



Note on “Common Fixed Point Results for Noncommuting Mappings Without Continuity in Cone Metric Spaces”¹

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Abstract : In this work, we prove a common fixed point theorem by using the generalized distance in a cone metric space. Our theorem extend some results of Abbas and Jungck [M. Abbas, G. Jungck, Common fixed point results for noncommuting mappings without continuity in cone metric spaces, J. Math. Anal. Appl. 341 (2008) 416–420] and Cho et al. [Y.J. Cho, R. Saadati, S.H. Wang, Common fixed point theorems on generalized distance in ordered cone metric spaces, Comput. Math. Appl. 61 (2011) 1254–1260].

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1 Introduction

In 2007, Huang and Zhang [1] introduced the concept of the cone metric space, replacing the set of real numbers by an ordered Banach space, and proved some fixed point theorems of contractive type mappings in cone metric spaces. Afterward, several fixed and common fixed point results in cone metric spaces were

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introduced in [2–6] and the references contained therein. Also, the existence of fixed and common fixed points in partially ordered cone metric spaces was studied in [7–10].

In 1996, Kada et al. [11] defined the concept of w-distance in complete metric spaces. Later, many authors proved some fixed point theorems in complete metric spaces (see [12–14]). Also, note that Saadati et al. [15] introduced a probabilistic version of the w-distance of Kada et al. [11] in a Menger probabilistic metric space. In the sequel, Cho et al. [16], and Wang and Guo [17] defined a concept of the c-distance in a cone metric space, which is a cone version of the w-distance of Kada et al. [11] and proved some fixed point theorems in ordered cone metric spaces. Then, Sintunavarat et al. [18] generalized the Banach contraction theorem on c-distance of Cho et al. [16]. Moreover, Sintunavarat and Kumam [19] and Kaewkhao et al. [20] proved some common fixed point theorems on c-distance in cone metric spaces. Recently, also, Cho et al. [21], Sintunavarat et al. [22], and Sintunavarat and Kumam [23] proved coupled fixed point theorems under weak contractions in a cone metric space.

The aim of this paper is to generalize and unify the common fixed point theorems of Huang and Zhang [1], Abbas and Jungck [2], Abbas et al. [3], Song et al. [6], Cho et al. [16], Wang and Guo [17] and Hardy and Rogers [24] on c-distance in a cone metric space.

2 Preliminaries

Definition 2.1 (See [1, 25]). Let E be a real Banach space and 0 denote the zero element in E . A subset P of E is called a *cone* if and only if

- (a) P is closed, non-empty and $P \neq \{0\}$;
- (b) $a, b \in \mathbf{R}$, $a, b \geq 0$, $x, y \in P$ implies that $ax + by \in P$;
- (c) if $x \in P$ and $-x \in P$, then $x = 0$.

Given a cone $P \subset E$, we define a partial ordering \preceq with respect to P by $x \preceq y \iff y - x \in P$.

We shall write $x \prec y$ if $x \preceq y$ and $x \neq y$. Also, we write $x \ll y$ if and only if $y - x \in \text{int}P$ (where $\text{int}P$ is interior of P). If $\text{int}P \neq \emptyset$, the cone P is called solid. The cone P is named normal if there is a number $K > 0$ such that for all $x, y \in E$, $0 \preceq x \preceq y \implies \|x\| \leq K\|y\|$.

The least positive number satisfying the above is called the normal constant of P .

Definition 2.2 (See [1]). Let X be a nonempty set and E be a real Banach space equipped with the partial ordering \preceq with respect to the cone $P \subset E$. Suppose that the mapping $d : X \times X \rightarrow E$ satisfies:

- (d1) $0 \preceq d(x, y)$ for all $x, y \in X$ and $d(x, y) = 0$ if and only if $x = y$;
- (d2) $d(x, y) = d(y, x)$ for all $x, y \in X$;

(d3) $d(x, y) \preceq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then, d is called a cone metric on X and (X, d) is called a *cone metric space*.

Definition 2.3 (See [1]). Let (X, d) be a cone metric space, $\{x_n\}$ a sequence in X and $x \in X$.

- (i) $\{x_n\}$ converges to x if for every $c \in E$ with $0 \ll c$ there exist $n_0 \in \mathbf{N}$ such that $d(x_n, x) \ll c$ for all $n > n_0$, and we write $\lim_{n \rightarrow \infty} d(x_n, x) = 0$.
- (ii) $\{x_n\}$ is called a *Cauchy sequence* if for every $c \in E$ with $0 \ll c$ there exist $n_0 \in \mathbf{N}$ such that $d(x_n, x_m) \ll c$ for all $m, n > n_0$, and we write $\lim_{n, m \rightarrow \infty} d(x_n, x_m) = 0$.
- (iii) If every Cauchy sequence in X is convergent, then X is called a *complete cone metric space*.

Lemma 2.4 (See [1, 5]). Let (X, d) be a cone metric space and P be a normal cone with normal constant K . Also, let $\{x_n\}$ and $\{y_n\}$ be sequences in X and $x, y \in X$. Then the following hold:

- (c₁) If $x_n \rightarrow x$ and $y_n \rightarrow y$ as $n \rightarrow \infty$, then $d(x_n, y_n) \rightarrow d(x, y)$ as $n \rightarrow \infty$.
- (c₂) $\{x_n\}$ is a Cauchy sequence if and only if $d(x_n, x_m) \rightarrow 0$ as $n, m \rightarrow \infty$.

Lemma 2.5 (See [8, 26]). Let E be a real Banach space with a cone P in E . Then, for all $u, v, w, c \in E$, the following hold:

- (p₁) If $u \preceq v$ and $v \ll w$, then $u \ll w$.
- (p₂) If $0 \preceq u \ll c$ for each $c \in \text{int}P$, then $u = 0$.
- (p₃) If $u \preceq \lambda u$ where $u \in P$ and $0 < \lambda < 1$, then $u = 0$.
- (p₄) Let $c \in \text{int}P$, $x_n \rightarrow 0$ and $0 \preceq x_n$. Then there exists positive integer n_0 such that $x_n \ll c$ for each $n > n_0$.

Definition 2.6 (See [16, 17]). Let (X, d) be a cone metric space. A function $q : X \times X \rightarrow E$ is called a *c-distance on X* if the following are satisfied:

- (q₁) $0 \preceq q(x, y)$ for all $x, y \in X$;
- (q₂) $q(x, z) \preceq q(x, y) + q(y, z)$ for all $x, y, z \in X$;
- (q₃) for all $n \geq 1$ and $x \in X$, if $q(x, y_n) \preceq u$ for some $u = u_x$, then $q(x, y) \preceq u$ whenever $\{y_n\}$ is a sequence in X converging to a point $y \in X$;
- (q₄) for all $c \in E$ with $0 \ll c$, there exists $e \in E$ with $0 \ll e$ such that $q(z, x) \ll e$ and $q(z, y) \ll e$ imply $d(x, y) \ll c$.

Remark 2.7 (See [16]). Each *w-distance q* in a metric space (X, d) is a *c-distance* with $E = \mathbf{R}^+$ and $P = [0, \infty)$. But the converse does not hold. Thus, the *c-distance* is a generalization of the *w-distance*.

Example 2.8 (See [16–18]).

- (1) Let (X, d) be a cone metric space and P be a normal cone. Put $q(x, y) = d(v, y)$ for all $x, y \in X$, where $v \in X$ is a fixed point. Then q is a c -distance.
- (2) Let $E = \mathbf{R}$, $P = \{x \in E : x \geq 0\}$ and $X = [0, \infty)$. Define a mapping $d : X \times X \rightarrow E$ by $d(x, y) = |x - y|$ for all $x, y \in X$. Then (X, d) is a cone metric space. Define a mapping $q : X \times X \rightarrow E$ by $q(x, y) = y$ for all $x, y \in X$. Then q is a c -distance.
- (3) Let (X, d) be a cone metric space and P be a normal cone. Put $q(x, y) = d(x, y)$ for all $x, y \in X$. Then q is a c -distance.

Remark 2.9 (See [16–18]). From Example 2.8, we have two important results

- (i) For c -distance q , $q(x, y) = 0$ is not necessarily equivalent to $x = y$ for all $x, y \in X$.
- (ii) For c -distance q , $q(x, y) = q(y, x)$ does not necessarily hold for all $x, y \in X$.

Lemma 2.10 (See [16–18]). Let (X, d) be a cone metric space and let q be a c -distance on X . Also, let $\{x_n\}$ and $\{y_n\}$ be sequences in X and $x, y, z \in X$. Suppose that $\{u_n\}$ and $\{v_n\}$ are two sequences in P converging to 0. Then the following hold:

- (qp₁) If $q(x_n, y) \preceq u_n$ and $q(x_n, z) \preceq v_n$ for $n \in \mathbf{N}$, then $y = z$. Specifically, if $q(x, y) = 0$ and $q(x, z) = 0$, then $y = z$.
- (qp₂) If $q(x_n, y_n) \preceq u_n$ and $q(x_n, z) \preceq v_n$ for $n \in \mathbf{N}$, then $\{y_n\}$ converges to z .
- (qp₃) If $q(x_n, x_m) \preceq u_n$ for $m > n$, then $\{x_n\}$ is a Cauchy sequence in X .
- (qp₄) If $q(y, x_n) \preceq u_n$ for $n \in \mathbf{N}$, then $\{x_n\}$ is a Cauchy sequence in X .

Remark 2.11. Note that Dordević et al. [27] proved this lemma for a tvs -cone metric space, where tvs is a real Hausdorff topological vector space.

Definition 2.12 (See [2]). Let X be a nonempty set and $f, g : X \rightarrow X$ be two mappings. If $fw = gw = z$ for some $z \in X$, then w is named a *coincidence point* of f and g , and z is named a *point of coincidence* of f and g .

Definition 2.13 (See [2]). Let X be a nonempty set and f and g be two self-maps defined on a set X . Then f and g are said to be *weakly compatible* if they commute at every coincidence point, that is, if $fgw = gfw$ for all coincidence points w .

3 Main Results

Our main result is the following theorem. We prove a common fixed point theorem by using c -distance and we do not require that f and g are weakly compatible. The following theorem extends and improves Theorems 2.1 and 2.3 of [2], Theorem 2.1 of [17] and Corollary 2.11 of [3] under generalized distance in a cone metric space.

Theorem 3.1. *Let (X, d) be a cone metric space, P be a normal cone with constant K and q be a c -distance on X . Suppose that the mappings $f, g : X \rightarrow X$ satisfy the following two contractive conditions:*

$$q(fx, fy) \preceq \alpha_1q(gx, gy) + \alpha_2q(gx, fx) + \alpha_3q(gy, fy) + \alpha_4q(gx, fy) + \alpha_5q(gy, fx), \tag{3.1}$$

$$q(fy, fx) \preceq \alpha_1q(gy, gx) + \alpha_2q(fx, gx) + \alpha_3q(fy, gy) + \alpha_4q(fy, gx) + \alpha_5q(fx, gy) \tag{3.2}$$

for all $x, y \in X$, where α_i for $i = 1, 2, \dots, 5$ are nonnegative constants such that

$$\alpha_1 + \alpha_2 + \alpha_3 + 2(\alpha_4 + \alpha_5) < 1. \tag{3.3}$$

If the range of g contains the range of f , $g(X)$ is a complete subspace of X , f and g satisfy

$$\inf\{\|q(fx, y)\| + \|q(gx, y)\| + \|q(gx, fx)\| : x \in X\} > 0$$

for all $y \in X$ with $y \neq fy$ or $y \neq gy$, then f and g have a common fixed point in X . If $fx = gx = x$, then $q(x, x) = 0$.

Proof. Let $x_0 \in X$ be an arbitrary point. Since the range of g contains the range of f , there exists an $x_1 \in X$ such that $fx_0 = gx_1$. By induction, a sequence $\{x_n\}$ can be chosen such that $fx_n = gx_{n+1}$ for $n = 0, 1, 2, \dots$. Now, set $x = x_{n-1}$ and $y = x_n$ in (3.1). Thus, by (q₂), for any natural number n , we have

$$\begin{aligned} q(gx_n, gx_{n+1}) &= q(fx_{n-1}, fx_n) \\ &\preceq \alpha_1q(gx_{n-1}, gx_n) + \alpha_2q(gx_{n-1}, fx_{n-1}) + \alpha_3q(gx_n, fx_n) \\ &\quad + \alpha_4q(gx_{n-1}, fx_n) + \alpha_5q(gx_n, fx_{n-1}) \\ &= \alpha_1q(gx_{n-1}, gx_n) + \alpha_2q(gx_{n-1}, gx_n) + \alpha_3q(gx_n, gx_{n+1}) \\ &\quad + \alpha_4q(gx_{n-1}, gx_{n+1}) + \alpha_5q(gx_n, gx_n) \\ &\preceq \alpha_1q(gx_{n-1}, gx_n) + \alpha_2q(gx_{n-1}, gx_n) + \alpha_3q(gx_n, gx_{n+1}) \\ &\quad + \alpha_4[q(gx_{n-1}, gx_n) + q(gx_n, gx_{n+1})] \\ &\quad + \alpha_5[q(gx_n, gx_{n+1}) + q(gx_{n+1}, gx_n)]. \end{aligned} \tag{3.4}$$

Similarly, set $x = x_{n-1}$ and $y = x_n$ in (3.2). Thus, by (q₂), for any natural number n , we have

$$\begin{aligned} q(gx_{n+1}, gx_n) &\preceq \alpha_1q(gx_n, gx_{n-1}) + \alpha_2q(gx_n, gx_{n-1}) \\ &\quad + \alpha_3q(gx_{n+1}, gx_n) + \alpha_4[q(gx_{n+1}, gx_n) + q(gx_n, gx_{n-1})] \\ &\quad + \alpha_5[q(gx_n, gx_{n+1}) + q(gx_{n+1}, gx_n)]. \end{aligned} \tag{3.5}$$

Adding up (3.4) and (3.5), we get that

$$\begin{aligned} q(gx_n, gx_{n+1}) + q(gx_{n+1}, gx_n) &\preceq (\alpha_1 + \alpha_2 + \alpha_4)[q(gx_{n-1}, gx_n) + q(gx_n, gx_{n-1})] \\ &\quad + (\alpha_3 + \alpha_4 + 2\alpha_5)[q(gx_n, gx_{n+1}) \\ &\quad + q(gx_{n+1}, gx_n)]. \end{aligned} \tag{3.6}$$

Now, set $v_n = q(gx_n, gx_{n+1}) + q(gx_{n+1}, x_n)$ in (3.6). Thus, we have

$$v_n \preceq (\alpha_1 + \alpha_2 + \alpha_4)v_{n-1} + (\alpha_3 + \alpha_4 + 2\alpha_5)v_n.$$

So, $v_n \preceq hv_{n-1}$ for all $n \geq 1$ with

$$h = \frac{\alpha_1 + \alpha_2 + \alpha_4}{1 - (\alpha_3 + \alpha_4 + 2\alpha_5)} < 1,$$

since $\alpha_1 + \alpha_2 + \alpha_3 + 2(\alpha_4 + \alpha_5) < 1$. Repeating this process, we get $v_n \preceq h^n v_0$ for $n = 0, 1, 2, \dots$. Thus,

$$q(gx_n, gx_{n+1}) \preceq v_n \preceq h^n \left(q(gx_0, gx_1) + q(gx_1, gx_0) \right) \quad (3.7)$$

for all $n = 0, 1, 2, \dots$. Let $m > n$, then it follows from (3.7) and $h < 1$ that

$$\begin{aligned} q(gx_n, gx_m) &\preceq q(gx_n, gx_{n+1}) + q(gx_{n+1}, gx_{n+2}) + \dots + q(gx_{m-1}, gx_m) \\ &\preceq (h^n + h^{n+1} + \dots + h^{m-1}) \left(q(gx_0, gx_1) + q(gx_1, gx_0) \right) \\ &\preceq \frac{h^n}{1-h} \left(q(gx_0, gx_1) + q(gx_1, gx_0) \right). \end{aligned} \quad (3.8)$$

Lemma 2.10 implies that $\{gx_n\}$ is a Cauchy sequence in X . Since $g(X)$ is a complete subspace of X , there exists a point $x' \in g(X)$ such that $gx_n \rightarrow x'$ as $n \rightarrow \infty$. By (3.8) and (q_3)

$$q(gx_n, x') \preceq \frac{h^n}{1-h} \left(q(gx_0, gx_1) + q(gx_1, gx_0) \right), \quad n = 0, 1, 2, \dots$$

Since P is a normal cone with normal constant K , we get

$$\|q(gx_n, x')\| \leq K \left(\frac{h^n}{1-h} \right) \|q(gx_0, gx_1) + q(gx_1, gx_0)\|, \quad n = 0, 1, 2, \dots, \quad (3.9)$$

and

$$\|q(gx_n, gx_m)\| \leq K \left(\frac{h^n}{1-h} \right) \|q(gx_0, gx_1) + q(gx_1, gx_0)\|, \quad (3.10)$$

for all $m > n \geq 1$. If $fx' \neq x'$ or $gx' \neq x'$, then, by the hypothesis, (3.9) and (3.10) with $m = n + 1$, we get

$$\begin{aligned} 0 &< \inf \{ \|q(fx, x')\| + \|q(gx, x')\| + \|q(gx, fx)\| : x \in X \} \\ &\leq \inf \{ \|q(fx_n, x')\| + \|q(gx_n, x')\| + \|q(gx_n, fx_n)\| : n \geq 1 \} \\ &= \inf \{ \|q(gx_{n+1}, x')\| + \|q(gx_n, x')\| + \|q(gx_n, gx_{n+1})\| : n \geq 1 \} \\ &\leq \inf \left\{ K \left(\frac{h^{n+1}}{1-h} \right) \|q(gx_0, gx_1) + q(gx_1, gx_0)\| \right. \\ &\quad \left. + K \left(\frac{h^n}{1-h} \right) \|q(gx_0, gx_1) + q(gx_1, gx_0)\| \right. \\ &\quad \left. + K \left(\frac{h^n}{1-h} \right) \|q(gx_0, gx_1) + q(gx_1, gx_0)\| : n \geq 1 \right\} = 0. \end{aligned}$$

which is a contradiction. Hence $x' = fx' = gx'$. Also, suppose that $gz = gz = z$. Then, by (3.1) we have

$$\begin{aligned} q(z, z) &= q(fz, fz) \\ &\preceq \alpha_1 q(gz, gz) + \alpha_2 q(gz, fz) + \alpha_3 q(gz, fz) + \alpha_4 q(gz, fz) + \alpha_5 q(gz, fz) \\ &= (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5)q(z, z). \end{aligned}$$

Since $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 < \alpha_1 + \alpha_2 + \alpha_3 + 2(\alpha_4 + \alpha_5) < 1$, we get that $q(z, z) = 0$ by Lemma 2.5(p_3). This completes the proof. \square

Remark 3.2.

- (i) Obviously, our Theorem 3.1 has generalized and unified the Corollary 2.11 of Abbas et al. [3] and the Theorem 2.1 of Song et al. [6] on c -distance in a normal cone metric space.
- (ii) As corollary, we obtain common fixed point result for mappings f and g satisfying

$$q(fx, fy) \preceq \alpha_1 q(gx, gy) + \alpha_2 q(gx, fx) + \alpha_3 q(gy, fy) + \alpha_4 q(gx, fy)$$

for all $x, y \in X$, where α_i for $i = 1, 2, 3, 4$ are nonnegative constants such that

$$\alpha_1 + \alpha_2 + \alpha_3 + 2\alpha_4 < 1.$$

Therefore, our Theorem 3.1 has generalized the main theorem of Wang and Guo’s work [17] in a cone metric space.

- (iii) Theorems 2.1 and 2.3 of [2] cannot be applied to some examples, but by the following conditions

$$q(fx, fy) \preceq kq(gx, gy), \quad k \in [0, 1),$$

$$q(fx, fy) \preceq k\left(q(gx, fx) + q(gy, fy)\right), \quad k \in [0, \frac{1}{2})$$

for all $x, y \in X$ of Theorem 3.1, f and g have a common fixed point theorem. Thus, our Theorem 3.1 has generalized the main results of Abbas and Jungck’s work [2] on c -distance in a cone metric space.

Example 3.3. Let $E = \mathbf{R}$, $P = \{x \in E : x \geq 0\}$ and $X = [0, \infty)$. Define a mapping $d : X \times X \rightarrow E$ by $d(x, y) = |x - y|$ for all $x, y \in X$. Then (X, d) is a cone metric space. Define a mapping $q : X \times X \rightarrow E$ by $q(x, y) = y$ for all $x, y \in X$. Then q is a c -distance (by Example 2.8). Define the mapping $f : X \rightarrow X$ by $f(2) = \frac{5}{3}$ and $fx = \frac{2x}{3}$ for all $x \in X$ with $x \neq 2$ and the mapping $g : X \rightarrow X$ by $gx = x$ for all $x \in X$. Since $d(g(1), g(2)) = d(f(1), f(2))$, there is not $0 \leq k < 1$ such that $d(fx, fy) \leq kd(gx, gy)$ for all $x, y \in X$. Hence, Theorem 2.1 of [2] cannot be applied to this example. Observe that

- (a) the range of g contains the range of f and $g(X)$ is a complete subspace of X ;
- (b) if $y = 2$, then we have $q(fx, fy) = fy = \frac{5}{3} \leq kq(gx, gy) = kgy = 2k$. Thus, there is $k \in [0, 1)$ such that $q(fx, fy) \leq kq(gx, gy)$ for $x \in X$ and $y = 2$;
- (c) if $y \neq 2$, then we have $q(fx, fy) = fy = \frac{2y}{3} \leq kq(gx, gy) = kgy = ky$. Thus, there is $k \in [0, 1)$ such that $q(fx, fy) \leq kq(gx, gy)$ for $x \in X$ and $y \neq 2$;
- (d) for $y \neq fy$ or $y \neq gy$, i.e., $y \neq 0$, $\inf\{\|q(fx, y)\| + \|q(gx, y)\| + \|q(gx, fx)\| : x \in X\} > 0$.

So, the hypothesis is satisfied. From Theorem 3.1 with $\alpha_1 = k$ and $\alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0$, we conclude that f and g have a common fixed point. That is $x = 0$.

In Theorem 3.1, if $g = i_X$ is the identity map on X , then we get the following Corollary of Hardy-Rogers type on c-distance in a cone metric space.

Corollary 3.4. Let (X, d) be a complete cone metric space, P be a normal cone with constant K and q be a c-distance on X . Suppose that the mapping $f : X \rightarrow X$ satisfies the following two contractive conditions:

$$q(fx, fy) \preceq \alpha_1 q(x, y) + \alpha_2 q(x, fx) + \alpha_3 q(y, fy) + \alpha_4 q(x, fy) + \alpha_5 q(y, fx), \quad (3.11)$$

$$q(fy, fx) \preceq \alpha_1 q(y, x) + \alpha_2 q(fx, x) + \alpha_3 q(fy, y) + \alpha_4 q(fy, x) + \alpha_5 q(fx, y) \quad (3.12)$$

for all $x, y \in X$, where α_i for $i = 1, 2, \dots, 5$ are nonnegative constants such that

$$\alpha_1 + \alpha_2 + \alpha_3 + 2(\alpha_4 + \alpha_5) < 1. \quad (3.13)$$

If f satisfies

$$\inf\{\|q(fx, y)\| + \|q(x, y)\| : x \in X\} > 0$$

for all $y \in X$ with $y \neq fy$, then f has a fixed point in X . If $fz = z$, then $q(z, z) = 0$.

Remark 3.5.

- (i) Some special cases of the previous theorem, for example Banach-type and Kannan-type fixed point results, need only one condition:

$$q(fx, fy) \preceq kq(x, y), \quad k \in [0, 1), \quad (3.14)$$

$$q(fx, fy) \preceq k\left(q(x, fx) + q(y, fy)\right), \quad k \in [0, \frac{1}{2}), \quad (3.15)$$

respectively.

- (ii) Obviously, Theorems 1 and 3 in [1] are a special case of Corollary 3.4 on c -distance in a cone metric space by relations (3.14) and (3.15). Therefore, our Corollary 3.4 has generalized and unified the main results of Huang and Zhang’s work in [1]. Also, Theorem 2.3 in [16] is a special case of Corollary 3.4 and most of the examples in [1, 2, 16] will easily translate into c -distance in a cone metric space.

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