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$\tau_1\tau_2$-Q^* Closed Sets

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Abstract : In the present paper, we introduced Q^*-closed sets and studied some of their properties in bitopological spaces. It is proved that the arbitrary intersection of $\tau_1\tau_2$-Q^* closed sets is $\tau_1\tau_2$-Q^* closed. Also some relations are established with known generalized closed sets.

Keywords : $\tau_1\tau_2$-Q^* closed sets; $\tau_1\tau_2$-Q^* open sets; $\tau_1\tau_2$-δ set; nowhere $\tau_1\tau_2$-dense.
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1 Introduction

Maheshwari and Prasad [1] introduced semi open sets in bitopological spaces [2] in 1977. Further properties of this notion were studied by Bose [3] in 1981. Fukutake [4] defined one kind of semi open sets in bitopological spaces and studied their properties in 1989. He also extended the generalized closed sets [5] from topological spaces to bitopological spaces and introduced pairwise generalized closure operator [6] in bitopological spaces in 1986. A set A of a bitopological space (X, τ_1, τ_2) is $\tau_i\tau_j$-generalized closed set (briefly $\tau_i\tau_j$-g-closed) if τ_j-$cl(A) \subseteq U$ whenever $A \subseteq U$ and U is τ_i-open in X, $i, j = 1, 2$ and $i \neq j$. Also, he defined a new closure operator and strongly pairwise $T_{\frac{1}{2}}$-space.

Semi generalized closed sets and generalized semi closed sets are extended to bitopological settings by Khedr and Al-saadi [7]. They proved that the union of

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two ij-sg closed sets need not be ij-sg closed. This is an unexpected result. Also they defined that the ij-semi generalized closure of a subset A of a space X is the intersection of all ij-sg closed sets containing A and is denoted by ij-$sgcl(A)$. Rao and Mariasingam (2000) [8] defined and studied regular generalized closed sets in bitopological settings.

Rao and Kannan introduced semi star generalized closed sets [9–12], regular generalized star closed sets [13], regular generalized star star closed sets [14], generalized star regular closed sets [15] and generalized star star closed sets [16] in bitopological spaces. Moreover, Kannan, Sheik John and Sundaram introduced semi star star closed sets [17] and g*-closed sets [18] in bitopological spaces. Murugalingam and Lalitha [19] introduced Q^*-closed sets in topological spaces.

In the present paper, we introduced Q^*-closed sets and studied some of their properties in bitopological spaces. Also some relations are established with known generalized closed sets.

2 Preliminaries

A topological space (X, τ) is a hyperconnected space if X cannot be written as the union of two proper closed sets. Moreover, for a topological space X the following conditions are equivalent: (1) No two nonempty open sets are disjoint. (2) X cannot be written as the union of two proper closed sets. i.e) X is hyperconnected. (3) Every nonempty open set is dense in X. (4) The interior of every proper closed set is empty.

Throughout this paper, (X, τ_1, τ_2) or simply X denotes a bitopological space and $A \subseteq B$ means that A is the subset of B. For any subset $A \subseteq X$, τ_i-$int(A)$, τ_i-$cl(A)$ denote the interior, closure of a set A in X with respect to the topology τ_i. A^C or $X - A$ denotes the complement of A in X unless explicitly stated. We shall now require the following known definitions.

A set A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-semi open if there exists an τ_1-open set U such that $U \subseteq A \subseteq \tau_2$-$cl(U)$. Equivalently, a set A is $\tau_1\tau_2$-semi open if $A \subseteq \tau_2$-$cl[\tau_1$-$int(A)]$. Similarly, A is $\tau_1\tau_2$-semi closed if $X - A$ is $\tau_1\tau_2$-semi open. Equivalently, a set A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-semi closed if there exists an τ_1-closed set F such that τ_2-$int(F) \subseteq A \subseteq F$.

A set A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-generalized closed ($\tau_1\tau_2$-g closed){resp. $\tau_1\tau_2$-semi generalized closed ($\tau_1\tau_2$-sg closed), $\tau_1\tau_2$-generalized semi closed ($\tau_1\tau_2$-gs closed), $\tau_1\tau_2$-semi star generalized closed ($\tau_1\tau_2$-s^*g closed), $\tau_1\tau_2$-regular generalized closed($\tau_1\tau_2$-rg closed), $\tau_1\tau_2$-regular generalized star closed ($\tau_1\tau_2$-rg^* closed), $\tau_1\tau_2$-generalized star regular closed ($\tau_1\tau_2$-g^*r closed), $\tau_1\tau_2$-regular generalized star star closed ($\tau_1\tau_2$-rg^{**} closed), $\tau_1\tau_2$-generalized star star regular closed ($\tau_1\tau_2$-$g^{**}r$ closed)} if τ_2-$cl(A)$ {resp. τ_2-$scl(A)$, τ_2-$scl(A)$, τ_2-$cl(A)$, τ_2-$cl(A)$, τ_2-$rcl(A)$, τ_2-$rcl(A)$, τ_2-$cl[\tau_1$-$int(A)]$, τ_2-$cl[\tau_1$-$int(A)]$} $\subseteq U$ whenever $A \subseteq U$ and U is τ_1-open {resp. τ_1-semi open, τ_1-open, τ_1-semi open, $\tau_1\tau_2$-regular open, $\tau_1\tau_2$-regular open, τ_1-open, $\tau_1\tau_2$-regular open, τ_1-open } in X.

The complement of $\tau_1\tau_2$-generalized closed {resp. $\tau_1\tau_2$-semi generalized closed,

$\tau_1\tau_2$-generalized semi closed, $\tau_1\tau_2$-semi star generalized closed, $\tau_1\tau_2$-regular generalized closed, $\tau_1\tau_2$-regular generalized star closed, $\tau_1\tau_2$-generalized star regular closed, $\tau_1\tau_2$-regular generalized star star closed, $\tau_1\tau_2$-generalized star star closed} is called $\tau_1\tau_2$-generalized open {resp. $\tau_1\tau_2$-semi generalized open, $\tau_1\tau_2$-generalized semi open, $\tau_1\tau_2$-semi star generalized open, $\tau_1\tau_2$-regular generalized open, $\tau_1\tau_2$-regular generalized star open, $\tau_1\tau_2$-generalized star regular open, $\tau_1\tau_2$-regular generalized star star open, $\tau_1\tau_2$-generalized star star open} in X. A set A is called $\tau_1\tau_2$-δ set {resp. nowhere $\tau_1\tau_2$-dense} if τ_1-$int\{\tau_2$-$cl[\tau_1$-$int(A)]\} = \phi$ {resp. τ_2-$cl[\tau_1$-$int(A)] = \phi$}.

3 $\tau_1\tau_2$-Q^* Closed Sets

In this section, the concept of $\tau_1\tau_2$-Q^* closed sets is introduced and its basic properties in bitopological spaces are discussed. Recall that a set A of a topological space (X, τ) is called Q^*-closed if $int(A) = \phi$ and A is closed.

Definition 3.1. A set A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-Q^* closed if τ_1-$int(A) = \phi$ and A is τ_2-closed.

Example 3.2. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}\}$ and $\tau_2 = \{\phi, X, \{a\}, \{b, c\}\}$. Then ϕ, $\{b, c\}$ are $\tau_1\tau_2$-Q^* closed.*

Remark 3.3. *It is obvious that every $\tau_1\tau_2$-Q^* closed set is τ_2-closed, but the converse is not true in general. The following example supports our claim.*

Example 3.4. *In Example 3.2, $\{a\}$ is τ_2-closed, but not $\tau_1\tau_2$-Q^* closed.*

Remark 3.5. *Since every $\tau_1\tau_2$-Q^* closed set is τ_2-closed and every τ_2-closed set is $\tau_1\tau_2$-g closed, $\tau_1\tau_2$-sg closed, $\tau_1\tau_2$-gs closed and $\tau_1\tau_2$-s^*g closed, we have every $\tau_1\tau_2$-Q^* closed set is $\tau_1\tau_2$-g closed, $\tau_1\tau_2$-sg closed, $\tau_1\tau_2$-gs closed and $\tau_1\tau_2$-s^*g closed. But the converse is not true in general. The following example supports our claim.*

Example 3.6. *In Example 3.2, $\{a\}$ is $\tau_1\tau_2$-g closed, $\tau_1\tau_2$-sg closed, $\tau_1\tau_2$-gs closed and $\tau_1\tau_2$-s^*g closed, but not $\tau_1\tau_2$-Q^* closed in X.*

Theorem 3.7. *Every $\tau_1\tau_2$-Q^* closed set is $\tau_1\tau_2$-δ set.*

Proof. Let A be $\tau_1\tau_2$-Q^* closed. Then τ_1-$int(A) = \phi$ and A is τ_2-closed. Consequently, τ_1-$int\{\tau_2$-$cl[\tau_1$-$int(A)]\} = \tau_1$-$int\{\tau_2$-$cl[\phi]\} = \tau_1$-$int(\phi) = \phi$. Therefore, A is $\tau_1\tau_2$-δ set. □

The converse of the above theorem is not true in general as can be seen from the following example.

Example 3.8. *Let $X = \{a, b, c, d\}$, $\tau_1 = \{\phi, X, \{a, b\}\}$, and $\tau_2 = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$. Then $\{a\}$ is $\tau_1\tau_2$-δ set, but not $\tau_1\tau_2$-Q^* closed.*

Remark 3.9. *τ_2-regular closed sets and $\tau_1\tau_2$-Q^* closed sets are independent of each other in general. It is proved in the following example.*

Example 3.10. *In Example 3.8, $\{a,\ b,\ d\}$ is τ_2-regular closed, but not $\tau_1\tau_2$-Q^* closed and $\{a,\ d\}$ is $\tau_1\tau_2$-Q^* closed but not τ_2-regular closed.*

The following diagram is established now from the above results.

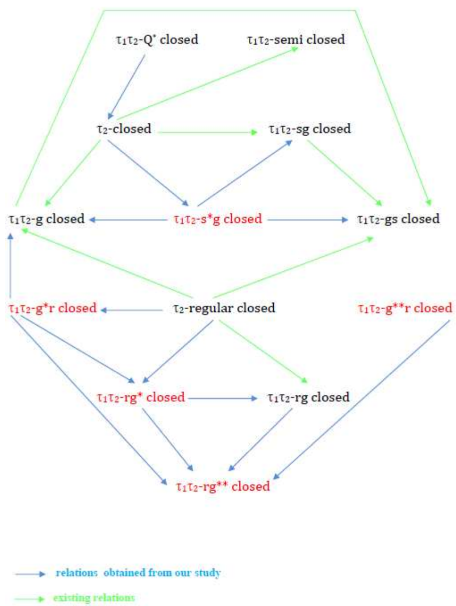

Figure 3.1: Relations between existing generalized closed sets and $\tau_1\tau_2$-Q^* closed sets

Theorem 3.11. *If A and B are $\tau_1\tau_2$-Q^* closed sets then so is $A \cup B$.*

Proof. Suppose that A and B are $\tau_1\tau_2$-Q^* closed sets. Then τ_1-$int(A) = \tau_1$-$int(B) = \phi$ and A, B are τ_2-closed. Consequently, $A \cup B$ is τ_2-closed. Now, $A^C \cap \tau_1$-$int(A \cup B) \subseteq \tau_1$-$int(A \cup B)$. Let $x \in A^C \cap \tau_1$-$int(A \cup B)$. Then $x \in A^C$ and $x \in \tau_1$-$int(A \cup B)$. Therefore, $x \in B$. Consequently, $A^C \cap \tau_1$-$int(A \cup B) \subseteq B$. But, τ_1-$int(B) = \phi$. Hence $A^C \cap \tau_1$-$int(A \cup B) = \phi$. But τ_1-$int(A) = \phi$. This implies that τ_1-$int(A \cup B) = \phi$. It follows that $A \cup B$ is $\tau_1\tau_2$-Q^* closed. □

Theorem 3.12. *The arbitrary intersection of $\tau_1\tau_2$-Q^* closed sets is $\tau_1\tau_2$-Q^* closed.*

Proof. Let $\{A_\alpha/\alpha \in I\}$ be a family of $\tau_1\tau_2$-Q^* closed sets. Then for each $\alpha \in I$, τ_1-$int\,(A_\alpha) = \phi$ and A_α is τ_2-closed. Since, τ_1-$int\,[\bigcap A_\alpha] \subseteq \tau_1$-$int\,(A_\alpha)$, we have τ_1-$int\,[\bigcap A_\alpha] = \phi$. Since the arbitrary intersection of τ_2-closed sets is τ_2-closed, we have $\bigcap [A_\alpha]$ is τ_2-closed. This completes the proof. □

Theorem 3.13. *If A is $\tau_1\tau_2$-Q^* closed then A is nowhere $\tau_1\tau_2$-dense*

Proof. Since A is $\tau_1\tau_2$-Q^* closed, we have τ_1-$int(A) = \phi$ and A is τ_2-closed. Therefore, τ_2-$cl\,[\tau_1$-$int(A)] = \phi$. Hence A is nowhere $\tau_1\tau_2$-dense. □

The converse of the above theorem is not true in general as can be seen from the following example.

Example 3.14. *In Example 3.8, $\{b\}$ is nowhere $\tau_1\tau_2$-dense, but not $\tau_1\tau_2$-Q^* closed.*

4 $\tau_1\tau_2$-Q^* Open Sets

Definition 4.1. A set A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-Q^* open if its complement A^C is $\tau_1\tau_2$-Q^* closed in X

Example 4.2. *In Example 3.2, $\{a\}$, X are $\tau_1\tau_2$-Q^* open.*

Theorem 4.3. *A set A of a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$-Q^* open if and only if τ_1-$cl(A) = X$ and A is τ_2-open.*

Proof. Necessity: Suppose that A is $\tau_1\tau_2$-Q^* open. Then A^C is $\tau_1\tau_2$-Q^* closed. Therefore, τ_1-$int\left(A^C\right) = [\tau_1$-$cl(A)]^C = \phi$ and A^C is τ_2-closed. Consequently, τ_1-$cl(A) = X$ and A is τ_2-open.

Sufficiency: Suppose that τ_1-$cl(A) = X$ and A is τ_2-open. Then $[\tau_1$-$cl(A)]^C = \tau_1$-$int\left(A^C\right) = \phi$ and A^C is τ_2-closed. Consequently, A^C is $\tau_1\tau_2$-Q^* closed. This completes the proof. □

Corollary 4.4. *A set A of a bitopological space (X, τ_1, τ_2) is $\tau_1\tau_2$-Q^* open if and only if A is τ_1-dense and τ_2-open.*

Theorem 4.5.

(a) X is not $\tau_1\tau_2$-Q^* closed;

(b) X is $\tau_1\tau_2$-Q^* open;

(c) ϕ is not $\tau_1\tau_2$-Q^* open;

(d) ϕ is $\tau_1\tau_2$-Q^* closed.

Remark 4.6. *It is obvious that every $\tau_1\tau_2$-Q^* open set is τ_2-open, but the converse is not true in general. The following example supports our claim.*

Example 4.7. *In Example 3.2, $\{b,\ c\}$ is τ_2-open, but not $\tau_1\tau_2$-Q^* open.*

Remark 4.8. *Since every $\tau_1\tau_2$-Q^* open set is τ_2-open and every τ_2-open set is $\tau_1\tau_2$-g open, $\tau_1\tau_2$-sg open, $\tau_1\tau_2$-gs open and $\tau_1\tau_2$-s^*g open, we have every $\tau_1\tau_2$-Q^* open set is $\tau_1\tau_2$-g open, $\tau_1\tau_2$-sg open, $\tau_1\tau_2$-gs open and $\tau_1\tau_2$-s^*g open. But the converse is not true in general. The following example supports our claim.*

Example 4.9. *In Example 3.2,* $\{b,\ c\}$ *is* $\tau_1\tau_2$*-g open,* $\tau_1\tau_2$*-sg open,* $\tau_1\tau_2$*-gs open and* $\tau_1\tau_2$*-*s^*g *open, but not* $\tau_1\tau_2$*-*Q^* *open in* X*.*

Remark 4.10. τ_2*-regular open sets and* $\tau_1\tau_2$*-*Q^* *open sets are independent of each other in general. It is proved in the following example.*

Example 4.11. *In Example 3.8,* $\{c\}$ *is* τ_2*-regular open, but not* $\tau_1\tau_2$*-*Q^* *open and* $\{b,\ c\}$ *is* $\tau_1\tau_2$*-*Q^* *open but not* τ_2*-regular open.*

Theorem 4.12. *Let* (X, τ_1) *be a hyperconnected topological space and* $A \subseteq X$*. If* A *is both* τ_1*-open and* τ_2*-open then* A *is* $\tau_1\tau_2$*-*Q^* *open in the bitopological space* (X, τ_1, τ_2)*.*

Proof. It is enough to prove that A is τ_1-dense. Suppose that $\tau_1\text{-}cl(A) \neq X$. Then $[\tau_1\text{-}cl(A)]^C \neq \phi$. Moreover, both A and $[\tau_1\text{-}cl(A)]^C$ are disjoint τ_1-open sets. This is contradiction to the fact that (X, τ_1) is a hyperconnected topological space. Hence A is τ_1-dense. □

Theorem 4.13. *If* A *and* B *are* $\tau_1\tau_2$*-*Q^* *open sets then so is* $A \cap B$*.*

Proof. Suppose that A and B are $\tau_1\tau_2$-Q^* open sets. Then A^C and B^C are $\tau_1\tau_2$-Q^* closed sets. Therefore, $A^C \cup B^C$ is $\tau_1\tau_2$-Q^* closed. But $A^C \cup B^C = (A \cap B)^C$. Hence $A \cap B$ is $\tau_1\tau_2$-Q^* open. □

Theorem 4.14. *The arbitrary union of* $\tau_1\tau_2$*-*Q^* *open sets is* $\tau_1\tau_2$*-*Q^* *open.*

Proof. Let $\{A_\alpha / \alpha \in I\}$ be a family of $\tau_1\tau_2$-Q^* open sets. Then for each $\alpha \in I$, $\tau_1\text{-}cl\,(A_\alpha) = X$ and A_α is τ_2-open. Since, $\tau_1\text{-}cl\,(A_\alpha) \subseteq \tau_1\text{-}cl\,[\bigcup A_\alpha]$, we have $\tau_1\text{-}cl\,[\bigcup A_\alpha] = X$. Since the arbitrary union of τ_2-open sets is τ_2-open, we have $\bigcup [A_\alpha]$ is τ_2-open. This completes the proof. □

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References

[1] S.N. Maheshwari, R. Prasad, Semiopen sets and semi continuous functions in bitopological spaces, Math. Note 26 (1977/78) 29–37.

[2] J.C. Kelly, Bitopological spaces, Proc. London Math. Society, 13 (1963) 71–89.

[3] S. Bose, Semi open sets, semi continuity and semi open mappings in bitopological spaces, Bull. Cal. Math. Soc. 73 (1981) 237–246.

[4] T. Fukutake, Semi open sets on bitopological spaces, Bull. Fukuoka Uni. Education 38 (3) (1989) 1–7.

[5] K. Kannan, On Levine's generalized closed sets: a survey, Research Journal of Applied Sciences, Engineering and Technology 4 (11) (2012) 1612–1615.

[6] T. Fukutake, On generalized closed sets in bitopological spaces, Bull. Fukuoka Univ. Ed. Part III 35 (1986) 19–28.

[7] F.H. Khedr, H.S. Al-saadi, On pairwise semi-generalized closed sets, JKAU: Sci. 21 (2) (2009) 269–295.

[8] K. Chandrasekhara Rao, M. Mariasingam, On bitopological spaces, Acta Ciencia Indica XXVI M (4) (2000) 283–288.

[9] K. Chandrasekhara Rao, K. Kannan, Semi star generalized closed sets and semi star generalized open sets in bitopological spaces, Varāhmihir Journal of Mathematical Sciences 5 (2) (2005) 473–485.

[10] K. Chandrasekhara Rao, K. Kannan, D. Narasimhan, Characterizations of $\tau_1\tau_2$-s^*g closed sets, Acta Ciencia Indica XXXIII (3) (2007) 807–810.

[11] K. Kannan, D. Narasimhan, K. Chandrasekhara Rao, Pairwise semi star generalized homeomorphisms, Journal of Engineering and Applied Sciences 7 (1) (2012) 86–89.

[12] K. Kannan, D. Narasimhan, K. Chandrasekhara Rao, On semi star generalized closed sets in bitopological spaces, Bol. Soc. Paranaense de Mat. (3s.) 28 (1) (2010) 29–40.

[13] K. Chandrasekhara Rao, K. Kannan, Regular generalized star closed sets, Thai J. Math. 4 (2) (2006) 341–349.

[14] K. Kannan, D. Narasimhan, K. Chandrasekhara Rao, R. Ravikumar, Regular generalized star star closed sets, World Academy of Science, Engineering and Technology I: Mathematical and Computational Sciences 5 (2) (2011) 67–69.

[15] K. Chandrasekhara Rao, K. Kannan, Generalized star regular closed sets, Antarctica J. Math. 4 (2) (2007) 139–146.

[16] K. Chandrasekhara Rao, K. Kannan, D. Narasimhan, Generalized star star regular closed sets, Natl. Acad. Sci. Lett. 31 (1 & 2) (2008) 31–33.

[17] K. Kannan, $\tau_1\tau_2$-semi star star generalized closed sets, International Journal of Pure and Applied Mathematics 76 (2) (2012) 277–294.

[18] M. Sheik John, P. Sundaram, g^*-closed sets in bitopological spaces, Indian J. Pure Appl. Math. 35 (1) (2004) 71–80.

[19] M. Murugalingam, N. Lalitha, Q^*-sets, Bulletin of Pure and Applied Sciences 29E (2) (2010) 369–376.

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