



Fuzzy Analysis Methods for the Estimation of Medical Service Value Model

Wiyada Kumam and Adisak Pongpullponsak¹

Department of Mathematics, Faculty of Science
King Mongkut's University of Technology Thonburi (KMUTT)
Bang Mod, Thung Kru, Bangkok 10140, Thailand
e-mail : wiyada.kum@mail.rmUTT.ac.th (W. Kumam)
adisak.pon@kmutt.ac.th (A. Pongpullponsak)

Abstract : This article develops an estimation of a model for non-surgical and surgical medical service value for informal workers, under the social security system in Thailand. By using the data for the year 2010, provided by the Social Security Office, we analyzed and established the medical service value model.

The data obtained from the fuzzy clustering analysis is used in creating a membership function in fuzzy logic. Subsequently, the result from this model, which is compensation for medical expenses, will be considered in the estimation of the monetary value of medical services, for informal workers. Moreover, the result of this method gave closer estimates to the real expenses comparing to the regression method.

Keywords : fuzzy clustering; fuzzy logic; informal workers; medical service value; regression.

2010 Mathematics Subject Classification : 94D05; 62J86; 62G08.

1 Introduction

Thailand has attempted to extend the social security program to cover informal workers. Their benefits as well as the costs of the medical service. In 2010, Pongpullponsak et al. [1] has redefined benefits on medical cost to cover informal

¹Corresponding author.

workers. Based on treatment of patients, the expenses of medical services into 3 groups composing of Group I: where patients are admitted to a hospital for treatment without surgery, Group II : where patients are admitted to a hospital for surgical treatment and Group III : where patients receive treatment but are not admitted to a hospital.

Great research has gone into creating an appropriate medical benefits for informal sector workers. In 1995, Baker and Krueger [2] studied a model for estimating costs of providing health care under workers compensation and traditional health insurance. In 2009, Ding and Zhu [3] determined a health care demand model based on the structure of health care delivery and health insurance systems in China. In 2011, Galbraith and Stone [4] argued that the theoretical basis of regression in the National Health Service allocation formulae has fallacious. In 2008, Nawata et al. [5] and [6] analyzed the length of stay at the hospital by the discrete-type proportional hazard model and that the duration of hospital stay that should be taken into account for medical service value model.

This research has previously developed a method called the Linear Regression is best on medical service value of informal workers in case of treatment without surgery, used for estimating medical costs of informal workers for the social security system in Thailand. Since the information used in the previous study is highly uncertain, we shall use the Fuzzy Clustering method, which is a more effective methodology that has been popularly used to deal with fuzzy or uncertain data. In 2011, Chen et al. [7] used the fuzzy clustering method in clustering the data of flood damage into dependent variables and independent variables. In 2012, using the fuzzy clustering method which is based on fuzzy equivalent relation, the data can be divided into three groups in which each data group is used in establishing, a logistic regression model by Kumam and Pongpullponsak [8] and [9]. In 1996, Peduzzi et al. [10] conducted a simulation study of the number of events per variable in logistic regression analysis, using fuzzy clustering method for data allocation. In 2012, Laura et al. [11] used logistic regression method to analyze the volume and nature of emergency medical calls, during severe weather events.

In analysis of medical service costs, it was found that if the uncertain data has been used in the study, this might yield unclear results. To escape such a problem, several researchers have adapted the principle of fuzzy for data analysis. In 1994, Hudson [12] proposed fuzzy logic analysis for medical expert systems. In 2011, Ho [13] developed a method for optimal evaluation of infectious medical waste disposal companies, by using the fuzzy analytic hierarchy process. In 2005, Bolotin [14] studied the fuzzification of linear regression models with indicator variables in medical decision making. In 2010, Stefan [15] designated three types of fuzzy predictions of the observed variable in the classical regression model. In 2011, Pourahmad [16] used fuzzy logistic regression based on the least squares approach with application in clinical studies.

Therefore, the purpose of this study is to develop the cost of medical service value for informal workers in case's of treatment without surgery or case's of treatment with surgery. By using the fuzzy clustering method as step1 and then step

2, using the fuzzy logic method. Subsequently, the estimates obtained from using the newly constructed model will be compared with the results from our previous model in order to select the most appropriate estimation method.

2 Methods

2.1 The Medical Service Value Model

Case 1: patients are admitted to a hospital for treatment without surgery

Pongpullponsak [1] deduced the expected value without surgical medical service of patients when they are admitted to a hospital for treatment ($E(C_{ws})$) as the following formula;

$$E(C_{ws}(t)) = \int_0^t \nu_i h_i(t) d\nu_i = d_t \int_0^t \nu_i \exp(\nu_i' \beta) d\nu_i, \tag{2.1}$$

where $E(C_{ws}(t))$ is the expected value of patients when admitted to a hospital for treatment without surgery, ν_i is random variable of medical expenses of patients i^{th} .

Then a conditional probability that the i^{th} patient staying in a hospital on the t^{th} day is function $h_i(t)$ by Nawata et al. [5, 6] in the following form:

$$p_i(t) = \begin{cases} h_i(t), & t = 1 \\ [\prod_{s=1}^{t-1} \{1 - h_i(s)\}] h_i(t), & t \geq 2, i = 1, 2, \dots, n, \end{cases} \tag{2.2}$$

and

$$h_i(t) = d_t \exp(\nu_i' \beta); \quad t = 1, 2, 3, \dots, T, \tag{2.3}$$

where n is number of patients, s is number of days staying in the hospital and $s = 1, 2, \dots, t - 1$, d_t is the rate of patient staying in a hospital on day t^{th} , and β is the regression coefficients of patient condition.

In 2000, Dennis and Wage [17] studied health insurance and pension plans to investigate the relationship between employee compensation and small business owner income by using regression analysis. From the equation (1) of Pongpullponsak et al. [1] reported, the estimation of $\nu_i' \beta$ using regression method but Kumam and Pongpullponsak [8] and [9] estimation $\nu_i' \beta$ by fuzzy regression.

Case 2: patients are admitted to a hospital for surgical treatment

Pongpullponsak [1] deduced the expected value of surgical medical service of patients when they are admitted to a hospital for treatment ($E(C_s)$) as the

following formula;

$$\begin{aligned} E(C_s(t)) &= E(v + u) = E(v) + E(u) \\ &= d_t \int_0^t \nu_i \exp(v'_i \beta) d\nu_i + E(u), \end{aligned} \quad (2.4)$$

where $E(v)$ is the expected value of non-surgical medical service of patients staying in a hospital. $E(u)$ is cost of surgery but will not pay more than 10,000 baht per one visit and u is normal distribution. From (2.4), the expected value of surgical medical service of patients, when they are admitted to a hospital for treatment as below;

$$E(C_s(t)) = d_t \int_0^t \nu_i \exp(v'_i \beta) d\nu_i + \int_0^{10000} \frac{u}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{u-\mu}{\sigma}\right)^2} du, \quad (2.5)$$

Since the information used in the study is uncertain, this study is separated to analyze the data of independent variables as in the previous study by fuzzy clustering method before using it to establish a new method by fuzzy logic. Fuzzy clustering method is an effective methodology that has been popularly used to deal with fuzzy or uncertain data. Chen et al. [7] used fuzzy clustering method in clustering the data of flood damage into dependent variables and independent variables. The result of fuzzy clustering analysis by Kumam and Pongpullponsak [8] and [9], the data can be divided into three groups and each data group is then used in establishing a triangular membership function and trapezoidal membership function of independent variable.

In 2010, Stefan [15] proposed three types of fuzzy predictions of the observed variable where unknown parameters and observations are crisp number. Therefore, the medical services value model in equation (2.1) and (2.5) define the membership function by interval estimation of fuzzy prediction.

2.2 The Fuzzy Logic

Definition 2.1 ([18]). (membership function) For a set A , we define a membership function μ_A such as

$$\mu_A(x) = \begin{cases} 1, & \text{if and only if } x \in A; \\ 0, & \text{if and only if } x \notin A. \end{cases}$$

We can say that the function μ_A maps the element in the universal set X to the set $\{0, 1\}$.

$$\mu_A : X \rightarrow \{0, 1\}.$$

membership function μ_A in crisp set maps whole members in universal set X to set $\{0, 1\}$

$$\mu_A : X \rightarrow \{0, 1\}.$$

Definition 2.2 ([18]). (membership function of fuzzy set) In fuzzy sets, each elements is mapped to $[0, 1]$ by membership function.

$$\mu_A : X \rightarrow \{0, 1\}.$$

where $[0, 1]$ mean real numbers between 0 and 1 (including 0, 1).

Definition 2.3 ([18]). (α -cut set) The α -cut set A_α is made up of members whose membership is not less than α .

$$A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}$$

note that α is arbitrary. This α - cut set is a crisp set.

Definition 2.4 ([18]). (Fuzzy number) If a fuzzy set is convex and normalized, and its membership function is defined in \mathfrak{R} and piecewise continuous, it is called as "fuzzy number". So fuzzy number (fuzzy set) represents a real number interval whose boundary is fuzzy (figure 1).

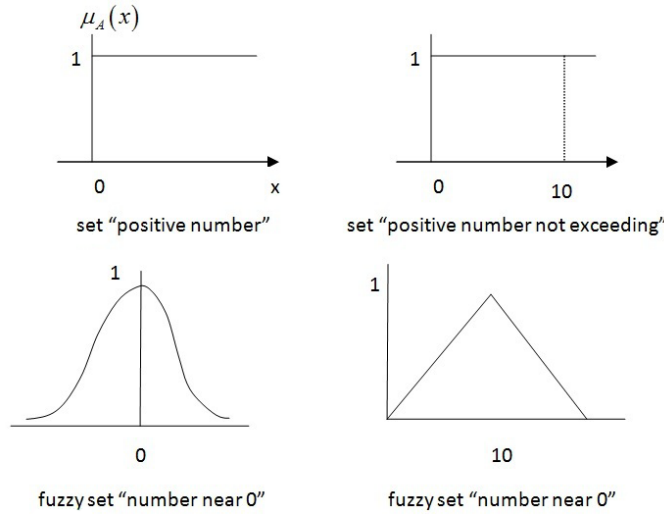
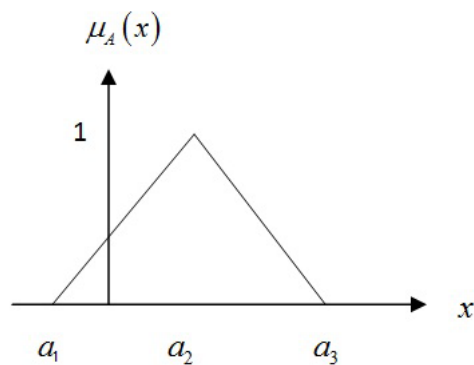


Figure 1: Sets denoting intervals and fuzzy numbers.

Definition 2.5 ([18]). (Triangular Fuzzy number) It is fuzzy number represented with three points as follows:

$A = (a_1, a_2, a_3)$ this representation is interpreted as membership functions (figure 2).

$$\mu_A(x) = \begin{cases} 0, & x < a_1 \\ \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_1 \leq x \leq a_2 \\ 0, & x > a_3. \end{cases}$$

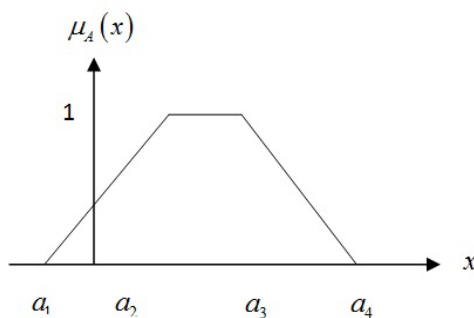
Figure 2: Triangular fuzzy number $A = (a_1, a_2, a_3)$.

Definition 2.6 ([18]). (Trapezoidal Fuzzy number) We can define trapezoidal fuzzy number A as

$$A = (a_1, a_2, a_3, a_4)$$

the membership functions of this fuzzy number will be interpreted as follows (figure 3).

$$\mu_A(x) = \begin{cases} 0 & , \quad x < a_1 \\ \frac{x-a_1}{a_2-a_1} & , \quad a_1 \leq x \leq a_2 \\ 1 & , \quad a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3} & , \quad a_3 \leq x \leq a_4 \\ 0 & , \quad x > a_4 \end{cases}$$

Figure 3: Trapezoidal fuzzy number $A = (a_1, a_2, a_3)$.

3 Main Results

Let X_1 represent the age, X_2 the gender, X_3 the income, X_4 the number of medical examinations, X_5 the number of family members, Y_1 the medical service value for case's of non-surgical medical expenses of patients staying in hospital on one day, Y_2 the medical service value for case's of non-surgical medical expenses of patients staying in a hospital on two days, Y_3 the medical service value for case's of non-surgical medical expenses of patients staying in a hospital on three days, Y_4 the medical service value for case's of surgical medical expenses of patients staying in a hospital on one day, Y_5 the medical service value for case's of surgical medical expenses of patients staying in a hospital on two days and Y_6 the medical service value for case's of surgical medical expenses of patients staying in a hospital on three days. This research defines a membership function of $X_1, X_2, X_3, X_4, X_5, Y_1, Y_2, Y_3, Y_4, Y_5$ and Y_6 as shown in Figure 4.

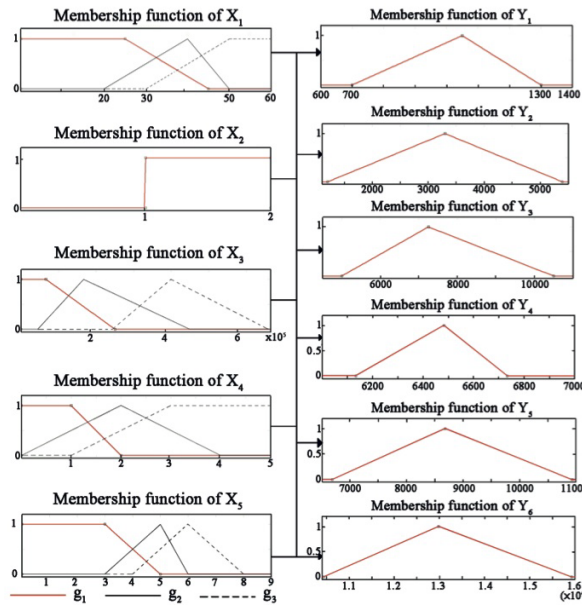


Figure 4: The Fuzzy rule of this study.

The medical services value model can be created based on fuzzy rule, where informal workers who are X_1 and X_2 and X_3 and X_4 and X_5 then Y_1 or Y_2 or Y_3 or Y_4 or Y_5 or Y_6 . Therefore, the results from medical services value model for estimation of hospital medical expenses of informal workers in Thailand can be established. The model was then used to predict the medical expenses as following. For informal workers who are 40 years of age and who are female and have an income of about 350,000 baht. Number of family members equal to 3

persons and number of medical examinations equal to 1 time then medical costs of patients are admitted to a hospital for treatment without surgery 1 day, 2 days and 3 days, the estimated medical services value will be 1010, 3340 and 7690 baht, respectively. Medical costs of patients admitted to hospital for treatment with surgery 1 day, 2 days and 3 days, the estimated medical services value will be 5740, 9070 and 12400 baht, respectively, as shown in Figure 5.

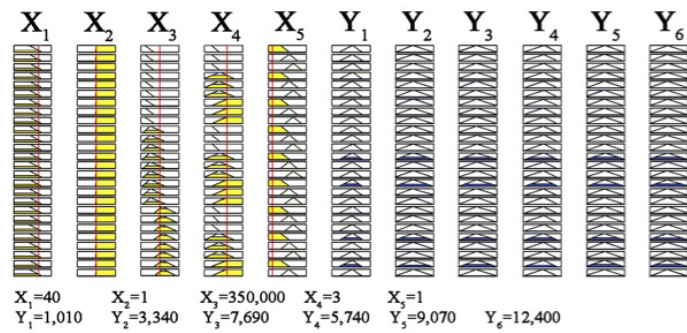


Figure 5: Calculation of medical services value by Centroid method.

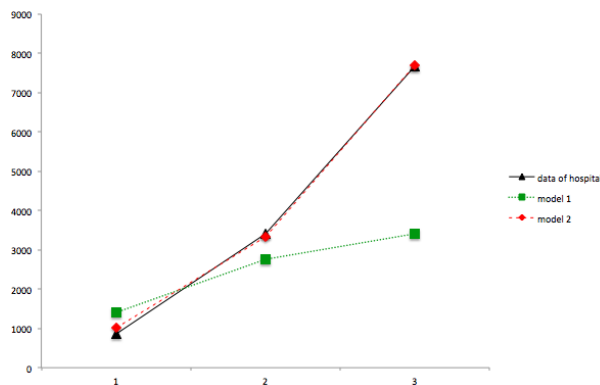


Figure 6: Comparisons of the non-surgical medical costs of informal workers between the actual expenses from data hospitals in Thailand, the estimates from the regression model (model 1) and from the fuzzy logic model (model 2).

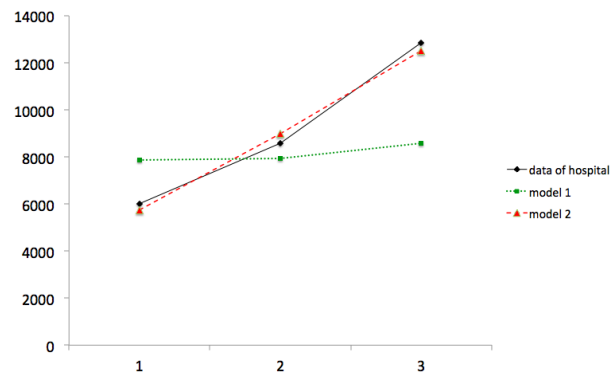


Figure 7: Comparisons of the surgical medical costs of informal workers between the actual expenses from data hospitals in Thailand, the estimates from the regression model (model 1) and from the fuzzy logic model (model 2).

From figure 6, the comparisons of the non-surgical medical costs and figure 7 comparisons of the surgical medical costs from each method with the actual expenses from data hospitals in Thailand revealed that using the model 2, the fuzzy logic method, gave the medical values a closer amount to the actual costs than those of model 1.

4 Conclusion

In this study, an analysis of the medical service value of informal workers in case of treatment without surgery and cases of treatment with surgery is shown. It is also presented that, in the estimation of fuzzy logic (model 2) has the best ability and closer to the actual medical expenses from data hospitals in Thailand than those of the regression method (model 1). For the future work, types of disease of the medical services value will be considered using the most suitable medical service value model.

Acknowledgement : The first author would like to thank the Rajamangala University of Technology Thanyaburi (RMUTT) for financial support for the Ph.D. program at King Mongkut's University of Technology Thonburi (KMUTT).

References

- [1] A. Pongpullponasak et al., Research to define the target group of the Social Security Office, Full report, 2010.

- [2] L.C. Baker, A.B. Krueger, Medical costs in workers' compensation insurance, *J. Health Economics* 14 (1995) 531–549.
- [3] J. Ding, M. Zhu, A theoretical investigation of the reformed public health insurance in urban China, Higher Education Press and Springer-Verlag 4 (2009) 1–29.
- [4] J. Galbraith, M. Stone, The abuse of regression in the National Health Service allocation formulae: response to the Department of Health's 2007 "resource allocation research paper", *J. of the Royal Statistical Society* 174 (2011) 517–528.
- [5] K. Nawata, A. Nitta, S. Watanabe, K. Kawabuchi, An analysis of hip fracture treatments in Japan by the discrete-type proportional hazard and ordered probit models, *Mathematics and Computers in Simulation* 78 (2008) 303–312.
- [6] K. Nawata, M. Ii, A. Ishiguro, K. Kawabuchi, An analysis of the length of hospital stay for cataract patients in Japan using the discrete-type proportional hazard model, *Mathematics and Computers in Simulation* 79 (2009) 2889–2896.
- [7] J. Chen, S. Zhao, H. Wang, Risk analysis of flood disaster based on fuzzy clustering, *Energy Procedia* 5 (2011) 1915–1919.
- [8] W. Kumam, A. Pongpullponsak, The linear regression and fuzzy logistic regression based medical service value models for informal workers in Thailand, *Archives Des Sciences* 65 (2012) 114–125.
- [9] W. Kumam, A. Pongpullponsak, The Fuzzy Clustering and Logistic Regression based on Medical Service Value Model of Social Security for Informal Workers, Thailand", *Proceeding of the Third KMITL-TKU Joint International Symposium on Mathematics and Applied Mathematics (MAM 2012)*, (2012) 132–140.
- [10] P. Peduzzi, J. Concato, E. Kemper, T.R. Holford, A.R. Feinstein, A simulation study of the number of events per variable in logistic regression analysis, *J. Clin Epidemiol* 49 (1996) 1373–1379.
- [11] A.M. Laura, E.L. Boone, J.P. Brooks, Analyzing the volume and nature of emergency medical calls during severe weather events using regression methodologies, *Socio-Economic Planning Sciences* 46 (2012) 55–66.
- [12] L.D. Hudson, Fuzzy logic in medical expert systems, *Engineering in Medicine and Biology Magazine* 13 (1994) 693–698.
- [13] C.C. Ho, Optimal evaluation of infectious medical waste disposal companies using the fuzzy analytic hierarchy process, *Waste Management* 31 (2011) 1553–1559.

- [14] A. Bolotin, Fuzzification of linear regression models with indicator variables in medical decision making, Proceedings of the 2005 International Conference on Computational Intelligence for Modeling, Control and Automation, and International Conference Intelligent Agents, Web Technologies and Internet Commerce (CIMCA-IAWTIC'05), 1 (2005) 572–576.
- [15] V. Stefan, Fuzzy predictions in regression models, J. Appl. Math. 3 (2010) 245–252.
- [16] S. Pourahmad, S.M.T. Ayatollahi, S.M. Taheri, Z.H. Agahi, Fuzzy logistic regression based on the least squares approach with application in clinical studies, Comp. Math. Appl. 62 (2011) 3353–3365.
- [17] W.J. Dennis, J. Wages, Health insurance and pension plans: the relationship between employee compensation and small business owner income, Small Business Economics 15 (2000) 247–263.
- [18] K.H. Lee, *First Course on Fuzzy Theory and Applications*, Springer Berlin Heidelberg New York, 2005.

(Received 7 March 2013)

(Accepted 29 April 2013)