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Examining the Consistence of Futures Margin Levels using Bivariate Extreme Value Copulas

X. Gong[†1], **H. T. Nguyen**[‡], **V. Kreinovich**[⋆] **and S. Sriboonchitta**[¶]

[†] [¶]Faculty of Economics,
Chiang Mai University, Chiang Mai 50200, Thailand
e-mail : gongxue.cmu@gmail.com; songsakecon@gmail.com
[‡]Department of Mathematical Sciences,
Las Cruces, New Mexico 88003, USA and
Faculty of Economics,
Chiang Mai University, Chiang Mai 50200, Thailand
e-mail : hunguyen@nmsu.edu
[⋆]Department of Computer Science,
University of Texas at El Paso, 500 W. University, El Paso, TX 79968, USA
e-mail : vladik@utep.edu

Abstract : This study examines the consistence of the futures margin levels of different commodities and combinations in the CME group by Extreme Value Copula (EVC). We find that if we ignore the co-movements of the commodities, the margins become consistent with each other, and the margin violation rates hover around 0.5%. However, if we consider the co-movement of the related commodities using EVC, the margin levels are found to be not consistent anymore, especially in the combinations of strongly related commodities which are in the same category. Therefore, we suggest that the CME group should try to harmonize the margins policy with respect to the dependence between the futures in the future.

Keywords : Bivariate Extreme Value Copula; Futures, Consistent Margin levels; CME group.

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[1]Corresponding author.

1 Introduction

Margin requirements are like cushions, which are customers' deposits to take long or short positions in financial markets. When losses occur, they protect the brokers and exchanges from customers' defaults. The exchanges must choose margin levels that balance the benefits that are received by attracting customers and executing transactions against the involved risk exposure. There are three different kinds of margins in the futures market, namely, initial, maintenance and spread margins. The intial margin refers to the equity required to initiate a futures position. The maintenance margin is "marked to market" every day, that is, when the losses make the margin levels lower than the maintenance margins which are lower than the initial margins, the traders must make up for the loss. For example, if an investor buys one contract of wheat, which contains 5,000 bushels, he should pay 3,713 USD for the initial margin requirement, when the futures price changes; in order to keep the account active, if the margin account drops below the 2,750 USD mark of the maintenance margin requirement, the investor will have to make up for the difference between the account balance and the initial margin requirement. Additionally, the "spread margin" is the margin that is set for an account which consists of two related commodities (Sriboonchita et al[1]).

The *consistence* of the futures margins (where the margins are sets either by the exchanges or the brokers) refers to the margin requirements that can be different for different futures commodities, but such that these futures are exposed to the same risk of defaulting (Gong and Sriboonchitta [2]), i.e., having the same probability of default across markets. The reason why we should set the margin requirements to the equal level is that, first, we have to make their commodities futures as competitive as others and, second, also the most important, we have to protect the whole exchange system (Longin [3]and Estrella [4]). Because the futures margins are set by the futures exchanges and brokers, and not by any external agencies, the problem of futures margins should receive more attention.

Currently, the Chicago Mercantile Exchange (CME Group) is the world's leading and largest derivatives market, which merged with the Chicago Board of Trade (CBOT) in July 2007, and the Commerce Exchange (COMEX) and New York Mercantile Exchange (NYMEX) in August 2008. The four exchanges have now become the divisions of the CME group, but conducting their responsibilities individually. Although the four exchanges are in charge of different kinds of futures and options, such as the CBOT which is mainly trading in agricultural products, CME's functions are primarily based on equity index and agricultural futures, and NYMEX trades mostly in the energy and metals futures. These four exchanges also offer the same or similar commodities futures.

The motivation for this paper comes from the problem that when the CME group officially merged, the customers under the same account could trade without the restriction of the exchanges. "Are futures margin levels consistent with each other across the four exchanges?", i.e., the same probability of default for futures in all four exchanges? Because of the co-movement of the commodity prices, the consistent margin requirement has several meanings: first, the weaker condition means that every single commodity position should have the same margin violation rate, secondly, a stronger condition is that the account which consists of two commodities (called "spread") has the same margin violation rate. We aim at answering these questions by using the extreme value theory within or beyond the division under the CME exchanges.

The paper is organized as follows. In section 2, we review the literature related to margin setting and extreme value theory. In section 3, we review the extreme value copulas. Section 4 will provide the empirical results and the last section concludes this study.

2 Brief Literature Review

Available literature reviews based on the EVT focus on margin-setting problems can be found, for example, in Longin [5] and Cotter and Kevin [6]. The method they used can be applied to test the margin consistence problem. However, most of the studies address the single commodity account instead of the multi-commodities account. Figlewski [7] first solved the margin-setting problems by using normal distribution for the different types of stock-related instruments, such as options and futures. However, since the margin-setting problem is directly influenced by the extreme price change, and the normal distribution ignores the extreme movements, Longin [5] tried to improve the accuracy of the risk exposures by the extreme value theory, he suggested that only the large price variations may lead to losses. Therefore, the EVT may be the best method for the modeling the margin setting.

Many authors did realize that there is extreme dependence or correlation between different futures. An example would be Ning [8], whose study examines the dependences in eight stock indices from North America, Europe, and East Asia. The findings imply that there are different degrees and structures of intra-continental and inter-continental dependence, therefore, he suggests that the co-movement in the financial markets should be taken into account in risk management. Our study tries to estimate the margin violation rates by capturing the dependence between the futures prices using extreme value theory.

3 Methodology

The key to solving the margin-setting problem is obtaining the exact form of the probability distributions of the futures prices. However, traditional methods of margin setting never considered the characteristics of the financial series, which follows a heavy-tailed distribution (Breymann, Dias, and Embrechts [9]). Also, only the extremes in the tail could lead to the margin defaults, that is, the extremes are critical in the margin setting due to the fact that they have direct connection with the margin violations. Suppose there are three commodities in one account,

$i = 1, 2, 3$. Let the margin level of commodity i be ML_i and let its price be X_i (in the short position, or in the long position).

If the three futures prices X_i are (mutually) independent, then the violation rate is

$$P(X_1 > ML_1, X_2 > ML_2, X_3 > ML_3) = P(X_1 > ML_1) \cdot P(X_2 > ML_2) \cdot P(X_3 > ML_3) \quad (3.1)$$

However, if they are dependent, the violation rate could be higher, for example when the price variables X_i are *mutually positively dependent*. Recall that two variables X_i, X_j are said to be positively dependent if for any Borel sets A, B of the real line $\mathbb{R}$, we have

$$P(X_i \in A | X_j \in B) > P(X_i \in A)$$

or, equivalently,

$$P(X_i \in A, X_j \in B) > P(X_i \in A)P(X_j \in B)$$

*Remark.*This definition is clearly symmetric. Indeed,

$$P(X_j \in A | X_i \in B) = \frac{P(X_j \in A, X_i \in B)}{P(X_i \in B)} > \frac{P(X_j \in A)P(X_i \in B)}{P(X_i \in B)} = P(X_j \in A)$$

Three variables X_1, X_2, X_3 are said to be *mutually positively dependent* if for any Borel sets A, B, C of $\mathbb{R}$, and $i \neq j \neq k$,

$$P(X_i \in A, X_j \in B | X_k \in C) > P(X_i \in A, X_j \in B)$$

In particular, for $B = \mathbb{R}$,

$$P(X_i \in A | X_k \in C) = P(X_i \in A, X_j \in \mathbb{R} | X_k \in C) > P(X_i \in A, X_j \in \mathbb{R}) = P(X_i \in A)$$

Thus, when X_1, X_2, X_3 are mutually positively dependent, we have

$$\begin{aligned} &P(X_1 > ML_1, X_2 > ML_2, X_3 > ML_3) = \\ &P(X_1 > ML_1, X_2 > ML_2 | X_3 > ML_3) \cdot P(X_3 > ML_3) > \\ &P(X_1 > ML_1, X_2 > ML_2) \cdot P(X_3 > ML_3) = \\ &P(X_1 > ML_1 | X_2 > ML_2).P(X_2 > ML_2) \cdot P(X_3 > ML_3) \\ &> P(X_1 > ML_1) \cdot P(X_2 > ML_2) \cdot P(X_3 > ML_3) \quad (3.2) \end{aligned}$$

*Remark.*In terms of copulas, positive dependence between X and Y means that the copula C of (X, Y) is such that $C(u, v) \geq uv$, for all $u, v \in [0, 1]$. In n dimensions, $C(u_1, u_2, ..., u_n) \geq u_{1.}u_2...u_n$.

Now observe that, in n days, the margin failure of a commodity i is related only to extreme price movements, namely

$$\pi_i = P(M_i > ML_i) \tag{3.3}$$

where M_i denotes the random variable $\max\{X_{i,1}, ..., X_{i,n}\}$. As such, for large n, the distribution of M_i can be approximated by an extreme value distribution (EVD), see e.g., Coles, 2000 [10], Beirlant, 2004 [13]. Let's elaborate a bit on this. We are concerned with unusual large values (called extremes) of a price variable X, or more specifically the distribution of a random variable Y whose values are "large" values of X. But like the concept "tail" (of a distribution), the concept "large values" is a fuzzy concept, i.e., having no sharply defined boundary (between large and not large). As such, it is not clear how to define Y. Without calling upon the theory of fuzzy sets, the usual approach in statistics is defuzzification, e.g., either "viewing" Y as the maximum of observations $X_1, X_2, ..., X_n$, drawn from X, with distribution function F (i.e., $Y = X_{(n)} = \max\{X_1, X_2, ..., X_n\}$) or as a variable taking values above some given "high" threshold. In this paper, we take $Y = X_{(n)}$ with distribution $F^n(.)$. Then the Fisher-Tippett-Gnedenko theorem says that if $\frac{X_{(n)} - a_n}{b_n}$ converges in distribution to some non-degenerate distribution G, then G is of the form $G_\gamma(ax + b)$ with $a > 0, b, \gamma \in \mathbb{R}$, where, for $\gamma \neq 0$,

$$G_\gamma(x) = \exp\{-(1 + \gamma x)^{-\frac{1}{\gamma}}\}1_{\{x:1+\gamma x>0\}}(x)$$

and for $\gamma = 0$,

$$G_\gamma(x) = \exp\{-e^{-x}\}1_{\mathbb{R}}(x)$$

The above generalized extreme value distribution (GEVD) can be put in a three-parameter family

$$G_{\gamma,\mu,\sigma}(x) = \exp\{-[1 + \gamma(\frac{x - \mu}{\sigma})]^{-\frac{1}{\gamma}}$$

for $\gamma \neq 0$, and for $\gamma = 0$, as $\exp\{-e^{-\frac{(x-\mu)}{\sigma}}\}$. For $\gamma = 0$, we get the Gumbel distribution $\exp\{-e^{-x}\}1_{\mathbb{R}}(x)$ which is not a heavy-tailed distribution since its tail $1 - \exp\{-e^{-x}\} \approx e^{-x}$ as $x \to \infty$. For $\gamma < 0$, we get the Weibul distribution $\exp\{-(1 + \gamma x)^{-\frac{1}{\gamma}}\}1_{\{x:1+\gamma x>0\}}$ which has a "short" tail (its right end point is $-\frac{1}{\gamma}$). For $\gamma > 0$, we get the Frechet distribution $\exp\{-x^{-\alpha}\}1_{(x>0)}(x)$ where the tail index $\alpha = \frac{1}{\gamma}$.Since its tail is equivalent to $(\gamma^{-\frac{1}{\gamma}})x^{-\frac{1}{\gamma}}$ as $x \to \infty$, it is a heavy-tailed distribution (see. e.g., Embrechts et al., [14]). *Remark.* The Frechet distribution (EVD) G corresponds to the normalizing sequences $a_n = 0, b_n = F^{-1}(1-\frac{1}{n})$, where $F^{-1}(.)$ is the quantile function of F. As an application in our problem, if we want to derive the margin level at 5% margin failure, then take $\pi_i = 5\%$, and set the margin level at $G^{-1}(0.95)$. Copulas constitute a powerful tool to link

the marginal distributions to obtain joint distributions (Sirisrisakulchai et al.[11]; Liu et al.[12]). The extreme value copula (EVC) provides an appropriate dependence structure between extreme events (Goegebeur and Segers[15]). Recall that in EVT we are concerned with extreme order statistics. Let $(X_i, Y_i), i = 1, 2, ..., n$ be i.i.d. (X, Y) whose copula is C. Let $X_{(n)} = \max\{X_i : i = 1, 2, ..., n\}$ and $Y_{(n)} = \max\{Y_i : i = 1, 2, ..., n\}$. The copula C_n of $(X_{(n)}, Y_{(n)})$ is said to be in the domain of attraction of C.

$$P(X_{(n)} \leq x, Y_{(n)} \leq y) = P(\text{all } X_i \leq x, \text{all } Y_i \leq y) = [P(X \leq x, Y \leq y)]^n$$
$$= [C(F(x), G(y))]^n = [C((F_{(n)}(x))^{\frac{1}{n}}, G_{(n)}(x))^{\frac{1}{n}})]^n \quad (3.4)$$

where $F_{(n)}(x) = P(X_{(n)} \leq x) = F^n(x)$, and similarly, $G_{(n)}(y) = G^n(y)$. Thus,

$$C_n(u, v) = C^n(u^{\frac{1}{n}}, v^{\frac{1}{n}})$$

A copula C^* is said to be an extreme value copula if there is a copula C such that

$$C^*(u, v) = \lim_{n\to\infty} C^n(u^{\frac{1}{n}}, v^{\frac{1}{n}})$$

Note that the limit of a sequence of copulas is clearly a copula. The Gumbel copula of (X, Y) is

$$C_\theta(u, v) = \exp\{-[(-\log u)^\theta + (-\log v)^\theta]^{\frac{1}{\theta}}\}$$

for $\theta > 1$. It is an Archimedean copula with (additive) generator $\varphi_\theta(t) = (-\log t)^\theta$. Since

$$\lim_{\alpha\nearrow 1} \frac{1 - 2\alpha + C(\alpha, \alpha)}{1 - \alpha} = 2 - 2^{\frac{1}{\theta}} \neq 0$$

X and Y are upper tail dependent. Now, for

$$C_\theta(u, v) = \exp\{-[(-\log u)^\theta + (-\log v)^\theta]^{\frac{1}{\theta}}\}$$

we have $C_\theta(u,v) = C_\theta^r(u^{\frac{1}{r}}, v^{\frac{1}{r}})$ for any real $r > 0$. Therefore, Gumbel copulas are extreme value copulas. The situation in higher dimensions is similar. Let $X_i = (X_{i1},...,X_{id})$, $i = 1, ..., n$, be i.i.d. random vectors with joint distribution function F and marginal distributions $F_1, ..., F_d$ and copula C_F.

4 Data Description

We select the data from four sub-exchanges of the CME group. They are divided into four groups: Group 1: Equity Index (S&P 500, Nasdaq 100, and Nikkei 225 futures); Group 2: Agriculture (live cattle, corn, and wheat futures); Group 3: Energy (ethanol, crude oil, and natural gas futures); and Group 4: Metals (platinum, gold, and silver futures). They are chosen from different sub-exchanges, which can be checked from Table 1. The data were collected from Datastream. Since we can only get the energy futures margins from 2008, the data are from August 2008 to June 2012, which is a total of 987 daily observations. When using the EVT theory, the data must be stationary; otherwise, the results will be invalid. In our studies here, after appropriate transformations, our data is indeed stationary. Note that the margin violations are related to the differences in the prices, and hence, we transform the price into return (logarithm of the price difference) in percentage of each future, that is, $log(P_t/P_{t-1}) * 100$. Also, for the margin level (ML), in order to uniform the return change, we transform the margin level by plugging into the equation $log(P_t + ML)/P_{t-1} * 100$ to represent the margin level.

5 Empirical Results

5.1 The Consistent Test of the Margin Violations in CME group

In this section, we examine the historical price data of different commodities in four sub-exchanges. The data are the price but not the return since we would like to know the general situation in CME group. The violations happen when the margin is greater than the price change during a day. In Table 1, the commodities are grouped by the categories, with the initial margin violations in Agriculture, Energy, Metals, and Equity Index being 0.71%, 0.624%, 2.35%, and 0.11%, respectively, and the maintenance margin violations being 2.27%, 2.09%, 3.33%, and 0.32%, respectively. The average of 0.71%, 0.624%, 2.35%, and 0.11% is 0.95% which is approximately 1%.

Table 1: Initial and Maintenance Margins Violation

Commodity	Total	Initial Margin		Maintenance Margin	
	Observation	Violation	Rate	Violation	Rate
Corn	1261	5	0.396	24	1.903
Wheat	1261	8	0.634	33	2.616
Live Cattle	1261	14	1.11	29	2.299
Total	3783	27	0.71	86	2.27
Crude Oil	881	2	0.227	3	0.34
Natural Gas	881	9	1.021	34	3.859
Total	1762	11	0.62	37	2.09
Platinum	881	52	5.902	58	6.583
Gold	881	5	0.567	8	0.908
Silver	881	5	0.567	22	2.497
Total	2643	62	2.35	88	3.33
S&P 500	1261	1	0.079	5	0.396
Nasdaq 100	1261	3	0.237	4	0.317
Nikkei 225	1261	0	0.00	3	0.237
Total	3783	4	0.11	12	0.32

Note: Data Source: CME group website, www.cmegroup.com; rate is short for violation rate, and the number in rate subcolumn is in percentage.

The chi-square test in this section is used to determine whether initial margins violations, maintenance margins violations, and non-violations are distributed identically across different futures. Generally speaking, the null hypothesis is:

H_0: *the probability of the* i^{th} *category (initial margins, maintenance margins violations and non-violations) is the same for each* j^{th} *futures.*
H_A: *at least one future does not have the same probability as the other futures for one category.*

In this study we pick up a significance level, α=0.05. The test statistic is as:

$$\chi^2 = \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$$

Note that O_{ij} is the observed frequency; E_{ij} is the expected frequency, that is, $E_{ij} = \frac{(rowtotal)\times(columntotal)}{total}$. The degree of freedom for this statistics is $df = (No.\ of rows - 1) \times (No.\ of columns - 1)$. The details of this test can be shown in a simple example. We wish to test whether the proportion of initial margin violation in Corn future is identical to the proportion in wheat future. Similarly, for maintenance margin violation and non-violation frequency. The null hypothesis and alternative hypothesis are:

H_0: $P_{corn\ initial\ margin\ violation} = P_{wheat\ initial\ margin\ violation}$
H_0: $P_{corn\ maintenance\ margin\ violation} = P_{wheat\ maintenance\ margin\ violation}$
H_0: $P_{corn\ margin\ non-violation} = P_{wheat\ margin\ non-violation}$
H_a: *At least one of the null hypothesis statements is false.*

Finally, our results show that the probability of Chi-square test is calculated by $P(\chi^2 > 1.54) = 0.462$, which the null hypothesis is not rejected in our test.

The results of each pair test can be shown in Table 2. The degree of freedom is 2 in each case, and the test statistics values are in the each block. The numbers in bold are the ones which do not reject the null hypothesis, therefore they are the pairs which has consistent margin violations. We can say that from the historical price information, most of the margins are not consistent with each other. And later we check the different groups, we get the chi-square of agriculture, energy, and metal, and equity, they are: 7.27, 27.16, 85.36, 5.25, the probability of chi-square tests are 0.87, 0.00, 0.00, 0.262. In both of agriculture and equity group, the margin levels are consistent with each other, but the discrepancies appear in energy and metal group.

Finally, we test for whether whole exchange margin level is consistent, the chi-square statistic is 386.857 with 20 degree of freedom, and the probability of chi-square test is 0.000. It turns out the conclusion that in the aspect of historical price change, the margin levels are not consistent within the whole CME group.

5.2 Are margin levels of single commodity consistent with each other?

Since the four exchanges have merged with the CME group, the risk exposure should be the same for every commodity margin, i.e., margin

Table 2: The Consistent Test of Futures Margin Violations

Corn	CN										
Wheat	**1.54(0.46)**	WT									
Live Cattle	4.74(0.09)	**4.14(0.13)**	LC								
Crude Oil	11.37(0.00)	17.04(0.00)	13.66(0.00)	CO							
Natural Gas	7.78(0.02)	**2.69(0.26)**	7.71(0.02)	27.17(0.00)	NG						
Platinum	63.29(0.00)	58.31(0.00)	41.02(0.00)	51.65(0.00)	42.30(0.00)	PM					
Gold	7.25(0.03)	10.89(0.00)	6.27(0.04)	**2.30(0.32)**	18.82(0.00)	41.23(0.00)	GD				
Silver	**0.89(0.64)**	**0.05(0.98)**	**3.63(0.16)**	15.72(0.00)	**2.75(0.25)**	44.79(0.00)	9.913(0.01)	SR			
S&P 500	12.59(0.00)	20.97(0.00)	17.87(0.00)	**1.73(0.42)**	34.83(0.00)	74.59(0.00)	**4.44(0.11)**	18.39(0.00)	SP500		
Nasdaq 100	16.86(0.00)	24.77(0.00)	19.62(0.00)	**0.06(0.97)**	38.86(0.00)	72.66(0.00)	**3.42(0.18)**	22.89(0.00)	**2.80(0.25)**	N100	
Nikkei 225	16.81(0.00)	25.64(0.00)	22.27(0.00)	**3.29(0.19)**	40.39(0.00)	79.00(0.00)	7.38(0.03)	23.32(0.00)	**1.14(0.56)**	**4.00(0.14)**	N225

Note: The number in bold is consistent pair according to the Chi-square test.

levels of single commodities should be consistent with each other. Suppose the absolute return of commodities "i" in continuous days "t" is x_{it}; the probability of the margin violation is:

$$P(x_i < ML_n) = 1 - P(x_i < ML_n) = 1 - \{P(x_i^{max} < ML_i)\}^{1/k} \quad (5.1)$$

where "k" is the number of the observations in every block. We select maxima x_i^{max} from each block, and fit the data into the GEV distribution, thus computing the risk exposure in each commodity. Here, we use both the parametric and non-parametric methods to measure the risk exposures.

Because the margin level is "marked to market", the maintenance margin is not so important for margin requirement problems, therefore the analysis will not address this part. As shown in Table 3, with respect to the sub-exchanges, the averages of the margin violations of CME, CBOT, NYMEX, and COMEX exchanges in the short position are 0.372, 0.229, 0.787, and 0.136 in percentages, respectively, and the averages of the margin violations in the long position are 0.503, 0.239, 0.479, and 0.147 in percentages, respectively. From our calculations, we can see that the margin violations of the NYMEX are slightly higher than those of the other three sub-exchanges. With regard to the futures products, the averages of the short margin violations of Equity Index, Agriculture, Energy, and Metals futures are 0.392, 0.234, 0.656, and 0.321 in percentages, respectively, and those of the long margin violations are 0.462, 0.348, 0.374, and 0.303 in percentages, respectively. It is obvious from the results of the calculations that the differences between the margin violations are quite small.

Now we wish to test for the consistence of margin level, we use the GEV distribution results due to we believe that it can best describe the tail distribution. Since the Chi-square test needs the count data, we count the number (O_i) for the i^{th} interval is nP_i, where n is the number of sample size and P_i is equal to the probability of initial margin violation (p) and non-violation (1-p). The results of the pair consistency are shown in Table 4. Only four pairs of null hypothesis are rejected, which are in bold. And also we test for the twelve initial future margins are consistent. In long and short position, the chi-square statistics is 17.068 and 16.075, the degree of freedom is 11, the p-value is 0.106 and 0.138, the results show that the margin levels are consistent among the four sub-exchange.

Therefore, in this section, we come to the following conclusions:

First, most of the margin violation rates in the short position are much higher than those in the long position, which implies that the left tails are

Table 3: Margin Violation Rates according to Sub-exchanges and Commodities Group

By Products	By Exchanges	GEV (Short)	Empirical (Short)	GEV (Long)	Empirical (Long)
Equity Index	S&P 500	0.294	0.423	0.320	0.534
	Nasdaq 100	0.543	0.765	0.623	0.648
	Nikkei 225	0.339	0.423	0.443	0.765
Agriculture	Live Cattle	0.313	0.423	0.628	0.765
	Corn	0.322	0.314	0.313	0.423
	Wheat	0.067	0.102	0.104	0.207
Energy	Ethanol	0.300	0.534	0.300	0.534
	Crude Oil	0.710	1.007	0.710	0.885
	Natural Gas	0.958	1.133	0.113	0.102
Metals	Platinum	0.692	0.765	0.615	0.534
	Gold	0.212	0.423	0.160	0.314
	Silver	0.06	0.102	0.134	0.314

Note: 1. We present the results using percentages. 2. The sub-exchanges from top to bottom are CME, CBOT, NYMEX, and COMEX, respectively.

more likely to be heavy-tailed. This means that the investors who are in the long position have more chances to default.

Second, there is no significant difference between the single margin violation rates among the different categories and exchanges.

Third, the averages of the margin violations range from 0.06% to 0.958%, which confirms the announcement made by the CME group that the margin requirement covers about 99.9% of the risk exposure.

Hence, we can say that the harmonized margin policy of the CME group is successful, based on the risk exposure of every single selected commodity.

5.3 Are margin violation rates of an account which consist of two closely correlated commodities consistent with each other?

We keep asking the following questions: "How about the risk exposures of the margins violation rates for an account with two closely correlated commodities? Are they consistent with each other?" We address the commodities which are dependent on each other. The problem can be expressed in the following mathematic notations: We may want to figure out the margin violations of two commodities. Suppose the price change is between x_1

Table 4: The Consistent Test of Margin Level by Chi-square Test

	SP500	N100	N225	LC	CN	WT
S&P500	X	0.67(0.41)	0.10(0.75)	0.75(0.38)	0.00(1.00)	0.80(0.37)
Nasdaq100	0.43(0.51)	X	0.07(0.79)	0.00(1.00)	0.75(0.38)	**4.01(0.04)**
Nikkei 225	0.00(1.00)	0.22(0.64)	X	0.07(0.79)	0.10(0.75)	2.29(0.13)
Live Cattle	0.00(1.00)	0.34(0.56)	0.00(1.00)	X	0.75(0.39)	4.01(0.05)
Corn	0.00(1.00)	0.29(0.58)	0.00(1.00)	0.00(1.00)	X	0.80(0.37)
Wheat	0.76(0.38)	3.26(0.07)	1.16(0.28)	0.92(0.34)	1.00(0.32)	X
Ethanol	0.00(1.00)	0.25(0.62)	0.00(1.00)	0.00(1.00)	0.00(1.00)	0.59(0.44)
Crude Oil	1.14(0.28)	0.04(0.84)	0.79(0.37)	0.99(0.32)	0.92(0.34)	**4.69(0.03)**
Natural Gas	2.96(0.09)	0.74(0.39)	2.40(0.12)	2.72(0.09)	2.60(0.11)	**7.59(0.01)**
Platinum	1.04(0.31)	0.02(0.88)	0.71(0.39)	0.88(0.34)	0.82(0.36)	**4.48(0.03)**
Gold	0.00(1.00)	0.70(0.40)	0.02(0.89)	0.00(0.98)	0.00(0.95)	0.097(0.76)
Silver	0.49(0.48)	2.26(0.13)	0.77(0.38)	0.61(0.44)	0.67(0.42)	0.00(1.00)
	Ethanol	Crude Oil	Natural Gas	Platinum	Gold	Silver
S&P500	0.00(1.00)	0.79(0.37)	0.25(0.61)	0.29(0.59)	0.26(0.61)	0.26(0.61)
Nasdaq 100	0.39(0.53)	0.00(1.00)	2.23(0.14)	0.00(1.00)	2.23(0.14)	2.23(0.14)
Nikkei 225	0.02(0.89)	0.11(0.74)	1.13(0.29)	0.00(1.00)	1.13(0.29)	1.13(0.29)
Live Cattle	0.39(0.53)	0.00(1.00)	2.23(0.14)	0.00(1.00)	2.23(0.14)	2.23(0.14)
Corn	0.00(1.00)	0.79(0.37)	0.26(0.61)	0.29(0.59)	0.26(0.61)	0.26(0.61)
Wheat	0.76(0.39)	**4.07(0.04)**	0.00(1.00)	2.85(0.09)	0.00(1.00)	0.00(1.00)
Ethanol	X	0.45(0.50)	0.25(0.62)	0.13(0.72)	0.25(0.62)	0.25(0.62)
Crude Oil	0.77(0.38)	X	2.29(0.13)	0.00(1.00)	2.29(0.13)	2.29(0.13)
Natural Gas	2.09(0.15)	0.09(0.76)	X	1.51(0.22)	0.00(1.00)	0.00(1.00)
Platinum	0.69(0.41)	0.00(1.00)	0.13(0.72)	X	1.51(0.22)	1.51(0.22)
Gold	0.00(1.00)	1.42(0.23)	3.03(0.08)	1.32(0.25)	X	0.00(1.00)
Silver	0.39(0.53)	3.31(0.07)	5.35(0.02)	3.16(0.08)	0.05(0.83)	X

Note: The numbers in bold text are not consistent pair. The lower triangle is the short position, and the upper triangle is the long position.

and x_2, and the margin levels set by the corresponding exchanges or brokers are ML_1 and ML_2. We have

$$\begin{aligned} P(x_1 > ML_1, x_2 > ML_2) &= 1 - P(x_1 < ML_1) - P(x_2 < ML_2) + \\ &\quad P(x_1 < ML_1, x_2 < ML_2) \\ &= 1 - [G_1(ML_1)]^{1/k} - [G_2(ML_2)]^{1/k} + [G_3(ML_1, ML_2)]^{1/k} \quad (5.2) \end{aligned}$$

where k is the observation in every block; F_1, F_2, and F_3 are the distributions of the general prices; G_1 and G_2 are the probability distributions of maxima m_1 and m_2, that is, the GEV distribution; and G_3 is the joint distribution of the maxima. Consider that the co-movement of the two commodities exists; we model G_3 using the extreme value copula which was introduced in the last section. To confirm the results, we adopt both the parametric and nonparametric multivariate EVT methods. As the univariate EVT suggests, the block maxima of the price change M_1, M_2 followed the GEV distribution; hence,

$$\begin{aligned} F_3(ML_1, ML_2) &= C_F(F_1(ML_1), F_2(ML_2)) \\ &= P^{1/k}(M_1 < ML_1, M_2 < ML_2) \\ &= [C(G_1(ML_1), G_2(ML_2)]^{1/k} \end{aligned} \tag{5.3}$$

First, we consider the parametric method, assume that the margins follow the GEV distribution and the joint distribution follows the Gumbel model. We proceed by extracting the $n = 47$ monthly maxima, in our case, and by fitting every pair of data into the copula model. Therefore, the corresponding joint probability density function can be expressed as follows:

$$f_3(x_1, x_2) = f_1(x_1; \mu_1, \sigma_1, \xi_1) \cdot f_1(x_1; \mu_1, \sigma_1, \xi_1) \cdot c^n(F_1(x_1)), F_2(x_2); \theta) \tag{5.4}$$

where θ are a vector of parameters from extreme value copula. By maximum likelihood, after obtaining the estimated parameters $\hat{\theta}$, $\hat{\mu_1}$, $\hat{\sigma_1}$, $\hat{\xi_1}$, $\hat{\mu_2}$, $\hat{\sigma_2}$, $\hat{\xi_2}$ a as well as the copula function, we plug the margin levels ML_1 and ML_2 into the copula function, and calculate the margin violation rates.

Second, for comparison purposes, the nonparametric estimation is also introduced. To illustrate clearly, we only explain the nonparametric way by using the bivariate case of the EV copula. Suppose the marginal distribution functions F_1 and F_2 are GEV distributions, then we transform the data from X_i, Y_i into $U_i = F_1(X_i)$ and $V_i = F_2(Y_i)$, and then let $S_i = -logU_i$, $T_i = -logV_i$. We can say that S_i and T_i are standard exponential random variables. For $t \in [0, 1]$, let

$$\xi_i(t) = min(\frac{S_i}{1-t}, \frac{T_i}{t}) \tag{5.5}$$

and hence

$$\frac{1}{\hat{A}^p(t)} = \frac{1}{n} \sum_{i=1}^{n} \xi_i(t) \tag{5.6}$$

Thus, the dependence function can be calculated using the above equation which is derived from (5.6).

It can be read from Table 5 that almost all the commodities are dependent on each other, according to the estimation using the Gumbel model. The numbers around the diagonal line across the table are generally higher than the others, which shows that futures in the same groups, such as Corn and Wheat, S&P 500, and Nikkei 225, are more dependent on each other than the commodities between different groups. For instance, the dependence between the maxima of S&P 500 and Nikkei 225 is 0.53, which is much greater than the kendall tau between S&P 500 and wheat.

Table 5: Estimated Kendall Tau between Each Pair

	SP	N100	N225	LC	CN	WT	EL	CO	NG	PN	GD	SR
S&P 500	X	0.40	0.53	0.11	0.08	0.00	0.09	0.15	0.15	0.18	0.20	0.10
Nasdaq 100	0.46	X	0.35	0.03	0.03	0.00	0.06	0.10	0.17	0.16	0.10	0.11
Nikkei 225	0.06	0.02	X	0.06	0.00	0.00	0.06	0.10	0.12	0.14	0.09	0.15
Live Cattle	0.23	0.22	0.12	X	0.00	0.09	0.35	0.36	0.36	0.32	0.01	0.31
Corn	0.16	0.12	0.00	0.15	X	0.04	0.13	0.17	0.01	0.00	0.14	0.00
Wheat	0.04	0.04	0.00	0.12	0.00	X	0.07	0.01	0.00	0.03	0.00	0.16
Ethanol	0.05	0.03	0.00	0.27	0.01	0.23	X	0.49	0.38	0.37	0.04	0.27
Crude Oil	0.12	0.13	0.03	0.31	0.00	0.18	0.59	X	0.36	0.31	0.02	0.26
Natural Gas	0.17	0.19	0.09	0.53	0.05	0.10	0.23	0.22	X	0.79	0.00	0.52
Platinum	0.18	0.18	0.06	0.42	0.05	0.12	0.21	0.20	0.79	X	0.00	0.47
Gold	0.07	0.00	0.04	0.26	0.16	0.06	0.04	0.09	0.12	0.11	X	0.00
Silver	0.15	0.17	0.12	0.42	0.09	0.11	0.19	0.21	0.66	0.65	0.14	X

Note: The upper triangle is the estimated Kendall tau between the maxima, and the lower triangle is the estimated dependence Kendall tau between the minima.

Upon checking Table 5 carefully, we also notice something interesting: the Ethanol futures is highly related to the commodities not only in the energy group but also in the agriculture group. The possible reason is that the raw materials of the biofuel are mostly obtained from agricultural products. Since most of the commodities are dependent on each other in varying degrees, what becomes of the risk exposure of each pair? Does each pair have equal margin violation for the CME group? As we mentioned earlier, the margin violation rates of single commodities are consistent with each other; if they are not dependent, the margin violations for the portfolio of multi-commodities will be consistent. For this reason, the test will focus on the dependent pair, that is, the commodities in the same group.

Table 6 gives the margin violation rates by parametric method, and Table 7 presents the margin violations by parametric method. In both tables, the four 3 by 3 squares are the margin violations within the same group, and the other four squares are the margin violations in the different sub-exchanges. We know that it is only the dependence that makes the margin level inconsistent, given that all of the single commodities have almost equal margin violations. Therefore, our analysis focuses on the data in the diagonal because of the fact that these pairs have the most dependent parameters, according to Table 5. To explain the results carefully, we calculate the average of each square. Categorized according to the group, the results indicate that the margin violations may be not as consistent as expected, as the Equity Index futures are observed to have a higher violation rate (0.264%) than the other kinds such as the agricultural (0.056%), energy (0.048%), and metal (0.076%) futures. Additionally, the results in the different sub-exchanges vary from 0.03% to 0.08% except in the CME exchange (0.287%). The coincidence happens due to the fact that all of the equity indexes are extracted from the CME exchange; hence, the results are not surprising.

Table 6: Parametric Estimation of Short and Long Position

		Equity Index			Agriculture			Energy			Metals		
		SP500	N100	N225	LC	CN	WT	EL	CO	NG	PN	GD	SR
CME	S&P 500	X	0.24	0.00	0.00	-	-	-	-	-	-	-	-
	Nasdaq 100	0.24	X	0.17	0.00	-	-	-	-	-	-	-	-
	Nikkei 225	0.24	0.35	X	0.10	-	-	-	-	-	-	-	-
	Live Cattle	0.07	0.09	0.001	X	0.04	0.01	-	-	-	-	-	-
CBOT	Corn	-	-	-	0.08	X	0.11	0.27	-	-	-	-	-
	Wheat	-	-	-	0.02	0.07	X	0.047	-	-	-	-	-
	Ethanol	-	-	-	-	0.03	0.01	X	0.04	0.00	-	-	-
NYMEX	Crude Oil	-	-	-	-	-	-	0.07	X	0.01	0.08	-	-
	Natural Gas	-	-	-	-	-	-	0.000	0.05	X	0.05	-	-
	Platinum	-	-	-	-	-	-	-	0.09	0.00	X	0.03	0.00
COMEX	Gold	-	-	-	-	-	-	-	-	-	0.01	X	0.03
	Silver	-	-	-	-	-	-	-	-	-	0.06	0.16	X

Note: the numbers are in percentage.

Actually, on the basis of the earlier analysis, we suppose that the violation for the single commodity is 0.005. If there is no dependence, the violation rate for two commodities should be $2.5 * 10^{-5}$; however, in our parametric calculations, the greatest violation between the Nasdaq 100 and the Nikkei 225 is $3.5 * 10^{-3}$, which is roughly 140 times of the independent case.

We use the chi-square test to test for both the short and long position. First, we separate above combinations into different categories, the Chi-square of equity index, agriculture, energy and metals in short position are 0.382, 0.839, 1.000, 3.502 with p value are 0.826, 0.657, 0.606, 0.173, re-

Table 7: Non-parametric Estimation of Short and Long Position

		Equity Index			Agriculture			Energy			Metals		
		SP500	N100	N225	LC	CN	WT	EL	CO	NG	PN	GD	SR
CME	S&P 500	X	0.28	0.15	0.00	-	-	-	-	-	-	-	-
	Nasdaq 100	0.31	X	0.18	0.00	-	-	-	-	-	-	-	-
	Nikkei 225	0.26	0.37	X	0.04	-	-	-	-	-	-	-	-
	Live Cattle	0.05	0.03	0.00	X	0.05	0.04	-	-	-	-	-	-
CBOT	Corn	-	-	-	0.09	X	0.06	0.19	-	-	-	-	-
	Wheat	-	-	-	0.01	0.07	X	0.06	-	-	-	-	-
	Ethanol	-	-	-	-	0.00	0.01	X	0.08	0.00	-	-	-
NYMEX	Crude Oil	-	-	-	-	-	-	0.06	X	0.05	0.01	-	-
	Natural Gas	-	-	-	-	-	-	0.02	0.08	X	0.01	-	-
	Platinum	-	-	-	-	-	-	-	0.04	0.05	X	0.09	0.02
COMEX	Gold	-	-	-	-	-	-	-	-	-	0.11	X	0.04
	Silver	-	-	-	-	-	-	-	-	-	0.09	0.11	X

Note: the numbers are in percentage.

spectively. And for long position, the chi-square statistics are 4.495, 0.684, 1.103, 0.49 with probability 0.808, 0.710, 0.576 and 0.782, which implies that the margins within the same category are consistent. For all combinations, the test statistics are 42.849 and 35.409 with degree of freedom is 18, and the probability is 0.000 and 0.008, which means the null hypothesis are rejected. Hence, we can conclude that due to the dependence between different commodities, there exists discrepancy between the margin violations of the different combinations in the CME group, which implies that investors who long or short futures will have a higher chance of defaulting.

6 Conclusion

The CME group has merged the CBOT in July 2007, and also completed the takeover of the NYMEX and COMEX exchanges in August 2008. At the present time, although the four exchanges have become the sub-divisions of the CME group, each exchange remains a separate self-regulatory organization. For providing a common regulatory framework for customers, the rules of CME, CBOT, and NYMEX have been substantially harmonized. How does this biggest exchange in the world functions since its merger half a decade ago is worth a thought. One of the most important policies for an exchange is the margin rate; therefore, it is reasonable that we would like to raise questions such as “Is the margin level consistent with each other since the merger?" or "Is the adjustment of the margin level since the merger enough?" This paper try to answer the questions and aims to test the futures margin levels of different commodities and combinations in the CME group by extreme value theory. We observed that if we ignore the co-movements of the commodities, the margin is consistent with each other, that is, the margin levels are well harmonized by the CME group with respect to the margin risk exposure. However, if we consider the co-movement of the related commodities, the margin is not consistent anymore, especially for the commodities in the same group which are strongly related. We suggest that to avoid losses due to unforeseen financial crises, the CME group should try to harmonize the margins policy by taking into consideration the dependence between the futures in the future.

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