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Economic Forecasting Based on Copula Quantile Curves and Beliefs

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Abstract : This paper applies belief functions-based copula quantile curves model to capture dependence structure between crude oil and corn returns, and quantify uncertainty of the corn returns at one step period. We employ the time-varying copulas, including Gaussian, T and Clayton, which can be used to capture dynamic correlations between variables. We forecast their correlation ahead of one period, and the uncertainty of corn returns ahead of one period is measured under p-th copula quantile curves. The empirical results show the range of corn returns and its uncertainties under 5% and 95% copula quantile curves. In addition, the time-varying T copula describes the dependence structure between crude oil and corn returns quite well.

Keywords : Dynamic copulas; Belief functions; Quantile; Plausibility
2010 Mathematics Subject Classification : 62P20; 91B84 (2010 MSC)

1 Introduction

The copula method is widely used in finance and econometrics, particularly time-varying copulas for studying dynamic dependence and predicting dependence structure in the future (see Patton [1], Wu et al. [2] and Sriboonchitta et al. [3]). However, with time-varying copulas, it is difficult to determine the forcing variable for explaining the dynamic characteristics [1] [3] [5], and copulas cannot be used

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to forecast exchange rates, future prices and agricultural product prices, among others. Fortunately, Bouye and Salmon [4] proposed copula p-th quantile curves that can be used obtained the non-linear relationship between two variables for the p-quantile. Chen et al. [6] introduced copula-based quantile autoregression models that permit the estimated parameters to vary with quantiles, and forecast one period ahead. However, a forecast cannot be trusted unless it is accompanied by some measure of uncertainty. Generally, forecast uncertainty is described by subjective probabilities or prediction intervals. Recently, Kanjanatarakul et al. [11] proposed a method to quantify uncertainty on statistical forecasts using the formalism of belief functions. So, we attempt to combine time-varying copula p-th quantile curves with belief functions thereby achieving the purposes of forecasting dependence and uncertainty of one variable at one step period.

Rising biofuel production, particularly in the production of corn-based ethanol, is likely to have made corn and crude oil markets more connected. Wu et al. [15] found evidence of significant spillovers from crude oil prices to corn cash and futures prices, and that these spillover effects are time-varying. Also, the corn markets have become much more connected to crude oil markets after the introduction of the Energy Policy Act of 2005. Natanelov et al. [16] found that the interaction between crude oil and corn is relatively stronger through the biofuel production linkage. Therefore, it is necessary that we study the dependence structure between corn and crude oil returns.

In this study, we combine the time-varying copula quantile curves with belief functions to forecast the uncertainty of corn returns at t+1 period. To provide insight on recognizing and forecasting the uncertainty of corn index returns, the belief and plausibility functions are used to measure uncertainties. The purpose of this paper is to apply belief function theory with copula to statistical forecasting problems, as follows: (1) using time-varying copulas to replace static copulas in copula quantile curves; (2) combining time-varying copula quantile curves with belief and plausibility functions to forecast uncertainties of corn index returns. Our study represents two main contributions. First, we use time-varying copula-GARCH models to capture time-varying dependence between corn and crude oil returns. The results show that non-normality and asymmetry are significant in corn and crude oil returns, and the dependence structure is time-varying. We find that the time-varying T copula exhibits a better explanatory ability than the other dependence structures, and the dependence has a high degree of persistence between crude oil and corn returns. Second, there are no previous papers using belief functions-based copula quantile curves model to forecast the uncertainty of the corn returns at one step period, this paper will fill this gap.

The paper is organized as follows. Section 2 briefly describes the data, copula quantile curves and time-varying copulas. Section 3 conducts the belief functions-based copula quantile curves model. Section 4 provides an application of the model. Finally, section 5 offers a conclusion.

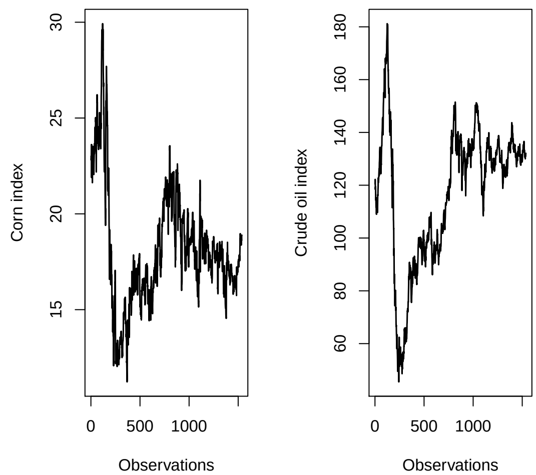

Figure 1: Corn and crude oil indices

2 Data and methodology

2.1 Data

We study two sets of daily time series data: a corn index and a crude oil index, for the period from January 2, 2008 to March 27, 2014. Each index contains 1537 observations. All data are obtained from Thomson Reuters ECOWIN. The asset return of each index is calculated using the differences between its logarithmic closing prices. The graphs of corn and crude oil indices are illustrated in Figure 1. It shows the corn and crude oil indices almost simultaneously reach peak and trough, and there exists obvious rank correlation between them in big fluctuation periods.

2.2 Copula quantile curves

Let $F_{X|Y=y}(.)$ be the conditional distribution of X given $Y = y$. The $p \in [0, 1]$ quantile of the distribution $F_{X|Y=y}(.)$ is defined as usual, namely $F^{-1}_{X|Y=y}(p) = inf\{x \in \mathbb{R} : F_{X|Y=y}(x) \geq p\}$.

We can express $F_{X|Y=y}(.)$ in terms of the copula C of (X, Y), with X,Y being continuous variables, as follows. First, recall that, if $H(., .)$ denotes the joint distribution function of (X, Y) with marginal distributions,

$$F(x) = H(x, \infty) = P(X \leq x), G(y) = H(\infty, y) = P(Y \leq y), \tag{2.1}$$

then according to Skar's theorem [12], there is a unique copula C such that, for any $x, y \in \mathbb{R}$,

$$H(x,y) = C(F(x), F(y)). \tag{2.2}$$

Now,

$$\begin{aligned} F_{X|Y=y}(x) &= P(X \le x | Y = y) = \lim_{\epsilon \to 0} P(X \le x | y \le Y \le y + \epsilon) \qquad (2.3) \\ &= \lim_{\epsilon \to 0} \frac{P(X \le x, y \le Y \le y+\epsilon)}{P(y \le Y \le y + \epsilon)} \\ &= \lim_{\epsilon \to 0} \frac{P(X \le x, y \le Y \le y+\epsilon) - P(X \le x, Y \le y)}{P(Y \le y+\epsilon) - P(Y \le y)} \\ &= \lim_{\epsilon \to 0} \frac{H(x, y+\epsilon) - H(x,y)}{G(y+\epsilon) - G(y)} \\ &= \lim_{\epsilon \to 0} \frac{C(F(x), G(y+\epsilon)) - C(F(x), G(y))}{G(y+\epsilon) - G(y)} \\ &= \lim_{\epsilon \to 0} \frac{(C(F(x), G(y+\epsilon)) - C(F(x), G(y))/\epsilon}{(G(y+\epsilon) - G(y))/\epsilon} \\ &= \frac{\partial C(u,v)}{\partial v} | u = F(x), v = G(y). \end{aligned}$$

So, we can write $F_{X|Y=y}(x) = C_1(F(x), G(y))$, where $C_1 : [0,1] \times [0,1] \to [0,1]$ is $C_1(u,v) = \frac{C(u,v)}{\partial v}$.

In terms of the partical derivative C_1 of the copula C, the $p-th$ copula quantile curve of X given Y=y is defined by the implicit equation [4]

$$p = C_1(F(x), G(y)) \tag{2.4}$$

or more specifically, for given y, it is $u = F(x) = C_1^{-1}(p, G(y))$.

There are three copulas, Gaussian, T and Clayton copulas, which are used in this study. Gaussian copula is tail independent, while the Student's t copula exhibits symmetric lower and upper tail dependence. We also use the Clayton copula due to its ability to parameterize lower tail dependence across asset returns. The first order partial derivative of the Gaussian copula is written as:

$$p = C_{1,gau}(u, v; \rho) = \Phi((\Phi^{-1}(u) - \rho\Phi^{-1}(v))/\sqrt{(1-\rho^2)}) \tag{2.5}$$

with ρ the linear correlation and Φ the univariate Gaussian distribution. Then $p - th$ conditional quantile function of u given v can be given as:

$$u = C_{1,gau}^{-1}(p, v; \rho) = \Phi(\rho\Phi^{-1}(v) + \sqrt{(1-\rho^2)}\Phi^{-1}(p)), \tag{2.6}$$

$$x = F^{-1}(u) = F^{-1}(\Phi(\rho\Phi^{-1}(G(y)) + \sqrt{(1-\rho^2)}\Phi^{-1}(p))) = h_{gau}(p, y; \rho). \tag{2.7}$$

For financial data, a Gaussian copula may be not a good choice, because the Gaussian copula cannot capture tail dependence. However, T and Clayton copulas are better than Gaussian copula in terms of which can capture tail dependence.

For T copula, the $p-th$ copula quantile curve of u given v follows from Chen et al. [6]:

$$u = C_T^{-1}(p, v; \rho, \nu) = T_\nu(\rho T_\nu^{-1}(G(y)) + \sigma((G(y))T_{\nu+1}^{-1}(p), \quad (2.8)$$
$$x = F^{-1}(u) = F^{-1}(T_\nu(\rho T_\nu^{-1}(G(y)) + \sigma(G(y))T_{\nu+1}^{-1}(p)) = h_T(p, y; \Theta), \quad (2.9)$$

where $\sigma(G(y)) = \sqrt{[\nu + T_\nu^{-1}(G(y))^2(1-\rho^2)]/(\nu+1)}$, T_ν is student-t distribution with the parameter degree of freedom ν, Θ is the parameter vector that includes ρ and ν. Last, the $p-th$ copula quantile curve of u given v of Clayton copula can be expressed as

$$u = C_{1,Cl}^{-1}(p, v; \theta) = [(p^{-\theta/(1+\theta)} - 1)v^{-\theta} + 1]^{-1/\theta}, \quad (2.10)$$
$$x = F^{-1}(u) = F^{-1}([(p^{-\theta/(1+\theta)} - 1)(G(y))^{-\theta} + 1]^{-1/\theta}) = h_{Cl}(p, y; \theta). \quad (2.11)$$

Bouye and Salmon [4] proposed copula quantile regression approach enables us to examine the dependency between assets at any given quantile, including extreme quantiles. We may in fact not often be interested in dependence strucure between assets but are more interested in forecasting uncertainty of asset returns. Thus, we make an attempt on constructing belief functions-based time-varying copula quantile curves model. The time-varying copulas and belief functions are illustrated next section below.

2.3 Time-varying copulas

The most dependence structures for financial data are not basically time-invariant [1] [2] [3]. Time-varying copulas might be considered as the dynamic generalizations of a Pearson correlation or Kendall's tau; it is still difficult to find causal variables to explain such dynamic characteristics [13]. We followed the concept of Wu [2] by assuming that the dependence parameters rely on past dependence and the previous historical information $(u_{t-1} - 0.5) * (v_{t-1} - 0.5)$. Then, some examples of time-varying copulas can be expressed as:

(1) time-varying Gaussian copula

$$\rho_t^\star = \omega + \beta * \rho_{t-1}^\star + \alpha * (u_{t-1} - 0.5) * (v_{t-1} - 0.5), \quad (2.12)$$

where $\rho_t^\star = -ln[(1-\rho_t)/(1+\rho_t)]$, which is used to ensure that the correlation falls within (-1, 1). In addition, the correlation parameter ρ_t in time-varying T copula is used. The formula is consistent with the time-varying Gaussian copula to capture dynamic characteristic, and the degree of freedom parameter is not considered as time-varying.

(2) time-varying Clayton copula

$$\tau_t^\star = \omega + \beta * \tau_{t-1}^\star + \alpha * (u_{t-1} - 0.5) * (v_{t-1} - 0.5), \quad (2.13)$$

where τ_t is the rank correlation Kendall's tau, and $\tau_t^{\star} = -ln[(1-\tau_t)/(1+\tau_t)]$, which is similar with the correlation parameter in the time-varying Gaussian copula.

Through the time-varying copulas, the dependence between corn and crude oil returns can be predicted at t+1 period. Thus, the predicted values of corn returns can be obtained by the different p-th copula quantile curves.

3 The belief functions-based copula quantile curves model

The Dempster-Shafer theory (DST) was first developed by Dempster [7] and Shafer [8]. DST offers an alternative to traditional probabilistic theory for the mathematical representation of uncertainty. In general, belief function and its dual, plausibility function are used to measure uncertainty. It is tempting to consider Bel(A) resp. Pl(A), as lower, resp. upper, bound of the "true" probability of A. A detailed introduction about DST can be found in Shafer [8] [9]. The definition of belief function is described as follows. Assume (Θ, A) is a measurable space, where A is a non-empty subset of 2^{Θ} closed under complementation and countable union. A belief function Bel: $A \rightarrow [0,1]$ is a belief function if and only if it satisfies the following conditions: $Bel(\varnothing) = 0$, $Bel(\Theta) = 1$, and for any $k \geq 2$ and any collection $A_1, ..., A_k$ of elements of A,

$$Bel(\cup_{i=1}^{k} A_i) \geq \sum_{\varnothing \neq I \subseteq 1,\dots,k} (-1)^{|I|+1} Bel(\cap_{i \in I} A_i). \tag{3.1}$$

Also, a plausibility function can be defined as a function $Pl : A \rightarrow [0,1]$ such that $Pl(\varnothing) = 0$, $Pl(\Theta) = 1$, and for any $k \geq 2$ and any collection $A_1, ..., A_k$ of elements of A,

$$Pl(\cap_{i=1}^{k} A_i) \leq \sum_{\varnothing \neq I \subseteq 1,\dots,k} (-1)^{|I|+1} Pl(\cup_{i \in I} A_i). \tag{3.2}$$

There exists a relationship between $Bel(A)$ and $Pl(A)$,$Bel(A) \leq Pl(A)$, $Pl(A) = 1 - Bel(\overline{A})$, where $\overline{A}$ denotes the complement of A. The complete information about the measure of belief in A can be represented by the interval $[Bel(A), Pl(A)]$, where $Pl(A) - Bel(A)$ is a natural expression of the ignorance concerning A. Recently, a likelihood-based belief function was suggested by Denoeux [7], and this method follows three basic principles: likelihood principle, compatibility with Bayesian inference and least commitment principle [10] [11] [14]. On the basis of likelihood-based belief function, a general approach to quantify uncertainty of statistical forecasts using belief functions was proposed by Kanjanatarakul et al [11].

Forecasting model was conducted by using the belief functions with time-varying copula quantile curves, and applied them to forecast the uncertainty under given different conditional quantile scenarios. Take time-varying T copula as an example, the belief function-based copula quantile curves model is introduced as follows:

Suppose that the random variable X at t+1 period can be written as

$$X_{t+1} = \hat{c} + \sum_{i=0}^{p-1} \hat{\phi}_i x_{t-i} + \sum_{i=0}^{q-1} \hat{\psi}_i \varepsilon_{t-i} + \hat{\sigma}_{t+1} h(p, Y_{t+1}; \omega, \alpha, \beta, \nu), \tag{3.3}$$

where $Y_{t+1} = \hat{c} + \sum_{i=0}^{p-1} \hat{\phi}_i y_{t-i} + \sum_{i=0}^{q-1} \hat{\psi}_i \varepsilon_{t-i} + \hat{\sigma}_{t+1} \eta_{t+1}$, η_{t+1} represents standardized residuals that are from a known distribution, the estimates $\hat{c}$, $\hat{\phi}_i$, $\hat{\psi}_i$, and $\hat{\sigma}_{t+1}$ for variables X and Y are from ARMA-GARCH model, respectively. Our model can be constructed by minimizing and maximizing the X at $t+1$ period; this is subject to a contour function that is greater than or equal to a random value z from uniform distribution [0, 1]. The detailed model can be expressed as:

$$x^L(z, y_{t+1}) = \min_{pl(\omega,\alpha,\beta,\nu) \geq z} \{\hat{c} + \sum_{i=0}^{p-1} \hat{\phi}_i x_{t-i} + \sum_{i=0}^{q-1} \hat{\psi}_i \varepsilon_{t-i} + \hat{\sigma}_{t+1} h(p, y_{t+1}; \omega, \alpha, \beta, \nu)\} \tag{3.4}$$

and

$$x^U(z, y_{t+1}) = \max_{pl(\omega,\alpha,\beta,\nu) \geq z} \{\hat{c} + \sum_{i=0}^{p-1} \hat{\phi}_i x_{t-i} + \sum_{i=0}^{q-1} \hat{\psi}_i \varepsilon_{t-i} + \hat{\sigma}_{t+1} h(p, y_{t+1}; \omega, \alpha, \beta, \nu)\}, \tag{3.5}$$

where pl is the likeliood ratio from past data, the pl is defined as follows

$$pl(\omega, \alpha, \beta, \nu) = \frac{L(\omega, \alpha, \beta, \nu; x, y)}{L(\hat{\omega}, \hat{\alpha}, \hat{\beta}, \hat{\nu}; x, y)}. \tag{3.6}$$

We independently randomize η_{t+1} and z N times (for example, 1000 times). Then the N pairs of lower and upper bounds of the predicted value X can be estimated under the given conditional quantile p. The quantities $Bel_p^X(A)$ and $Pl_p^X(A)$ are approximated by:

$$\hat{Bel}_p^X(A) = \frac{1}{N} \#\{i \in \{1, ..., N\} | [x^L(z_i, y_{t+1,i}), x^U(z_i, y_{t+1,i})] \subseteq A\} \tag{3.7}$$

and

$$\hat{Pl}_p^X(A) = \frac{1}{N} \#\{i \in \{1, ..., N\} | [x^L(z_i, y_{t+1,i}), x^U(z_i, y_{t+1,i})] \cap A \neq \varnothing\}, \tag{3.8}$$

where A is the real line, we may define the lower and upper predictive cdfs of X as, respectively,

$$F_p^L(x) = Bel_p^X((-\infty, x]), \tag{3.9}$$
$$F_p^U(x) = Pl_p^X((-\infty, x]), \tag{3.10}$$

Table 1: Data Description and Statistics

Returns	mean	min.	max.	s.d.	skew.	kur.	K-S
Corn	0.00	-0.23	0.21	0.03	0.26**	11.56***	3.60***
Crude oil	0.00	-0.10	1.12	0.02	-0.07*	5.45***	3.15***

Note: *, ** and *** denote rejection of the null hypothesis at the 10%, 5%, and 1% levels, respectively. For mean and skewness, the hypotheses are: mean and skewness= 0. For Kurtosis, the hypothesis is Kurtosis = 3. For Kolmogorov-Smirnov(K-S), the hypothesis is that the variable follows a normal distribution.

for any $x \in \mathbb{R}$. Both of the functions $Bel_p^X(A)$ and $Pl_p^X(A)$ describe the uncertainties of X on the event A, given the conditional quantile p. $Bel_p^X(A)$ presents a degree of belief that supports event A, while $Pl_p^X(A)$ presents the degree to which one fails to doubt A. Therefore, we use the belief interval $[Bel_p^X(A), Pl_p^X(A)]$ to describe the uncertainty of the predicted X_{t+1}.

4 An application

Before we apply the time-varying copula-GARCH model to study the relationship between corn and crude oil returns, we first represent the descriptive statistics of the indices in Table 1. From Table 1, we find that the mean return of each index is not significantly different from zero. The estimate of the skewness of corn return is strongly significantly positive, while the estimate of the skewness of crude oil return is strongly significantly negative. The estimates of the kurtosis, the flattening coefficient of the distribution, are significantly positive for both indices, implying that the distributions of the returns of all the indices are fatter than a normal distribution. The results of the (KS) normality tests and the tests for the skewness and kurtosis coefficients confirm that the distributions of the returns of both indices are not normal.

Figure 2 shows the normal Q-Q plots of corn and crude oil returns. if the corn and crude oil returns come from normal populations whose distributions differ only by a shift inlocation, the points should lie along a straight line that is displaced either up or down from the theoretical reference straight line. We can find that the normal Q-Q plots indicate departures from normality, and they are heavy-tailed. It is seen to be that the skewed student-t marginals are more appropriate in our study.

We select an ARMA (1, 0)-GARCH (1, 1) time series filter with skewed student-t innovation distribution, as the marginal distributions. By estimating ARMA(1, 0)-GARCH(1, 1) model, the standardized residuals can be obtained by normalizing residuals. Figure 3 and Figure 4 show the empirical densities of standardized residuals. It can be seen that skewed student-t densities (sstd) fit very

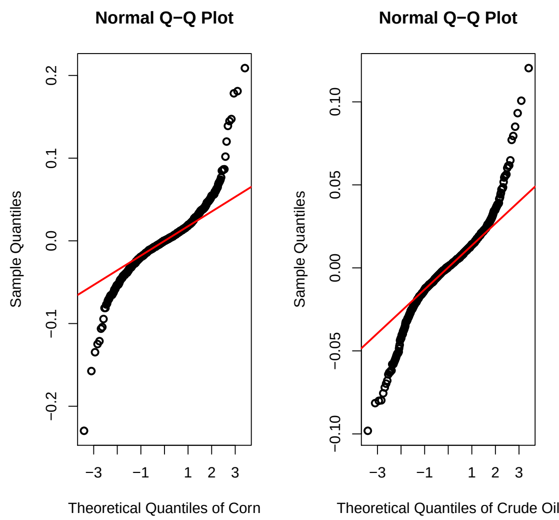

Figure 2: Q-Q plots of corn and crude oil returns

well, and it outperforms the normal densities.

Upon selection, we use a maximum log-likelihood estimation method to estimate time-varying copulas. The results of the estimated parameters are shown in Table 2. In terms of the values of AIC and BIC, the T copula exhibits a better explanatory ability than the other dependence structures. Also, the autoregressive parameter was close to 1, which implies a high degree of persistence in the dependence structure between crude oil and corn returns. Moreover, with T copula, the predicted correlation between crude oil and corn returns equals 0.24 at t+1 period. Figure 5 describes the correlation estimates from time-varying Gaussian, T and Clayton copulas. The linear correlations are captured by time-varying Gaussian and T copulas, and the nonlinear correlations, Kendall's tau, are measured by time-varying Clayton copula. Basically, they have the same shape, and generally decreased as the degree of the energy crisis is less.

Let the predicted correlation and the estimator of degree of freedom be known as the information, then the marginal distribution and the growth rate of corn index can be obtained given different quantiles and the marginal distribution of oil index. We extracted 1000 random numbers of crude oil returns at t+1 period by using ARMA(1, 0)-GARCH(1, 1) with skewed student-t distribution, and the correlation and degree of freedom are fixed at t+1 period as well. Then the different $p-th$ copula quantile curves and the forecasting values of corn index can be performed by formulas (2.8) and (2.9). Figure 6 shows the $p-th$ T copula quantile curves and the growth rates of corn index (for p=0.05, 0.1, 0.2,..., 0.9, 0.95 from bottom to top). We can find that the different quantiles have different

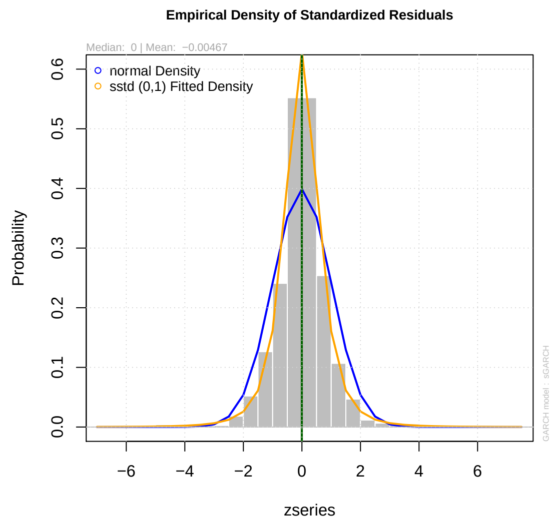

Figure 3: Empirical density of standardized residuals of ARMA-GARCH model for corn returns

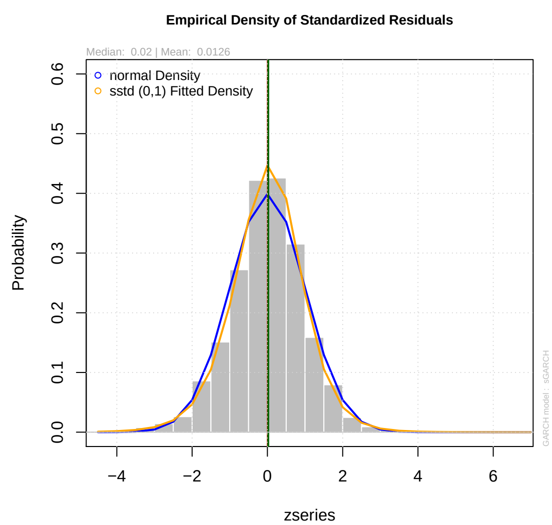

Figure 4: Empirical density of standardized residuals of ARMA-GARCH model for crude oil returns

Table 2: The results of time-varying copulas

Copulas	ω	β	α	ν	logL	AIC	BIC
Gaussian	0.001	0.98***	0.24**	—	62.10	-118	-122
	(0.002)	(0.006)	(0.09)				
T	0.001	0.98***	0.27*	15.23*	65.42	-123	-129
	(0.002)	(0.006)	(0.10)	(6.24)			
Clayton	0.28***	0.92***	-1.76***	—	51.14	-115	-120
	(0.08)	(0.03)	(0.50)				

Note: Signif. codes are as follows: 0 *** 0.001 ** 0.01 * 0.05 .0.1. The numbers in the parentheses are the standard deviations.

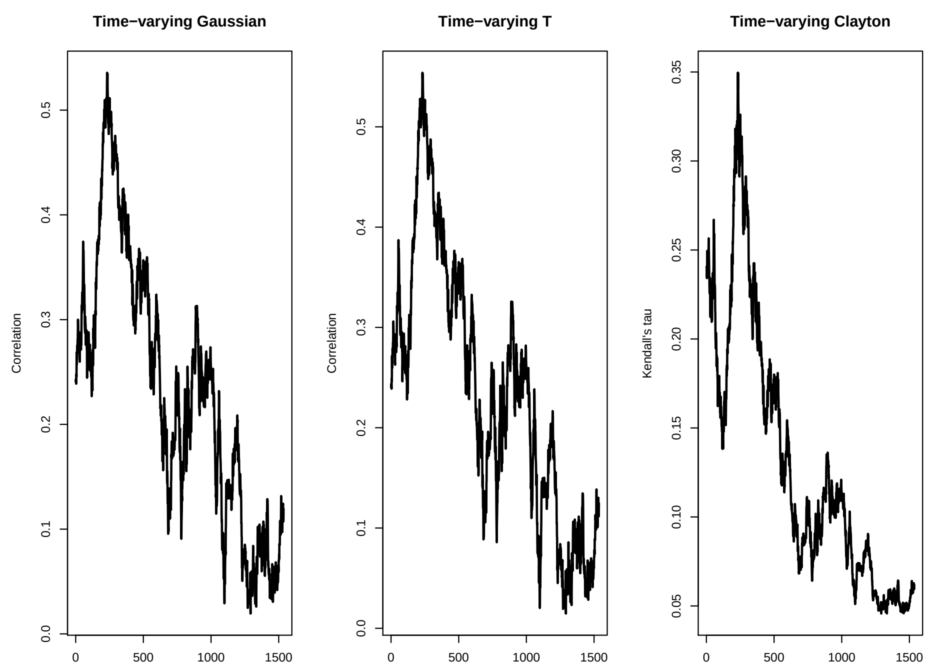

Figure 5: Correlation estimates from time-varying copulas

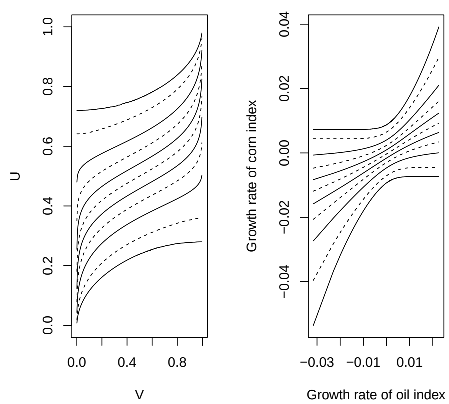

Figure 6: The different pth copula quantile curves and both of growth rates at t+1 period

curve. With the increasing of the quantiles, the growth rates of corn index increase in general as well. If the quantiles are greater than 0.8, the growth rates of corn index are always positive regardless of what the growth rates of crude oil index is. While the growth rates of corn keep negative if the quantiles are less than 0.3.

Figure 7 plots the cumulative belief and plausibility of the growth rate of the corn index under 5% and 95% copula quantiles, respectively. First, the growth rate ranges of the corn index drop in [-8%, 0] and [0, 8%], conditional on the 5% and 95% copula quantile curves, respectively. Second, the cumulative belief/plausibility measures the uncertainties at many points, in terms of the belief function. For instance, the growth rate of the corn index conditional on the 5% copula quantile curve is less than -2%, the believability of which is 0.38, while the plausibility of this event is 0.62. Similarly, at 95% copula quantile curve, Bel([0, 2%]) equals to 0.41, and Pl([0, 2%]) equals to 0.66, then Bel((2%,+8%]) is 0.34.

5 Conclusions

This paper demonstrates the model of measuring uncertainties in terms of belief functions. The approach uses the time-varying copulas to capture the dynamic dependence structure, and then applies the belief functions-based copula quantile curves to measure uncertainties at t+1 period. For time-varying copulas, we cannot guarantee that the causal variables are appropriate in explaining the dynamic characteristics between variables, which means the predicted value has the property of uncertainty. Then, we proposed a belief functions-based copula quantile curves model to describe the uncertainty information. In addition, we applied

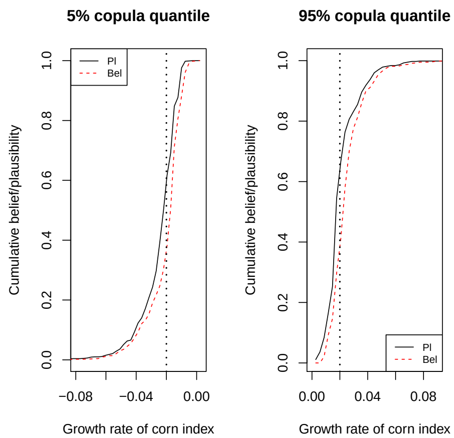

Figure 7: The cumulative belief/plausibility of the growth rate of the corn index under 5% and 95% copula quantiles

this function to the empirical data, demonstrating the feasibility of the model. This methodology may also be used in several contexts, such as vine copulas or Value-at-Risk computation in a non-normal framework.

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