addition, the student-t copula, as regards both the static copula and the time-varying copula, is the best for the explanation of both the dependences. The main results of this empirical finding are that we have proven that if MPI increases or economic growth enhances, it results in increased income of the Thai people, and then they will buy more imported goods. Our findings suggest that these copulas could also be used for calculation in financial markets such as stock, price, exchange rates, etc.

Acknowledgement(s) The authors acknowledge the financial support provided by the Prince of Songkla University Scholarship for Chakorn Praprom's PhD study. We also thank Dr. Chanagun Chitmanat for reviewing the manuscript.

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(Received 30 May 2014)
(Accepted 10 September 2014)