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Impulsive Periodic Control System with Parameter Perturbation

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Abstract : In this paper, we study the existence of periodic solution for impulsive periodic control system with parameter perturbation on infinite dimensional space, in these cases where the differential operator involved is the infinitesimal generator of C_0 semigroup.

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1 Introduction

The impulsive differential equations appear to a natural framework for mathematical modelings of several real world phenomena. For instance, systems with impulse effects have applications in physics, in biotechnology, in population dynamics, in optimal control and so on. For an introduction to the theory of impulsive systems, we refer the reader to see in [4]. In the framework of impulsive differential equations, some existence result of periodic solutions for impulsive periodic control systems with parameter perturbation on finite dimensional space has been studied by many authors in [2] and [6].

However, the investigation of the existence of periodic solutions for impulsive periodic control systems with parameter perturbation on infinite dimensional space have not been study. We apply the semigroup theory (see [1] and [5]) and fixed point theorems (see [3] and [7]) for impulsive systems, we establish conditions for ensuring that the system has a unique periodic solution.

The organization of this paper is as follows. Firstly, in Section 2, we introduce some definition of impulsive evolution operator and prove the existence of periodic solution for homogeneous linear impulsive periodic system by using fixed point theorem and Fredholm alternative theorem. In Section 3, nonhomogeneous linear impulsive periodic control system is investigated, we prove the existence of periodic solution by using properties of compact operators and boundedness of solution. Finally, in Section 4, we prove the existence of periodic solution for impulsive periodic control system with parameter perturbation by using fixed point theorems.

2 Impulsive Evolution Operator and Homogeneous Linear Impulsive Periodic System

Throughout this paper X will denote a Banach space with norm $\|\cdot\|_X$ and $\mathcal{L}(X)$ denote the space of all bounded linear operators on X. Let $PC([0,T_0]\,;X)$ be the space of all functions $x:[0,T_0]\to X$, $x(t)$ is continuous at $t\neq\tau_k$, left continuous at $t=\tau_k$ and the right limit $x(\tau_k^+)$ exists for $k=1,2,\ldots,\sigma$, where $0=\tau_0<\tau_1<\tau_2<\ldots<\tau_{\sigma-1}<\tau_\sigma=T_0<\infty$, which is a Banach space with the norm

$$\|x\|_{PC}=\sup_{t\in[0,T_0]}\|x(t)\|_X\,.$$

In this paper, we study the existence of periodic solutions for impulsive periodic control systems with parameter perturbation on infinite dimensional space,

$$\begin{cases}\dot{x}(t)=Ax(t)+u(t)+p(t,x,\xi), & t\neq\tau_k,\\ \Delta x(t)=B_kx(t)+c_k+q_k(x,\xi), & t=\tau_k,\ \ k\in\mathbb{N}\end{cases}\tag{2.1}$$

where $\Delta x(\tau_k)=x(\tau_k^+)-x(\tau_k^-)$. Suppose that the system (2.1) satisfy the following assumptions (A1), (A2) and (A3).

(A1.1) $0=\tau_0<\tau_1<\tau_2<\ldots<\tau_k<\ldots$, $\tau_k\to\infty$ as $k\to\infty$ and there exists a positive integer σ such that $\tau_{k+\sigma}=\tau_k+T_0$ for all $k\in\mathbb{N}$.

(A1.2) A is the infinitesimal generator of a C_0-semigroup $\{T(t),t\geq 0\}$ in X.

(A1.3) $B_k\in\mathcal{L}(X)$ such that $B_{k+\sigma}=B_k$ for all $k\in\mathbb{N}$.

(A2.1) $u\in PC([0,\infty)\,,X)$ such that $u(t+T_0)=u(t)$ for all $t\geq 0$.

(A2.2) $c_k\in X$ such that $c_{k+\sigma}=c_k$ for all $k\in\mathbb{N}$.

(A3.1) For each $\rho>0$ and $x\in\mathcal{B}_\rho:=\{x\in X\mid\|x\|_X\leq\rho\}$. $p(\cdot,x,\xi)\in PC([0,\infty),X)$ such that $p(t+T_0,x,\xi)=p(t,x,\xi)$ for all $(t,x,\xi)\in[0,\infty)\times\mathcal{B}_\rho\times[0,\xi_0]$.

(A3.2) $q_k\in C(\mathcal{B}_\rho\times[0,\xi_0],X)$ such that $q_{k+\sigma}(x,\xi)=q_k(x,\xi)$ for all $k\in\mathbb{N}$ and $(x,\xi)\in\mathcal{B}_\rho\times[0,\xi_0]$.

(A3.3) there exists a nonnegative function $\chi(\xi)$ such that

$$\|p(t,x,\xi)\|_X\leq\chi(\xi),\ \ \|q_k(x,\xi)\|_X\leq\chi(\xi)\quad\text{and}\quad\lim_{\xi\to 0}\chi(\xi)=\chi(0)=0\tag{2.2}$$

for all $k\in\mathbb{N}$ and $(t,x,\xi)\in[0,\infty)\times\mathcal{B}_\rho\times[0,\xi_0]$.

For the system (2.1), we give the following definition.

Definition 2.1 Let Assumption (A1) hold. An operator value function $U(t,s)$ with values in $\mathcal{L}(X)$, defined on the triangle $\Delta \equiv \{0 \le s \le t \le a\}$ with $t,s \in (\tau_{k-1}, \tau_k]$ for all $k \in \mathbb{N}$, given by

$$U(t,s) = \begin{cases} T(t-s), & \tau_{k-1} \le s \le t \le \tau_k, \\ T(t-\tau_k)(I+B_k)T(\tau_k - s), & \tau_{k-1} < s \le \tau_k < t \le \tau_{k+1}, \\ T(t-\tau_k)\left[\prod_{j=i+1}^{k} (I+B_j)T(\tau_j - \tau_{j-1})\right](I+B_i)T(\tau_i - s), & \\ \qquad \text{for } i<k,\ \tau_{i-1} < s \le \tau_i < \ldots < \tau_k < t \le \tau_{k+1}. \end{cases} \tag{2.3}$$

is called an *impulsive evolution operator.*

Proposition 2.2 *Let assumption (A1) hold and* $\{U(t,s), 0 \le s \le t \le a\}$ *be a family of impulsive evolution operators. For each fixed* $T_0 = \tau_\sigma > 0$*, then the following are satisfied :*

(i) $U(t,t) = I$, *the identity operator on* X *;*

(ii) $U(t,s) = U(t,r)U(r,s)$ *for all* $0 \le s \le r \le t \le a$ *;*

(iii) $U(t+KT_0, s+KT_0) = U(t,s)$ *for all* $K \in \mathbb{N}$ *and* $0 \le s \le t \le T_0$ *with* $T_0 \le a$.

(iv) $U(t,0) = U(\bar{t},0)[U(T_0,0)]^M$ *where* $t = \bar{t} + MT_0$ *for all* $\bar{t} \in [0,T_0]$ *and* $M \in \mathbb{N} \cup \{0\}$.

Corollary 2.3 *Let assumption (A1) hold and* $\{U(t,s) : 0 \le s \le t \le a\}$ *be a family of impulsive evolution operators, then*

$$\sup_{0 \le s \le t \le a} \|U(t,s)\|_{\mathcal{L}(X)} < \infty \quad \textit{for all} \quad a > 0.$$

Definition 2.4 A function $x \in PC([0,\infty);X)$ is said to be *a mild solution* of the system (2.1) with initial condition $x(0) = x_0$ if x is given by

$$x(t) = U(t,0)x_0 + \int_0^t U(t,s)[u(s) + p(s,x,\xi)]ds + \sum_{0 \le \tau_k < t} U(t,\tau_k)[c_k + q_k(x,\xi)]. \tag{2.4}$$

Definition 2.5 A function $x \in PC([0,\infty);X)$ is said to be *a periodic solution* of the system (2.1) if there exists $T_0 > 0$ such that $x(t+T_0) = x(t)$ for all $t \ge 0$.

Definition 2.6 Function $x \in PC([0,\infty);X)$ is said to be *a* T_0*-periodic solution* of the system (2.1) if $x(t+T_0) = x(t)$ for all $t \ge 0$.

First, we consider the homogeneous linear impulsive periodic system,

$$\begin{cases} \dot{x}(t) = Ax(t), & t \neq \tau_k, \\ \Delta x(t) = B_k x(t), & t = \tau_k,\ k \in \mathbb{N}. \end{cases} \tag{2.5}$$

where $\Delta x(\tau_k) = x(\tau_k^+) - x(\tau_k^-)$ and satisfies the assumption (A1).

For the system (2.5), we give the following definition.

Definition 2.7 A function $x \in PC([0,\infty)\,;X)$ is said to be *a mild solution* of the system (2.5) with initial condition $x(0) = x_0$ if x is given by

$$x(t) = U(t,0)x_0$$

where

$$U(t,0) = \begin{cases} T(t), & 0 \leq t \leq \tau_1, \\ T(t-\tau_k)\left[\prod_{j=1}^{k}(I+B_j)T(\tau_j - \tau_{j-1})\right], & \tau_k < t \leq \tau_{k+1}, \end{cases} \tag{2.6}$$

for all $k \in \mathbb{N}$.

Remark 2.8 If $\{T(t), t > 0\}$ is a compact semigroup in X, then $U(t,0)$ is a compact operator. Particularly, $U(T_0,0)$ is also a compact operator.

Theorem 2.9 *Let assumption (A1) hold. The system (2.5) has a periodic solution if and only if the operator $U(T_0,0)$ has a fixed point $x_0 \in X$.*

Proof. Let $x(t)$ be a periodic solution of system (2.5). Suppose $x(0) = x_0$ be the initial condition of system (2.5), then $x(T_0) = x(0) = x_0$. Since $x(T_0) = U(T_0,0)x_0$, then $x_0 = U(T_0,0)x_0$. That is, the operator $U(T_0,0)$ has a fixed point $x_0 \in X$. Conversely, assume that x_0 be a fixed point of $U(T_0,0)$. Use x_0 as the initial condition of system (2.5), then the solution is $x(t) = U(t,0)x_0$ where $t = \bar{t} + MT_0$ for all $\bar{t} \in [0,T_0]$ and $M \in \mathbb{N} \cup \{0\}$. By assumption and Proposition 2.2 (4), we have $x(t) = x(\bar{t} + MT_0) = U(\bar{t},0)[U(T_0,0)]^M x_0 = U(\bar{t},0)x_0 = x(\bar{t})$. Hence x is a periodic solution of system (2.5). □

Theorem 2.10 *Let assumption (A1) hold. Furthermore, assume that A is the infinitesimal generator of a compact semigroup $\{T(t), t > 0\}$ in X. Then system (2.5) either has a unique trivial solution or have finitely many linearly independent nontrivial periodic solutions in $PC([0,\infty)\,, X)$.*

Proof. Since $U(T_0,0) : X \to X$ is a compact linear operator, then by applying Fredholm alternative theorem (see[3]), we obtain $U(T_0,0)$ satisfy Fredholm alternative that either (a) or (b) holds: (a)The homogenous equations $[I - U(T_0,0)]x = 0$

have only the trivial solution $x = 0$. That is, $U(T_0, 0)$ has only a unique fixed point $x = 0$ (i.e., by theorem 2.9, this means that system (2.5) has a unique trivial solution). (b) The homogenous equations $[I - U(T_0, 0)]x = 0$ have nontrivial solutions, then all of linearly independent nontrivial solutions are finite. Suppose all of nontrivial solutions $x_0^1, x_0^2, \ldots, x_0^m$ be such that $[I - U(T_0, 0)]x_0^i = 0$, $i = 1, 2, \ldots, m$. So $x_0^1, x_0^2, \ldots, x_0^m$ are fixed points of $U(T_0, 0)$. Again by Theorem 2.9, this means that system (2.5) have periodic solutions, say $x^1, x^2, \ldots, x^m$ where x^i are the solutions of system (2.5) corresponding to initial conditions $x^i(0) = x_0^i$, $i = 1, 2, \ldots, m$. Hence the number of linearly independent nontrivial periodic solutions of system (2.5) are finite. □

3 Nonhomogeneous Linear Impulsive Periodic Control System

We consider the following nonhomogeneous linear impulsive periodic control system,

$$\begin{cases} \dot{x}(t) = Ax(t) + u(t), & t \neq \tau_k, \\ \Delta x(t) = B_k x(t) + c_k, & t = \tau_k,\ k \in \mathbb{N} \end{cases} \tag{3.1}$$

where $\Delta x(\tau_k) = x(\tau_k^+) - x(\tau_k^-)$ and A is the infinitesimal generator of a compact semigroup $\{T(t), t > 0\}$ in X. Suppose that system (3.1) satisfy the assumptions (A1) and (A2).

For system (3.1), we give the following definition.

Definition 3.1 A function $x \in PC([0, \infty), X)$ is said to be a *mild solution* of system (3.1) with initial condition $x(0) = x_0$ if x is given by

$$x(t) = U(t, 0)x_0 + \int_0^t U(t, s)u(s)ds + \sum_{0 \leq \tau_k < t} U(t, \tau_k)c_k, \tag{3.2}$$

for all $k \in \mathbb{N}$.

To be able to apply the method in Pazy [5], we also need the following lemma.

Lemma 3.2 ([5]) *Consider the nonhomogeneous initial value problem*

$$\begin{cases} \dot{x}(t) = Ax(t) + u(t), & t > 0; \\ x(0) = x_0. \end{cases} \tag{3.3}$$

If $u \in L^1([0, T_0], X)$, then for every $x_0 \in X$ the initial value problem (3.3) has a unique solution which satisfies

$$x(t) = T(t)x_0 + \int_0^t T(t - s)u(s)ds, \quad 0 \leq t \leq T_0. \tag{3.4}$$

Theorem 3.3 *If assumptions (A1) and (A2) hold, then system (3.1) has a unique mild solution* $x \in PC([0, T_0], X)$.

Proof. For $t \in [0, \tau_1]$, Lemma 3.2 implies that system

$$\dot{x}(t) = Ax(t) + u(t), \quad 0 \le t \le \tau_1, \quad x(0) = x_0, \tag{3.5}$$

has a unique mild solution on $I_1 = [0, \tau_1]$ which satisfies

$$x_1(t) = \quad T(t)x_0 + \int_0^t T(t-s)u(s)ds, \quad t \in [0, \tau_1]. \tag{3.6}$$

Now, define

$$x_1(\tau_1) = T(\tau_1)x_0 + \int_0^{\tau_1} T(\tau_1 - s)u(s)ds, \tag{3.7}$$

so that $x_1(\cdot)$ is left continuous at τ_1.
Next, on $I_2 = (\tau_1, \tau_2]$, consider system

$$\dot{x}(t) = Ax(t) + u(t), \quad \tau_1 < t \le \tau_2, \quad x_1(\tau_1^+) = (I + B_1)x_1(\tau_1) + c_1, \tag{3.8}$$

Since $x_1 \in X$, we can use Lemma 3.2 again to get a unique mild solution on $(\tau_1, \tau_2]$ which satisfying

$$x_2(t) = T(t - \tau_1)\left[(I + B_1)x_1(\tau_1) + c_1\right] + \int_{\tau_1}^t T(t-s)u(s)ds. \tag{3.9}$$

Now, define $x_2(\tau_2)$ accordingly so that $x_2(\cdot)$ is left continuous at τ_2.

It is easily seen that Lemma 3.2 can be applied to interval $(\tau_1, \tau_2]$ to verify that $x_2(\tau_2) \in X$. It is also easily seen that this procedure can be repeated on $I_k = (\tau_{k-1}, \tau_k],\ k = 3, 4, \ldots, \sigma\ (\tau_\sigma = T_0)$ to get a mild solutions

$$x_k(t) = \quad T(t - \tau_{k-1})\left[(I + B_{k-1})x_{k-1}(\tau_{k-1}) + c_{k-1}\right] + \int_{\tau_{k-1}}^t T(t-s)u(s)ds.$$

for $t \in (\tau_{k-}, \tau_k]$ and define $x_k(\tau_k)$ accordingly with $x_k(\cdot)$ left continuous at τ_k and $x_k(\tau_k) \in X,\ k = 1, 2, \ldots, \sigma$.

Thus we obtain $x \in PC([0, T_0], X)$ is a unique mild solution of system (3.1) and given by.

$$x(t) = \begin{cases} x_1(t), & 0 \le t \le \tau_1, \\ \\ x_k(t), & \tau_{k-1} < t \le \tau_k,\ k = 2, 3, \ldots, \sigma. \end{cases}$$

□

Next, by mathematical induction to show that (3.2) is satisfied on $[0, T_0]$. First, (3.2) is satisfied on $[0, \tau_1]$. If (3.2) is satisfied on $(\tau_{k-1}, \tau_k]$, then for $t \in (\tau_k, \tau_{k+1}]$,

$$
\begin{aligned}
x(t) &= x_{k+1}(t) = T(t-\tau_k)\left[(I+B_k)x_k(\tau_k)+c_k\right] + \int_{\tau_k}^{t} T(t-s)u(s)ds \\
&= T(t-\tau_k)(I+B_k)x(\tau_k) + T(t-\tau_k)c_k + \int_{\tau_k}^{t} T(t-s)u(s)ds \\
&= T(t-\tau_k)(I+B_k)\Big[U(\tau_k,0)x_0 + \int_0^{\tau_k} U(\tau_k,s)u(s)ds + \sum_{0\le\tau_i<\tau_k} U(\tau_k,\tau_i)c_i\Big] \\
&\quad + T(t-\tau_k)c_k + \int_{\tau_k}^{t} T(t-s)u(s)ds \\
&= U(t,0)x_0 + \int_0^{\tau_k} U(t,s)u(s)ds + \int_{\tau_k}^{t} U(t,s)u(s)ds \\
&\quad + \textstyle\sum_{0\le\tau_i<\tau_k} U(t,\tau_i)c_i + U(t,\tau_k)c_k \\
&= U(t,0)x_0 + \int_0^{t} U(t,s)u(s)ds + \sum_{0\le\tau_i<t} U(t,\tau_i)c_i.
\end{aligned}
$$

Thus (3.2) is also true on $(\tau_k, \tau_{k+1}]$. Therefore (3.2) is true on $[0, T_0]$.

If $x(t)$ is T_0-periodic solution of system (3.1), then we have $x(T_0) = x(0)$; namely,

$$
[I - U(T_0,0)]x(0) = \int_0^{T_0} U(T_0,s)u(s)ds + \sum_{0\le\tau_k<T_0} U(T_0,\tau_k)c_k. \tag{3.10}
$$

We consider into 2 cases.

Case 1 : $[I - U(T_0,0)]^{-1}$exists

Theorem 3.4 *Let assumptions (A1) and (A2) hold. Assume that* $[I-U(T_0,0)]^{-1}$ *exists and system (2.5) has no nontrivial periodic solution , then system (3.1) has a unique* T_0*-periodic solution*

$$
\begin{aligned}
x_{T_0}(t) = U(t,0)[I - U(T_0,0)]^{-1} &\left(\int_0^{T_0} U(T_0,s)u(s)\, ds \right. \\
&\left. + \sum_{0\le\tau_k<T_0} U(T_0,\tau_k)c_k \right) + \int_0^{t} U(t,s)u(s)ds \\
&+ \sum_{0\le\tau_k<t} U(t,\tau_k)c_k.
\end{aligned} \tag{3.11}
$$

Proof. Suppose that $[I - U(T_0, 0)]^{-1}$ exists and system (2.5) has only trivial solution. Then (3.10) gives

$$x(0) = [I - U(T_0, 0)]^{-1}\left(\int_0^{T_0} U(T_0, s)u(s)ds + \sum_{0\le\tau_k<T_0} U(T_0, \tau_k)c_k\right) := x_0.$$

Substitute $x(0) = x_0$ into equation (3.2), we get

$$\begin{aligned} x(t) = U(t, 0)[I - U(T_0, 0)]^{-1}&\left(\int_0^{T_0} U(T_0, s)u(s)\ ds\right.\\ &\left.+ \sum_{0\le\tau_k<T_0} U(T_0, \tau_k)c_k\right) + \int_0^t U(t, s)u(s)ds \\ &+ \sum_{0\le\tau_k<t} U(t, \tau_k)c_k. \end{aligned} \tag{3.12}$$

which is a mild solution of system (3.1).

Next, we want to show that a mild solution is unique and is T_0-periodic. Suppose that $y(t) = x(t + T_0)$ is a mild solution of system (3.1).
By Proposition 2.2(3), we obtain

$$\begin{aligned} y(t) = x(t + T_0) &= U(t + T_0, 0)x_0 + \int_0^{t+T_0} U(t + T_0, s)u(s)ds \\ &+ \sum_{0\le\tau_k<t+T_0} U(t + T_0, \tau_k)c_k \\ &= U(t + T_0, T_0)U(T_0, 0)x_0 + \int_0^{T_0} U(t + T_0, s)u(s)ds + \sum_{0\le\tau_k<T_0} U(t + T_0, \tau_k)c_k \\ &+ \int_{T_0}^{t+T_0} U(t + T_0, s)u(s)ds + \sum_{T_0\le\tau_k<t+T_0} U(t + T_0, \tau_k)c_k \\ &= U(t, 0)U(T_0, 0)x_0 + \int_0^{T_0} U(t + T_0, T_0)U(T_0, s)u(s)ds \\ &+ \sum_{0\le\tau_k<T_0} U(t + T_0, T_0)U(T_0, \tau_k)c_k + \int_0^t U(t, s)u(s)ds + \sum_{0\le\tau_k<t} U(t, \tau_k)c_k \\ &= U(t, 0)U(T_0, 0)x_0 + U(t, 0)\int_0^{T_0} U(T_0, s)u(s)ds + U(t, 0)\sum_{0\le\tau_k<T_0} U(T_0, \tau_k)c_k \\ &+ \int_0^t U(t, s)u(s)ds + \sum_{0\le\tau_k<t} U(t, \tau_k)c_k \end{aligned}$$

$$
\begin{aligned}
&= U(t,0)\left[U(T_0,0)x_0 + \int_0^{T_0} U(T_0,s)u(s)ds + \sum_{0\le\tau_k<T_0} U(T_0,\tau_k)c_k\right] \\
&\quad + \int_0^t U(t,s)u(s)ds + \sum_{0\le\tau_k<t} U(t,\tau_k)c_k \\
&= U(t,0)x(T_0) + \int_0^t U(t,s)u(s)ds + \sum_{0\le\tau_k<t} U(t,\tau_k)c_k \\
&= U(t,0)y(0) + \int_0^t U(t,s)u(s)ds + \sum_{0\le\tau_k<t} U(t,\tau_k)c_k.
\end{aligned}
$$

This implies that $y(t)$ is also a solution. By Corollary 3.2 implies that $y(t) = x(t+T_0) = x(t)$ for all $t \ge 0$. So $x(t)$ is a T_0-periodic solution of system (3.1), which is exactly (3.11). This completes the proof. □

Case 2 : $[I - U(T_0,0)]^{-1}$ does not exists

In this case, system (2.5) has nontrivial T_0-periodic solutions. Let us construct the following adjoint equation of system (2.5),

$$
\begin{cases}
\dot{y}(t) = -A^* y, & t \ne \tau_k, \\
-\Delta y(t) = B_k^* y(t), & t = \tau_k,\ k = 1,2,\ldots,\sigma
\end{cases}
\tag{3.13}
$$

where A^* is the adjoint operator of A, $0 < \tau_1 < \tau_2 < \ldots < \tau_{\sigma-1} < \tau_\sigma = T_0$ and $\Delta y(\tau_k) = y(\tau_k^+) - y(\tau_k^-)$. Suppose that system (3.13) satisfies the following assumption (A4).

(A4.1) A^* is the infinitesimal generator of the adjoint semigroup $\{T^*(t), t \ge 0\}$ in X^*;

(A4.2) $B_k^* \in \mathcal{L}(X^*)$ such that $B_{k+\sigma}^* = B_k^*$ for all $k \in \mathbb{N}$.

Definition 3.5 A function $y \in PC([0,T_0],X)$ is said to be a *periodic solution* of system (3.13) with initial condition $y(T_0) = y(0) := y_0$ if y is given by

$$
y(t) = U^*(T_0,t)y_0, \quad 0 \le t \le T_0, \tag{3.14}
$$

where

$$
U^*(T_0,t) = \begin{cases}
T^*(T_0 - t), & \tau_{\sigma-1} < t \le \tau_\sigma = T_0, \\
T^*(\tau_i - t)(I + B_i^*)\left[\prod_{j=i+1}^{k} (I+B_j)T(\tau_j - \tau_{j-1})\right]^* T^*(T_0 - \tau_k), & \\
& 0 \le \tau_{i-1} < t \le \tau_i \le \tau_\sigma = T_0,
\end{cases}
\tag{3.15}
$$

for all $i = 1,2,\ldots,\sigma-1$.

Theorem 3.6 *Let assumptions (A1) and (A2) hold. Furthermore, assume that X is a Hilbert space and $u \in L^1_{loc}([0,\infty)\,,X)$. If system (2.5) have m linearly independent periodic solutions x^1, $x^2,\ldots,$ x^m with $1 \le m \le n$ where x^i are periodic solutions of system (2.5) corresponding to initial conditions $x^i(0) = x^i_0$ for all $i = 1, 2, \ldots, m$, then*

(i) *the adjoint system (3.13) also have m linearly independent T_0-periodic solutions $y^1, y^2, \ldots, y^m$;*

(ii) *system (3.1) has a T_0-periodic solution if and only if*

$$\langle\, y\,,\, z\,\rangle = 0, \tag{3.16}$$

where $\langle\, y\,,\, z\,\rangle$ the pairing of an element $y \in X^*$ with an element $z \in X$ such that

$$[I - U^*(T_0, 0)]y = 0 \tag{3.17}$$

and $z := \int_0^{T_0} U(T_0, s)u(s)ds + \sum_{0\le\tau_k<T_0} U(T_0, \tau_k)c_k$, *or if and only if*

$$\int_0^{T_0} \langle\, y(s),\, u(s)\,\rangle ds + \sum_{0\le\tau_k<T_0} \langle\, y(\tau_k),\, c_k\,\rangle = 0. \tag{3.18}$$

Furthermore, let $x_a(t)$ be a particular T_0-periodic solution of system (3.1), then each T_0-periodic solution of system (3.1) has the form

$$x(t) = x_a(t) + \sum_{i=1}^{m} \alpha_i x^i(t),$$

where $\alpha_i,\ i = 1, 2, \ldots, m$ are constants.

Proof. (i) Suppose system (2.5) have m linearly independent periodic solutions x^1, $x^2,\ldots,$ x^m with $1 \le m \le n$ where x^i are periodic solutions of system (2.5) corresponding to initial conditions $x^i(0) = x^i_{_0}$, for all $i = 1, 2, \ldots, m$. By Theorem 2.9, this means that the equations

$$[I - U(T_0, 0)]x^i_{_0} = 0 \tag{3.19}$$

have fixed points $x^1_{_0}, x^2_{_0}, \ldots, x^m_{_0}$. Then from Theorem 8.6-3 [3], we know that the following adjoint equations of (3.19)

$$[I - U^*(T_0, 0)]y^i_{_0} = 0 \tag{3.20}$$

also have m linearly independent solutions $y^1_{_0}$, $y^2_{_0}, \ldots,$ $y^m_{_0}$. So $y^1_{_0}$, $y^2_{_0}, \ldots,$ $y^m_{_0}$ are fixed points of $U^*(T_0, 0)$. Again by Theorem 2.9, this means that system (3.13) have T_0-periodic solutions, say y^1, $y^2, \ldots,$ y^m where y^i are T_0-periodic solutions of system (3.13) corresponding to initial conditions $y^i(0) = y^i_0$, for all $i = 1, 2, \ldots, m$.

(ii) System (3.1) has a T_0-periodic solution $x(t)$ if and only if the equation

$$[I - U(T_0, 0)]x(0) = \int_0^{T_0} U(T_0, s)u(s)ds + \sum_{0 \le \tau_k < T_0} U(T_0, \tau_k)c_k := z \tag{3.21}$$

has a solution $x(0)$. It follows from Theorem 8.5-1 [3], that the above condition is equivalent to

$$\langle\, y\,,\, z\,\rangle = 0, \tag{3.22}$$

for all $y \in X^*$ satisfying

$$[I - U^*(T_0, 0)]y = 0 \tag{3.23}$$

From equation (3.22), we obtain

$$\begin{aligned}
\langle\, y\,,\, z\,\rangle = 0 \quad &\Leftrightarrow \quad \langle\, y, \int_0^{T_0} U(T_0, s)u(s)ds + \sum_{0 \le \tau_k < T_0} U(T_0, \tau_k)c_k \,\rangle = 0 \\
&\Leftrightarrow \quad \int_0^{T_0} \langle\, y, U(T_0, s)u(s)\,\rangle ds + \sum_{0 \le \tau_k < T_0} \langle\, y, U(T_0, \tau_k)c_k \,\rangle = 0 \\
&\Leftrightarrow \quad \int_0^{T_0} \langle\, U^*(T_0, s)y, u(s)\,\rangle ds + \sum_{0 \le \tau_k < T_0} \langle\, U^*(T_0, \tau_k)y, c_k \,\rangle = 0 \\
&\Leftrightarrow \quad \int_0^{T_0} \langle\, y(s), u(s)\,\rangle ds + \sum_{0 \le \tau_k < T_0} \langle\, y(\tau_k), c_k \,\rangle = 0,
\end{aligned}$$

from which we immediately have (3.18). This completes the proof. □

The following theorem guarantee the existence of periodic solution. The proof is based on boundedness property.

Theorem 3.7 *If system (3.1) has a bounded solution, then it has at least one T_0-periodic solution.*

Proof. Assume that $x(t)$ is a bounded solution of system (3.1). Then for any $t \ge 0$, we have

$$x(t) = U(t, 0)x_0 + \int_0^t U(t, s)u(s)ds + \sum_{0 \le \tau_k < t} U(t, \tau_k)c_k,$$

where $x(0) = x_0$ and

$$x(T_0) = U(T_0, 0)x_0 + \int_0^{T_0} U(T_0, s)u(s)ds + \sum_{0 \le \tau_k < T_0} U(T_0, \tau_k)c_k.$$

Define $z := \int_0^{T_0} U(T_0,s)u(s)ds + \sum_{0\le\tau_k<T_0} U(T_0,\tau_k)c_k$, then

$$x(T_0) = U(T_0,0)x_0 + z.$$

We know that the function $x(t+T_0)$ is also a solution of system (3.1) for $t\in[0,T_0]$ and its value at $t=0$ is $x(T_0)$. So

$$x(t+T_0) = U(t,0)x(T_0) + \int_0^t U(t,s)u(s)ds + \sum_{0\le\tau_k<t} U(t,\tau_k)c_k$$

and

$$x(2T_0) = U(T_0,0)x(T_0) + z = U^2(T_0,0)x_0 + [U(T_0,0)+I]z.$$

Proceeding by this way, we get

$$x(mT_0) = U^m(T_0,0)x_0 + \sum_{i=0}^{m-1} U^i(T_0,0)z \quad \text{for all } m\in\mathbb{N}. \tag{3.24}$$

By contradiction, we assume that (3.1) has no T_0-periodic solution. This means that the periodicity condition

$$x(T_0) = U(T_0,0)x_0 + z = x_0 \tag{3.25}$$

has no solution, i.e., the equation

$$[I - U(T_0,0)]x = z \tag{3.26}$$

has no solution. Then from Theorem 8.5-1 [3], we know that there is $y\in X^*$ such that

$$[I - U^*(T_0,0)]y = 0 \quad \text{and} \quad \langle y\,,\,z\rangle \neq 0. \tag{3.27}$$

The first condition means that $U^*(T_0,0)y = y$, hence

$$U^{*^m}(T_0,0)y = y, \quad \text{for all } m\in\mathbb{N}. \tag{3.28}$$

Assume that $\langle y\,,\,z\rangle = \gamma \neq 0$. Then from equation (3.24), we have

$$\begin{aligned}
\langle y\,,\,x(mT_0)\,\rangle &= \langle y\,,\,U^m(T_0,0)x_0\,\rangle + \sum_{i=0}^{m-1}\langle y\,,\,U^i(T_0,0)z\rangle \\
&= \langle U^{*^m}(T_0,0)y\,,\,x_0\,\rangle + \sum_{i=0}^{m-1}\langle U^{*^i}(T_0,0)y\,,\,z\rangle \\
&= \langle y\,,\,x_0\,\rangle + \sum_{i=0}^{m-1}\langle y\,,\,z\rangle \\
&= \langle y\,,\,x_0\,\rangle + m\gamma.
\end{aligned}$$

Letting $m \to \infty$, then

$$\lim_{m\to\infty} \langle\, y\,,\, x(mT_0)\,\rangle = \infty. \tag{3.29}$$

Since $x(t)$ is bounded solution and $y \in X^*$, then

$$|\langle\, y\,,\, x(mT_0)\,\rangle| \;\le\; \|y\|_{X^*}\|x(mT_0)\|_X \;\le\; M\|y\|_{X^*} \;<\; \infty.$$

It's contradiction to (3.29). Consequently, the assumption is not true and system (3.1) has at least one T_0-periodic solution. □

Corollary 3.8

(i) *Assume that system (3.1) has no T_0-periodic solution, then all of its solutions are unbounded for $t \ge 0$.*

(ii) *Assume that system (3.1) has a unique bounded solution for $t \ge 0$, then this solution is T_0-periodic.*

4 Impulsive Periodic Control System with Parameter Perturbation

In this section, we will find sufficient conditions for the existence of T_0-periodic solutions of system (2.1), by using the fixed point theorems of an operator acting in a Banach space (see [7]). We assume that system (2.5) has only trivial solution. Let $\xi = 0$, then system (2.1) has the same form as system (3.1) because it follows from (2.2) that $p(t, x, 0) = 0$ and $q_k(x, 0) = 0$. It follows from Theorem 3.4, that system (2.1) has a T_0-periodic solution ;

$$\begin{aligned} x_{T_0}(t) \;&= U(t,0)[I - U(T_0,0)]^{-1}\left(\int_0^{T_0} U(T_0,s)u(s)\,ds \right.\\ &\quad \left. + \sum_{0\le\tau_k<T_0} U(T_0,\tau_k)c_k\right) + \int_0^t U(t,s)u(s)ds \\ &\quad + \sum_{0\le\tau_k<t} U(t,\tau_k)c_k, \end{aligned} \tag{4.1}$$

where $U(t,s)$ is defined in (2.3). Then we have the following theorem to show that for small ξ system (2.1) has a T_0-periodic solution which is closed to $x_{T_0}(t)$.

Theorem 4.1 *Under assumption (A1)-(A3). Let A be the infinitesimal generator of a compact semigroup $\{T(t), t > 0\}$ in X. Assume that*

(i) the T_0-periodic solution $x_{T_0}(t)$ satisfies

$$\rho_0 = \sup_{t\in[0,T_0]} \|x_{T_0}(t)\|_X < \rho \tag{4.2}$$

where ρ be any positive real number ;

(ii) *system (2.5) has only trivial solution ;*

(iii) $p(t,x,\xi)$ *and* $q_k(x,\xi)$ *satisfy Lipschitz conditions, i.e. for any* (t,x,ξ), $(t,y,\xi)\in[0,\infty)\times\mathcal{B}_\rho\times[0,\xi_0]$, *there exists a constant* $N(\xi)$ *such that*

$$\|p(t,x,\xi)-p(t,y,\xi)\|_X\le N(\xi)\|x-y\|_X$$

and
$$\|q_k(x,\xi)-q_k(y,\xi)\|_X\le N(\xi)\|x-y\|_X.$$

Then there exists $\xi_0>0$ *such that for* $\xi\in[0,\xi_0]$ *system (2.1) has a unique* T_0*-periodic mild solution* $x^\xi_{T_0}(t)$ *satisfying*

$$\|x^\xi_{T_0}(t)-x_{T_0}(t)\|_X<\rho-\rho_0 \tag{4.3}$$

and

$$\lim_{\xi\to0}x^\xi_{T_0}(t)=x_{T_0}(t) \tag{4.4}$$

uniformly on t.

Proof. Let $PC_{T_0}([0,\infty),X):=\big\{x\in PC([0,\infty),X)\mid x(t+T_0)=x(t),\forall t\ge0\big\}$. Moreover, $PC_{T_0}([0,T_0],X)$ is a Banach space with the norm

$$\|x\|_{PC_{T_0}}=\sup_{t\in[0,T_0]}\|x(t)\|_X.$$

Let us define

$$\begin{aligned}\mathcal{B}&:=\mathcal{B}(x_{T_0},\rho_1)=\{x\in PC_{T_0}([0,T_0],X)\mid \|x-x_{T_0}\|_{PC_{T_0}}\le\rho_1:=\rho-\rho_0\}\\ L_1&=\sup_{0\le s\le t\le T_0}\|U(t,s)\|_{\mathcal{L}(X)}\\ L_2&=\|[I-U(T_0,0)]^{-1}\|_{\mathcal{L}(X)}\end{aligned} \tag{4.5}$$

and an operator $\Omega:\mathcal{B}\to PC_{T_0}([0,T_0],X)$ such that

$$\begin{aligned}\Omega(x)(t):=\;&U(t,0)[I-U(T_0,0)]^{-1}\left(\int_0^{T_0}U(T_0,s)[u(s)+p(s,x(s),\xi)]\,ds\right.\\&\left.+\sum_{0\le\tau_k<T_0}U(T_0,\tau_k)[c_k+q_k(x(\tau_k),\xi)]\right)+\int_0^tU(t,s)[u(s)\\&+p(s,x(s),\xi)]ds+\sum_{0\le\tau_k<t}U(t,\tau_k)[c_k+q_k(x(\tau_k),\xi)].\end{aligned} \tag{4.6}$$

From (4.2) and (4.5), we know that if $x\in\mathcal{B}$, then

$$\|x\|_{PC_{T_0}}\le\|x-x_{T_0}\|_{PC_{T_0}}+\|x_{T_0}\|_{PC_{T_0}}\le\rho_1+\rho_0=\rho. \tag{4.7}$$

For any $x, y \in \mathcal{B}$, we have

$$\begin{aligned}
\|\Omega(x)-\Omega(y)\|_{PC_{T_0}} = & \sup_{t\in[0,T_0]} \|U(t,0)[I-U(T_0,0)]^{-1} \\
& \left(\int_0^{T_0} U(T_0,s)[p(s,x(s),\xi)-p(s,y(s),\xi)]\,ds \right. \\
& \left. + \sum_{0\le\tau_k<T_0} U(T_0,\tau_k)[q_k(x(\tau_k),\xi)-q_k(y(\tau_k),\xi)]\right) \\
& + \int_0^t U(t,s)[p(s,x(s),\xi)-p(s,y(s),\xi)]ds \\
& + \sum_{0\le\tau_k<t} U(t,\tau_k)[q_k(x(\tau_k),\xi)-q_k(y(\tau_k),\xi)]\|_X \\
\le & \; LN(\xi)\|x-y\|_{PC_{T_0}},
\end{aligned} \tag{4.8}$$

where $\quad L = L_1^2L_2T_0 + L_1^2L_2\sigma + L_1T_0 + L_1\sigma \quad$ and

$$\begin{aligned}
\|\Omega(x_{T_0})-x_{T_0}\|_{PC_{T_0}} = & \sup_{t\in[0,T_0]} \|U(t,0)[I-U(T_0,0)]^{-1} \\
& \left(\int_0^{T_0} U(T_0,s)p(s,x_{T_0}(s),\xi)\,ds \right. \\
& \left. + \sum_{0\le\tau_k<T_0} U(T_0,\tau_k)q_k(x_{T_0}(\tau_k),\xi)\right) \\
& + \int_0^t U(t,s)p(s,x_{T_0}(s),\xi)ds \\
& + \sum_{0\le\tau_k<t} U(t,\tau_k)q_k(x_{T_0}(\tau_k),\xi)\|_X \\
\le & \; L\chi(\xi).
\end{aligned} \tag{4.9}$$

Let us choose $\xi_0 > 0$ such that

$$\eta = L\sup_{|\xi|\le\xi_0} N(\xi) < 1, \qquad L\sup_{|\xi|\le\xi_0}\chi(\xi) \le \rho_1(1-\eta). \tag{4.10}$$

Assume that $\xi \in [0,\xi_0]$, then it follows from (4.8), (4.9) and (4.10) that

$$\begin{aligned}
\|\Omega(x)-\Omega(y)\|_{PC_{T_0}} \le & \; \eta\|x-y\|_{PC_{T_0}}, \\
\|\Omega(x_{T_0})-x_{T_0}\|_{PC_{T_0}} \le & \; \rho_1(1-\eta).
\end{aligned} \tag{4.11}$$

This means that $\Omega : \mathcal{B} \to \mathcal{B}$ is a contraction mapping, so Ω has a unique fixed point $x^{\xi}_{T_0} \in \mathcal{B}$ satisfy

$$\begin{aligned} x^{\xi}_{T_0}(t) = \quad & U(t,0)[I-U(T_0,0)]^{-1}\left(\int_0^{T_0} U(T_0,s)[u(s)+p(s,x^{\xi}_{T_0}(s),\xi)]\,ds\right. \\ & \left. + \sum_{0\le\tau_k<T_0} U(T_0,\tau_k)[c_k+q_k(x^{\xi}_{T_0}(\tau_k),\xi)]\right) \\ & + \int_0^t U(t,s)[u(s),p(s,x^{\xi}_{T_0}(s),\xi)]ds \\ & + \sum_{0\le\tau_k<t} U(t,\tau_k)[c_k+q_k(x^{\xi}_{T_0}(\tau_k),\xi)]. \end{aligned} \tag{4.12}$$

It is clear that $x^{\xi}_{T_0}(t)$ is a T_0-periodic solution of system (2.1) and satisfies estimate (4.3). Since we know that $\Omega(x^{\xi}_{T_0})(t) = x^{\xi}_{T_0}(t)$ for all $t \in [0,T_0]$.

Then $\|x^{\xi}_{T_0}(t) - x_{T_0}(t)\|_X = \|\Omega(x^{\xi}_{T_0})(t) - x_{T_0}(t)\|_X \le L\chi(\xi)$.
Letting $\xi \to 0$, we obtain (4.4). This completes the proof. □

The following definition and lemma will be used in the proof of Theorem 4.4.

Definition 4.2 A set $\mathcal{S} \subset PC([0,T_0],X)$ is *quasiequicontinuous* in $[0,T_0]$ if for any $\delta > 0$ there exists $\varepsilon > 0$ such that if $x \in \mathcal{S}$, t_1, $t_2 \in (\tau_{k-1},\tau_k] \cap [0,T_0]$, $k \in \mathbb{N}$ and $|t_1 - t_2| < \varepsilon$, then $\|x(t_1) - x(t_2)\|_X < \delta$.

Lemma 4.3 *A set $\mathcal{S} \subset PC([0,T_0],X)$ is relatively compact if and only if*

(i) $\mathcal{S}$ is bounded for each $x \in \mathcal{S}$,

(ii) $\mathcal{S}$ is quasiequicontinuous in $[0,T_0]$.

Theorem 4.4 *Under assumption (A1)-(A3). Let A be the infinitesimal generator of a compact semigroup $\{T(t), t > 0\}$ in X. Assume that*

(i) the T_0-periodic solution $x_{T_0}(t)$ satisfies

$$\rho_0 = \sup_{t\in[0,\infty]} \|x_{T_0}(t)\|_X < \rho; \tag{4.13}$$

(ii) system (2.5) has only trivial solution.

Then there exists $\xi_0 > 0$ such that for $\xi \in [0,\xi_0]$ system (2.1) has a unique T_0-periodic mild solution $x^{\xi}_{T_0}(t)$ satisfying

$$\|x^{\xi}_{T_0}(t) - x_{T_0}(t)\|_X \le \rho - \rho_0. \tag{4.14}$$

Proof. As in the proof of Theorem 4.1, we determine successively the number $\rho_1 = \rho - \rho_0$, the Banach space $PC_{T_0}([0, T_0], X)$, the set $\mathcal{B} := \mathcal{B}(x_{T_0}; \rho_1)$ and the operator $\Omega : \mathcal{B} \to PC_{T_0}([0, T_0], X)$ as defined in (4.6). Obviously, $\mathcal{B}$ is a non-empty bounded closed and convex set. It follows from equation (4.7) that if $x \in \mathcal{B}$, then $\|x\|_{PC_{T_0}} \leq \rho$. For any $x \in \mathcal{B}$, we have

$$\begin{aligned}
\|\Omega(x_{T_0}) - x_{T_0}\|_{PC_{T_0}} = \; & \sup_{t\in[0,T_0]} \|U(t,0)[I - U(T_0,0)]^{-1} \\
& \left(\int_0^{T_0} U(T_0, s)p(s, x_{T_0}(s), \xi)\, ds \right. \\
& \left. + \sum_{0\leq \tau_k < T_0} U(T_0, \tau_k) q_k(x_{T_0}(\tau_k), \xi) \right) \\
& + \int_0^t U(t,s)p(s, x_{T_0}(s), \xi) ds \\
& + \sum_{0\leq \tau_k < T_0} U(t, \tau_k) q_k(x_{T_0}(\tau_k), \xi)\|_X \\
\leq \; & \Big(L_1^2 L_2 T_0 + L_1^2 L_2 \sigma + L_1 T_0 + L_1 \sigma \Big) \chi(\xi).
\end{aligned}$$

So

$$\|\Omega(x_{T_0}) - x_{T_0}\|_{PC_{T_0}} \leq L\chi(\xi). \tag{4.15}$$

where $L = L_1^2 L_2 T_0 + L_1^2 L_2 \sigma + L_1 T_0 + L_1 \sigma$.

Let us choose $\xi \in [0, \xi_0]$ such that

$$L \sup_{\xi\in[0,\,\xi_0]} \chi(\xi) \leq \rho_1. \tag{4.16}$$

Then for $\xi \in [0, \xi_0]$, we have

$$\|\Omega(x_{T_0}) - x_{T_0}\|_{PC_{T_0}} \leq L\chi(\xi) \leq \rho_1, \tag{4.17}$$

From which we know that $\Omega(x) \in \mathcal{B}$ and therefore $\Omega : \mathcal{B} \to \mathcal{B}$.
It follows from (4.1), (4.13) and (4.17) that

$$\|\Omega(x_{T_0})\|_{PC_{T_0}} \leq \|\Omega(x_{T_0}) - x_{T_0}\|_{PC_{T_0}} + \|x_{T_0}\|_{PC_{T_0}} \leq \rho_1 + \rho_0 = \rho. \tag{4.18}$$

That is, the set $\mathcal{B}$ is uniformly bounded.
Let $x \in \mathcal{B}_\rho$ and $t_1, t_2 \in (\tau_{i-1}, \tau_i] \cap [0, T_0]$, $i = 1, 2, \ldots, \sigma$, where $\tau_0 = 0$ and $\tau_\sigma = T_0$.

For $0 < \varepsilon < t_1 < t_2 \leq T_0$, then we have

$$\begin{aligned}
\|(\Omega x)(t_1) - (\Omega x)(t_2)\|_X \leq\ & \|U(t_1, 0) - U(t_2, 0)\|_{\mathcal{L}(X)} \|[I - U(T_0, 0)]^{-1}\|_{\mathcal{L}(X)} \\
& \left(\int_0^{T_0} \|U(T_0, s)\|_{\mathcal{L}(X)} \|u(s) + p(s, x(s), \xi)\|_X \, ds \right. \\
& \left. + \sum_{0 \leq \tau_k < T_0} \|U(T_0, \tau_k)\|_{\mathcal{L}(X)} \|c_k + q_k(x(\tau_k), \xi)\|_X \right) \\
& + \int_0^{t_1 - \epsilon} \|U(t_1, s) - U(t_2, s)\|_{\mathcal{L}(X)} \|u(s) + p(s, x(s), \xi)\|_X ds \\
& + \int_{t_1 - \epsilon}^{t_1} \|U(t_1, s) - U(t_2, s)\|_{\mathcal{L}(X)} \|u(s) + p(s, x(s), \xi)\|_X ds \\
& + \int_{t_1}^{t_2} \|U(t_2, s)\|_{\mathcal{L}(X)} \|u(s) + p(s, x(s), \xi)\|_X ds \\
& + \sum_{0 \leq \tau_k < t} \|U(t_1, \tau_k) - U(t_2, \tau_k)\|_{\mathcal{L}(X)} \|c_k + q_k(x(\tau_k), \xi)\|_X .
\end{aligned}$$

from which we know that for any $\delta > 0$, there exists $\varepsilon > 0$ such that if $t_1 - t_2 < \varepsilon$, then $\|\Omega(x)(t_1) - \Omega(x)(t_2)\|_X < \delta$. Thus $\mathcal{B}$ is quasiequicontinuous and by Lemma 4.3, we know that the following set is relatively compact in $\mathcal{B}$;

$$\mathcal{S} = \{y \in \mathcal{B} \mid y = \Omega(x),\ x \in \mathcal{B}\}. \tag{4.19}$$

Applying Schuader's fixed point theorem, it follows that the operator Ω has a fixed point $x^{\xi}_{T_0} \in \mathcal{B}$ and satisfies equation (4.12). It is clear that $x^{\xi}_{T_0}(t)$ is a T_0-periodic solution of system (2.1). □

References

[1] N. U. Ahmed, *Semigroup theory with applications to systems and control*, Pitman research notes in mathematics series 246(1991). Harlow, U.K. : Longman scientific and technical.

[2] D. D. Bainov and P.S. Simeonov, *Impulsive Differential Equations : Periodic Solutions and Applications*, Longman Scientific and Technical, Harlow, 1993.

[3] Erwin Kreyszig, *Introductory functional analysis with applications*, John Wiley & Sons, Inc, 1978.

[4] V. Lakshmikantham, D. D. Bainov and P. S. Simenov, *Theory of Impulsive Differential Equations*, Series in modern applied mathematics. Singapore and Teaneck, NJ : World Scientific, 1989.

[5] A. Pazy, *Semigroups of linear operators and applications to partial differential equations*, Applied mathematics sciences 44(1983). New York : Springer-Verlag.

[6] Tao Yang, *Impulsive Control Theory*, Springer, New York, 2001.

[7] E. Zeider, *Nonlinear Functional Analysis and Its Applications II*, Springer, New York, 1990.

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