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A Note on Homomorphisms and Anti-Homomorphisms on $*$-Ring[1]

Nadeem ur Rehman[†,2], Abu Zaid Ansari[†] and Claus Haetinger[‡]

[†]Department of Mathematics, Aligarh Muslim University
Aligarh-202002, India
e-mail : rehman100@gmail.com (N. Rehman)
ansari.abuzaid@gmail.com (A.Z. Ansari)

[‡]Center of Exact and Technological Sciences - CETEC
Univates University Center 95900-000, Lajeado-RS, Brazil
e-mail : chaet@univates.br

Abstract : In this paper we describe generalized left $*$-derivation $F : R \to R$ in $*$-prime ring and prove that if F acts as homomorphism or anti-homomorphism on R, then either R is commutative or F is a right $*$-centralizer on R. Analogous results have been proved for generalized left $*$-biderivation and Jordan $*$-centralizer on R.

Keywords : semiprime (prime) ring; involution; generalized left $*$-derivation; generalized left $*$-biderivation; Jordan $*$-centralizer.
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1 Introduction

Throughout the present paper R will denote an associative ring with center $Z(R)$. Recall that R is prime if $aRb = (0)$ implies that $a = 0$ or $b = 0$ and R is said to semiprime ring if $aRa = (0)$ implies that $a = 0$. As usual $[x, y]$ will denote the commutator $xy - yx$. We shall make an extensive use of commutator identities; $[x, yz] = [x, y]z + y[x, z]$ and $[xy, z] = [x, z]y + x[y, z]$. Let S be a

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[2]Corresponding author.

nonempty subset of R. A function $f : R \to R$ is said to be a centralizing on S if $[f(x), x] \in Z(R)$ for all $x \in S$. In particular, if $[f(x), x] = 0$ for all $x \in S$, f is said to be commuting on S. An additive mapping $* : R \to R$ is said to be an involution if it satisfies $(xy)^* = y^*x^*$ and $(x^*)^* = x$ for all $x, y \in R$. A ring R equipped with involution $*$ is called a ring with involution or $*$-ring. An additive mapping $\delta : R \to R$ is called a derivation (resp. Jordan derivation) if $\delta(xy) = \delta(x)y + x\delta(y)$ (resp. $\delta(x^2) = \delta(x)x + x\delta(x)$) holds for all $x, y \in R$. An additive mapping $H : R \to R$ is called a generalized derivation if there exists a derivation $\delta : R \to R$ such that $H(xy) = H(x)y + x\delta(y)$ holds for all $x, y \in R$. In 1990 Bresar and Vukman [1] introduced the concept of left derivation as follows: An additive mapping $d : R \to R$ is called left derivation if $d(xy) = xd(y) + yd(x)$ holds for all $x, y \in R$. They proved that a prime ring which admits a nonzero left derivation is commutative. Obviously in a commutative ring, derivations (resp. generalized derivations) act as a left derivations (resp. generalized left derivations). However in noncommutative ring, the case is quite different in general.

According to [2], an additive mapping $F : R \to R$ is called a generalized left derivation (resp. generalized Jordan left derivation) if there exists a left derivation (resp. Jordan left derivation) $d : R \to R$ such that $F(xy) = xF(y) + yd(x)$ (resp. $F(x^2) = xF(x) + xd(x)$) holds for all $x, y \in R$. In [3] Bresar and Vukman introduced the concept of $*$-derivation as follows: Let R be a $*$-ring. An additive mapping $\delta : R \to R$ is said to be a $*$-derivation on R if $\delta(xy) = \delta(x)y^* + x\delta(y)$ holds for all $x, y \in R$. An additive mapping $H : R \to R$ is called generalized $*$-derivation if there exists a $*$-derivation δ such that $H(xy) = H(x)y^* + x\delta(y)$ for all $x, y \in R$.

Let S be a subring of a ring R. A mapping $B : R \times R \to R$ is said to be symmetric if $B(x, y) = B(y, x)$ for all $x, y \in R$. Following [4], a biadditive map $B : R \times R \to R$ is called a biderivation on S if it is a derivation in each argument, i.e., for every $x \in S$, maps $y \mapsto B(x, y)$ and $y \mapsto B(y, x)$ are derivations of S into R (viz. [5], where biderivations satisfying some special properties are studied). Typical examples are mappings of the form $(x, y) \mapsto c[x, y]$ where c is an element of the center of R. The notion of biderivation arises naturally in the study of additive commuting maps, since every commuting additive map $f : S \to R$ gives to rise a biderivation of S. Namely, linearization of the $[f(x), x] = 0$ for all $x \in S$ yields that $[f(x), y] = [x, f(y)]$ for all $x, y \in S$. Therefore, we note that the map $(x, y) \mapsto [f(x), y]$ is a biderivation. The concept of biderivation was introduced by Maska [6]. Further, Bresar [4] showed that every biderivation B of a noncommutative prime ring R is of the form $B(x, y) = \lambda[x, y]$ for some $\lambda \in C$, the extended centroid of R. In 2011 Shakir [7] introduced the concept of left biderivations and generalized left biderivations, which are defined as follows: a biadditive mapping $B : R \times R \to R$ is called a left biderivation (resp. Jordan left biderivation) if $B(xy, z) = xB(y, z) + yB(x, z)$ and $B(z, xy) = xB(z, y) + yB(z, x)$(resp. $B(x^2, z) = 2xB(x, z)$) and $B(z, x^2) = 2xB(z, x)$) hold for all $x, y, z \in R$. A biadditive mapping $G : R \times R \to R$ is called a generalized left biderivation (resp. generalized Jordan left biderivation) if there exists a left biderivation (Jordan left biderivation) $B : R \times R \to R$ such that $G(xy, z) = xG(y, z) + yB(x, z)$ and $G(z, xy) = xG(z, y) + yB(z, x)$ (resp.

$G(x^2, z) = xG(x, z) + xB(x, z)$ and $G(z, x^2) = xG(z, x) + xB(z, x)\big)$ hold for all $x, y, z \in R$.

2 Generalized Left $*$-Derivation

Motivated by the definition of $*$-derivation and generalized $*$-derivation, we introduce the notions of left $*$-derivation and generalized left $*$-derivation as follows: let R be a $*$-ring. An additive mapping $d : R \to R$ is said to be a left $*$-derivation if $d(xy) = x^*d(y) + yd(x)$ for all $x, y \in R$. An additive mapping $F : R \to R$ is said to be a generalized left $*$-derivation if there exists a left $*$-derivation d such that $F(xy) = x^*F(y) + yd(x)$ for all $x, y \in R$. The concept of generalized left $*$-derivations cover the concept of left $*$-derivations. Moreover, a generalized left $*$-derivation with $d = 0$ includes the concept of right $*$-centralizer (or right $*$-multiplier) i.e., an additive mapping $T : R \to R$ satisfying $T(xy) = x^*T(y)$ for all $x, y \in R$. Bell and Kappe [8] discussed derivations acting as a homomorphism or an anti-homomorphism on a nonzero right ideal of a prime ring. Recall that an additive mapping f from a ring R into itself is said to act as a homomorphism or as an anti-homomorphism on S, an additive subgroup of R, if for each pair $x, y \in S$, either $f(xy) = f(x)f(y)$ or $f(xy) = f(y)f(x)$ holds. Certainly the concept of mappings acting as a homomorphism on S can be defined in similar way. In [7], Shakir proved some results taking generalized left derivation of a prime ring R which acts either as s homomorphism or as an anti-homomorphism on a certain well behaved subset of R. The aim of the present section is to extend the previous results in the setting of generalized left $*$-derivation which acts either as a homomorphism or as an anti-homomorphism on prime $*$-ring R. More precisely, we prove the following:

Theorem 2.1. *Let R be a $*$-prime ring. Suppose that $F : R \to R$ is a generalized left $*$-derivation with associated left $*$-derivation on R.*

- (i) *If F acts as a homomorphism on R, then either R is commutative or F is right $*$-centralizer on R.*
- (ii) *If F acts as an anti-homomorphism on R, then either R is commutative or F is right $*$-centralizer on R.*

Proof. (i) Since F acts as a homomorphism on R, $F(xy) = F(x)F(y)$ for all $x, y \in R$ and also from the definition of generalized left $*$-derivation, we have $F(xy) = x^*F(y) + yd(x)$ for all $x, y \in R$, where d is a left $*$-derivation of R.
This yields that

$$\begin{aligned} F(xyz) = F(x(yz)) &= x^*F(yz) + yzd(x) \\ &= x^*F(y)F(z) + yzd(x) \text{ for all } x, y, z \in R. \end{aligned} \tag{2.1}$$

On the other hand

$$\begin{aligned} F(xyz) = F((xy)z) &= F(xy)F(z) \\ &= x^*F(y)F(z) + yd(x)F(z) \text{ for all } x, y, z \in R. \end{aligned} \tag{2.2}$$

Now, combining the relations (2.1) and (2.2), we obtain $x^*F(y)F(z) + yzd(x) = x^*F(y)F(z)+yd(x)F(z)$ for all $x, y, z \in R$. This yields that $y\big(zd(x)-d(x)F(z)\big) = 0$ for all $x, y, z \in R$. Multiplying left side by $zd(x)-d(x)F(z)$ to the above relation, we obtain $\big(zd(x) - d(x)F(z)\big)y\big(zd(x) - d(x)F(z)\big) = 0$ for all $x, y, z \in R$. Then primeness of R forces that

$$zd(x) - d(x)F(z) = 0 \text{ for all } x, z \in R. \tag{2.3}$$

Replacing x by xy in the above relation, we get

$$zd(xy) - d(xy)F(z) = 0 \text{ for all } x, y, z \in R.$$

This implies that

$$zx^*d(y) + zyd(x) - x^*d(y)F(z) - yd(x)F(z) = 0 \text{ for all } x, y, z \in R.$$

Using relation (2.3) in the above relation, we find that

$$zx^*d(y) + zyd(x) - x^*zd(y) - yzd(x) = 0 \text{ for all } x, y, z \in R.$$

This yields that

$$[z, x^*]d(y) + [z, y]d(x) = 0 \text{ for all } x, y, z \in R.$$

In particular, replacing z by x^* in the above relation, we find that

$$[x^*, y]d(x) = 0 \text{ for all } x, y \in R.$$

Putting yz for y in the above relation, we get

$$[x^*, y]zd(x) = 0 \text{ for all } x, y \in R.$$

i.e., $[x^*, y]Rd(x) = (0)$ for all $x, y \in R$. Now, consider $A = \{x \in R | [x^*, y] = 0\}$ for all $y \in R$ and $B = \{x \in R | d(x) = 0\}$. Then, each of A and B are additive subgroups of R and R is the set theoretic union of A and B. But a group can not be set theoretic union of its two proper subgroups. Hence, either $A = R$ or $B = R$. If $A = R$ then, $[x^*, y] = 0$ for all $x, y \in R$. Replacing x by x^* in the above relation, we get $[x, y] = 0$ for all $x, y \in R$. Therefore, R is commutative. Again, if $B = R$ then $d = 0$ on R. Hence, we get the required result.

(ii) Suppose that F acts as an anti-homomorphism on R. Then $F(xy) = F(y)F(x)$ for all $x, y \in R$ and also $F(xy) = x^*F(y) + yd(x)$ for all $x, y \in R$. Combining the above two relations, we get $x^*F(y)+yd(x) = F(y)F(x)$ for all $x, y \in R$. Replacing y by xy in the above relation, we obtain for all $x, y \in R$

$$x^*F(xy) + xyd(x) = F(xy)F(x)$$

i.e.,

$$x^*F(y)F(x) + xyd(x) = x^*F(y)F(x) + yd(x)F(x)$$

This implies that

$$xyd(x) = yd(x)F(x) \text{ for all } x, y \in R. \tag{2.4}$$

Replacing y by zy in the last relation, we get

$$xzyd(x) = zyd(x)F(x) \text{ for all } x, y, z \in R.$$

Using relation (2.4) in the above relation, we obtain

$$xzyd(x) = zxyd(x) \text{ for all } x, y, z \in R.$$

This yields that $[x, z]yd(x) = 0$ for all $x, y, z \in R$. Now, applying similar techniques as used in the last paragraph of the proof of (i) yields the required result. □

We immediately get the following corollary from the above theorem:

Corollary 2.2. *Let R be a $*$-prime ring. Suppose that $d : R \to R$ is a left $*$-derivation on R.*

(i) *If d acts as a homomorphism on R, then either R is commutative or d is right $*$-centralizer on R.*

(ii) *If d acts as an anti-homomorphism on R, then either R is commutative or d is right $*$-centralizer on R.*

3 Generalized Left $*$-Biderivation

Motivated by the definition of left biderivation and generalized left biderivation, we introduce the concept of left $*$-biderivation and generalized left $*$-biderivation which state as follows: A biadditive mapping $B : R \times R \to R$ is said to be a left $*$-biderivation if $B(xy, z) = x^*B(y, z) + yB(x, z)$ and $B(x, yz) = y^*B(x, z) + zB(x, y)$ for all $x, y, z \in R$. A biadditive mapping $G : R \times R \to R$ is said to be generalized left $*$-biderivation if there exists a left $*$-biderivation B on R such that $G(xy, z) = x^*G(y, z) + yB(x, z)$ and $G(x, yz) = y^*G(x, z) + zB(x, y)$ for all $x, y, z \in R$. If G is generalized left $*$-biderivation on R, and if $G(xy, z) = G(x, z)G(y, z)$ and $G(x, yz) = G(x, y)G(x, z)$ (resp. $G(xy, z) = G(y, z)G(x, z)$ and $G(x, yz) = G(z, x)G(y, x)$) for all $x, y, z \in R$, then G is said to be generalized left $*$-biderivation which acts as homomorphism (resp. anti-homomorphism) on S. In this section we study the notion of generalized left $*$-biderivation which acts as homomorphism or as an anti-homomorphism on S.

We begin our discussion with the following well known lemma due to Bresar [9]:

Lemma 3.1 ([9, Lemma 2.4])**.** *Let $G_1, G_2, \ldots, G_n$ be additive groups, R a semiprime ring. Suppose that mappings $S : G_1 \times G_2 \times \cdots \times G_n \to R$ and $T : G_1 \times G_2 \times \cdots \times G_n \to R$ are additive in each argument. If $S(a_1, a_2, \ldots, a_n)xT(a_1, a_2, \ldots, a_n) = 0$ for all $x \in R$, $a_i \in G_i$ $i = 1, 2, \ldots, n$, then $S(a_1, a_2, \ldots, a_n)xT(b_1, b_2, \ldots, b_n) = 0$ for all $x \in R, a_i, b_i \in G_i$, $i = 1, 2, \ldots, n$.*

Theorem 3.2. *Let R be a $*$-prime ring. Suppose that R admits a generalized left $*$-biderivation $G : R \times R \to R$ with associated left $*$-biderivation $B : R \times R \to R$.*

(i) *If G acts as a homomorphism on R, then either R is commutative or G is right $*$-bicentralizer on R.*

(ii) *If G acts as an anti-homomorphism on R, then either R is commutative or G is right $*$-bicentralizer on R.*

Proof. (i) By the definition of generalized left $*$-biderivation, we have $G(xy, z) = x^*G(y, z) + yB(x, z)$ for all $x, y, z \in R$. Since G acts as homomorphisms on R, then $G(xy, z) = G(x, z)G(y, z)$ for all $x, y, z \in R$. Now, consider

$$\begin{aligned} G(xyw, z) &= G(x(yw), z) \\ &= x^*G(yw, z) + ywB(x, z) \\ &= x^*G(y, z)G(w, z) + ywB(x, z) \text{ for all } x, y, z, w \in R. \end{aligned}$$

On the other hand

$$\begin{aligned} G(xyw, z) &= G((xy)w, z) \\ &= G(xy, z)G(w, z) \\ &= x^*G(y, z)G(w, z) + yB(x, z)G(w, z) \text{ for all } x, y, z, w \in R. \end{aligned}$$

Now, combining the above two relations, we get $ywB(x, z) - yB(x, z)G(w, z) = 0$ for all $x, y, z, w \in R$. This can be written as $y\big(wB(x, z) - B(x, z)G(w, z)\big) = 0$ for all $x, y, z, w \in R$. Now, multiplying left side by $wB(x, z) - B(x, z)G(w, z)$ to the above relation and using primeness of R, we obtain

$$wB(x, z) - B(x, z)G(w, z) = 0 \text{ for all } x, z, w \in R. \tag{3.1}$$

Replacing x by xy in the above relation, we get $wB(xy, z) - B(xy, z)G(w, z) = 0$ for all $x, y, z, w \in R$. This implies that $wx^*B(y, z) + wyB(x, z) - x^*B(y, z)G(w, z) - yB(x, z)G(w, z) = 0$ for all $x, y, z, w \in R$. Using relation (3.1) in the above relation, we get $wx^*B(y, z) + wyB(x, z) - x^*wB(y, z) - ywB(x, z) = 0$ for all $x, y, z, w \in R$. This can be written as $[x^*, w]B(y, z) + [y, w]B(x, z) = 0$ for all $x, y, z, w \in R$. In particular, putting $w = x^*$ in the above relation, we get $[y, x^*]B(x, z) = 0$ for all $x, y, z \in R$. Replacing y by yr in the above relation, we get $[y, x^*]rB(x, z) = 0$ for all $x, y, z, r \in R$. Then by Lemma 3.1, we get $[y, x^*]rB(t, z) = 0$ for all $x, y, z, r, t \in R$. Primeness of R forces that either $[x^*, y] = 0$ or $B(t, z) = 0$ for all $x, y, z, t \in R$. If $[x^*, y] = 0$ for all $x, y \in R$. Replacing x by x^*, we get that R is commutative and if $B(t, z) = 0$ for all $t, z \in R$, then G is right $*$-bicentralizer on R.

(ii) Since G acts as an anti-homomorphism on R, $G(xy, z) = G(y, z)G(x, z)$ for all $x, y, z \in R$ and also we have $G(xy, z) = x^*G(y, z) + yB(x, z)$ for all $x, y, z \in R$. Now, combining the above two relations, we get $x^*G(y, z) + yB(x, z) = G(y, z)G(x, z)$ for all $x, y, z \in R$. Replacing y by xy in the above relation, we get

$$x^*G(xy, z) + xyB(x, z) = G(xy, z)G(x, z) \text{ for all } x, y, z \in R.$$

This implies that

$$x^*G(y,z)G(x,z)+xyB(x,z) = x^*G(y,z)G(x,z)+yB(x,z)G(x,z) \text{ for all } x,y,z \in R.$$

That is,

$$xyB(x,z) = yB(x,z)G(x,z) \text{ for all } x,y,z \in R. \tag{3.2}$$

Replacing y by ty in the above relation, we get

$$xtyB(x,z) = tyB(x,z)G(x,z) \text{ for all } x,y,z,t \in R.$$

Using (3.2) in the above, we obtain

$$xtyB(x,z) = txyB(x,z) \text{ for all } x,y,z,t \in R.$$

This implies that

$$[x,t]yB(x,z) = 0 \text{ for all } x,y,z,t \in R.$$

Now, using Lemma 3.1, we get

$$[x,t]yB(s,z) = 0 \text{ for all } x,y,z,t,s \in R.$$

Using primeness of R, we get either R is commutative or G is right ∗-bicentralizer on R. □

Corollary 3.3. *Let R be a ∗-prime ring. Suppose that R admits a left ∗-biderivation $B : R \times R \to R$.*

- (i) *If B acts as a homomorphism on R, then either R is commutative or B is right ∗-bicentralizer on R.*
- (ii) *If B acts as an anti-homomorphism on R, then either R is commutative or B is right ∗-bicentralizer on R.*

4 Jordan ∗-Centralizer

Following [10], an additive mapping $T : R \to R$ is called a left (resp. right) centralizer of R if $T(xy) = T(x)y$ (resp. $T(xy) = xT(y)$) holds for all $x, y \in R$. An additive mapping $T : R \to R$ is called a Jordan left (resp. right) centralizer of R if $T(x^2) = T(x)x$ (resp. $T(x^2) = xT(x)$) holds for all $x \in R$. Obviously, every left (resp. right) centralizer is a Jordan left (resp. right) centralizer. The converse is in general not true. In [10], Zalar proved that every Jordan left (resp. right) centralizer on a 2-torsion-free semiprime ring is a left (resp. right) centralizer. Recall that an additive mapping $T : R \to R$ is said to be left Jordan ∗-centralizer (resp. right Jordan ∗-centralizer) if it satisfies $T(x^2) = T(x)x^*$ (resp. $T(x^2) = x^*T(x)$) for all $x \in R$, a ∗-ring. If T is both left as well right then T is said to be Jordan ∗-centralizer on a ∗-ring R. In the present section our aim is to study the behavior of Jordan ∗-centralizer which acts as a homomorphism or an anti-homomorphism on R.

For developing the proof of the main theorem we require the following lemma essentially proved in [11]:

Lemma 4.1 ([11, Proposition 2.1]). *Let R be a 2-torsion free semiprime ring with involution $*$. Suppose that $T : R \to R$ is an additive mapping satisfying $T(x^2) = T(x)x^*$ for all $x \in R$. Then $T(xy) = T(y)x^*$ for all $x, y \in R$.*

Theorem 4.2. *Let R be a $*$-ring. Suppose that $T : R \to R$ is a Jordan $*$-centralizer on R.*

- (i) *If R is semiprime and T acts as a homomorphism on R, then T maps R into $Z(R)$.*
- (ii) *If R is prime and T acts as an anti-homomorphism on R, then either $T = 0$ or T is an involution map.*

Proof. (i) Given that

$$T(xy) = T(x)T(y) \text{ for all } x, y \in R. \tag{4.1}$$

and also we have $T(x^2) = T(x)x^*$ for all $x \in R$. Then, using Lemma 4.1, we get $T(xy) = T(y)x^*$ for all $x, y \in R$. Combining the last expression with (4.1), we get

$$T(x)T(y) = T(y)x^* \text{ for all } x, y \in R. \tag{4.2}$$

Replacing y by zy in the above relation, we find that

$$T(x)T(zy) = T(zy)x^* \text{ for all } x, y, z \in R.$$

This implies that $T(x)T(y)z^* = T(y)z^*x^*$ for all $x, y, z \in R$. Using relation (4.2) in the previous relation, we obtain $T(y)x^*z^* = T(y)z^*x^*$ for all $x, y, z \in R$. This can be written as $T(y)[x^*, z^*] = 0$ for all $x, y, z \in R$. Replacing x by x^* and z by z^* in the above relation, we obtain $T(y)[x, z] = 0$ for all $x, y, z \in R$. Putting tx in place of x in the above relation, we obtain

$$T(y)t[x, z] = 0 \text{ for all } x, y, z, t \in R. \tag{4.3}$$

Replacing t by st in the above relation, we get

$$T(y)st[x, z] = 0 \text{ for all } x, y, z, t, s \in R. \tag{4.4}$$

Multiplying left side by s in (4.3), we obtain

$$sT(y)t[x, z] = 0 \text{ for all } x, y, z, t, s \in R. \tag{4.5}$$

Now, combining (4.4) and (4.5), we get

$$[T(y), s]t[x, z] = 0 \text{ for all } x, y, z, t, s \in R. \tag{4.6}$$

In particular, putting $s = z$ and $x = T(y)$ we find that

$$[T(y), z]t[T(y), z] = 0 \text{ for all } y, z, t \in R. \tag{4.7}$$

Semiprimeness of R forces that $[T(y), z] = 0$ for all $y, z \in R$. Hence, T maps R into $Z(R)$. Hence, we get the required result.

(ii) Now consider the case when T acts as an anti-homomorphism on R.

$$T(xy) = T(y)T(x) \text{ for all } x, y \in R. \tag{4.8}$$

Also, we have $T(x^2) = T(x)x^*$ for all $x \in R$. Then, using Lemma 4.1, we get $T(xy) = T(y)x^*$ for all $x, y \in R$. Combining the last expression with (4.8), we get

$$T(y)T(x) = T(y)x^* \text{ for all } x, y \in R. \tag{4.9}$$

This can be written as $T(y)(T(x) - x^*) = 0$ for all $x, y \in R$. Replacing y by zy in the above expression, we get $T(y)z^*(T(x) - x^*) = 0$ for all $x, y \in R$. Replacing z by z^* and using the primeness of R, we get either $T = 0$ or T is an involution. $\square$

In conclusion, it is tempting to conjecture as follows:

Conjecture 4.3. *Let R be a $*$-semiprime ring. Suppose that $T : R \to R$ is a Jordan $*$-centralizer on R and acts as an anti-homomorphism on R, then $T = 0$.*

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