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# On Ideal Convergent Sequences in $2-$Normed Spaces

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**Abstract :** In this paper, we investigate the relation between $\mathcal{I}$-cluster points and ordinary limit points of sequence in 2-normed space.

**Keywords :** $\mathcal{I}$-convergence, 2-normed space, $\mathcal{I}$-cluster point.
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## 1 Introduction

The notion of ideal convergence was introduced first by P. Kostyrko et al [6] as an interesting generalization of statistical convergence [1, 11]. Note that the concept of an $\mathcal{I}$-cluster point and $\mathcal{I}$-limit point of a sequence in a metric space was introduced and some results for the set of $\mathcal{I}$-cluster points and $\mathcal{I}$-limit points obtained in [7].

The concept of 2-normed spaces was initially introduced by Gähler [2] in the 1960's. Since then, this concept has been studied by many authors, see for instance [3, 10].

In a natural way, one may unite these two concepts, and study the relation between $\mathcal{I}$-cluster point set and ordinary limit point set of a given sequence in 2-normed spaces. This is actually what we offer in this article.

Throughout this paper $\mathbb{N}$ will denote the set of positive integers. Let $(X, \|.\|)$ be a normed space. Recall that a sequence $(x_n)_{n\in\mathbb{N}}$ of elements of $X$ is called to be *statistically convergent* to $\xi \in X$ if the set $A(\varepsilon) = \{n \in \mathbb{N} : \|x_n - \xi\| \geq \varepsilon\}$ has natural density zero for each $\varepsilon > 0$.

A family $\mathcal{I} \subset 2^Y$ of subsets a nonempty set $Y$ is said to be an ideal in $Y$ if

(i) $\emptyset \in \mathcal{I}$;

(ii) $A, B \in \mathcal{I}$ imply $A \cup B \in \mathcal{I}$;

(iii) $A \in \mathcal{I}$, $B \subset A$ imply $B \in \mathcal{I}$,

while an admissible ideal $\mathcal{I}$ of $Y$ further satisfies $\{x\} \in \mathcal{I}$ for each $x \in Y$ [7, 8].

Given $\mathcal{I} \subset 2^{\mathbb{N}}$ be a nontrivial ideal in $\mathbb{N}$. The sequence $(x_n)_{n\in\mathbb{N}}$ in $X$ is said to be $\mathcal{I}$-*convergent* to $\xi \in X$, if for each $\varepsilon > 0$ the set $A(\varepsilon) = \{n \in \mathbb{N} : \|x_n - \xi\| \geq \varepsilon\}$ belongs to $\mathcal{I}$ [6, 7].

An admissible ideal $\mathcal{I} \subset 2^{\mathbb{N}}$ is said to have the *property* (AP) if for any sequence $\{A_1, A_2, \ldots\}$ of mutually disjoint sets of $\mathcal{I}$ there is a sequence $\{B_1, B_2, \ldots\}$ of subsets of $\mathbb{N}$ such that each symmetric difference $A_i \Delta B_i$ $(i = 1, 2, \ldots)$ is finite and $B = \bigcup\limits_{i=1}^{\infty} B_i \in \mathcal{I}$, [7].

Let $X$ be a real vector space of dimension $d$, where $2 \leq d < \infty$. A 2-norm on $X$ is a function $\|.,.\| : X \times X \to R$ which satisfies

(i) $\|x, y\| = 0$ if and only if $x$ and $y$ are linearly dependent;

(ii) $\|x, y\| = \|y, x\|$ ;

(iii) $\|\alpha x, y\| = |\alpha| \, \|x, y\|$ , $\alpha \in \mathbb{R}$;

(iv) $\|x, y + z\| \leq \|x, y\| + \|x, z\|$ .

The pair $(X, \|.,.\|)$ is then called a 2-*normed space* [2]. As an example of a 2-normed space we may take $X = \mathbb{R}^2$ being equipped with the 2-norm $\|x, y\| :=$ the area of the parallelogram spanned by the vectors $x$ and $y$, which may be given explicitly by the formula

$$\|x, y\| = |x_1 y_2 - x_2 y_1| \,, \quad x = (x_1, x_2) \,, \quad y = (y_1, y_2) \,.$$

Recall that $(X, \|.,.\|)$ is a 2-Banach space if every Cauchy sequence in $X$ is convergent to some $x$ in $X$. Let $u = \{u_1, \ldots, u_d\}$ be a basis of $X$. We know the norm $\|.\|_\infty$ on $X$ is defined by

$$\|x\|_\infty := \max \{\|x, u_i\| : i = 1, \ldots, d\} \,.$$

Associated to the derived norm $\|.\|_\infty$ , we can define the (open) balls $B_u(x, \varepsilon)$ centered at $x$ having radius $\varepsilon$ by

$$B_u(x, \varepsilon) := \left\{y : \|x - y\|_\infty < \varepsilon\right\},$$

where $\|x - y\|_\infty := \max\left\{ \|x - y, u_j\| \,, \, j = 1, \ldots, d\right\}$.

## 2 The Relation Between $\mathcal{I}$-cluster Points and Ordinary Limit Points of 2-Normed Spaces

It is known that there is a strong connection between statistical cluster points and ordinary limit points of a given sequence. We will prove that these facts are satisfied for $\mathcal{I}$-cluster $\mathcal{I}$-limit point sets of a given sequences in 2-normed spaces.

**Definition 2.1** Let $\mathcal{I} \subset 2^{\mathbb{N}}$ be a nontrivial ideal in $\mathbb{N}$. The sequence $(x_n)$ of $X$ is said to be $\mathcal{I}$-convergent to $\xi$, if for each $\varepsilon > 0$ and nonzero $z$ in $X$ the set $A(\varepsilon) = \left\{n \in \mathbb{N} : \|x_n - \xi, z\| \geq \varepsilon\right\}$ belongs to $\mathcal{I}$.

If $(x_n)$ is $\mathcal{I}$-convergent to $\xi$ then we write $\mathcal{I}\text{-}\lim\limits_{n\to\infty} \|x_n - \xi, z\| = 0$ or $\mathcal{I}\text{-}\lim\limits_{n\to\infty} \|x_n, z\| = \|\xi, z\|$ . The number $\xi$ is $\mathcal{I}$-limit of the sequence $(x_n)$ .

**Remark 2.2** If $\{x_n\}$ is any sequence in $X$ and $\xi$ is any element of $X$, then the set

$$\{n \in \mathbb{N} : \|x_n - \xi, z\| \geq \varepsilon, \text{ for every } z \in X\} = \emptyset,$$

since if $z = \overrightarrow{0}$ (0 vector), $\|x_n - \xi, z\| = 0 \ngeq \varepsilon$ so the above set is empty.

Further we will give some examples of ideals and corresponding $\mathcal{I}$-convergences.

(I) Let $\mathcal{I}_f$ be the family of all finite subsets of $\mathbb{N}$. Then $\mathcal{I}_f$ is an admissible ideal in $\mathbb{N}$ and $\mathcal{I}_f$ convergence coincides with usual convergence in [2].

(II) Put $\mathcal{I}_\delta = \{A \subset \mathbb{N} : \delta(A) = 0\}$. Then $\mathcal{I}_\delta$ is an admissible ideal in $\mathbb{N}$ and $\mathcal{I}_\delta$ convergence coincides with the statistical convergence in [4].

Now we give an example of $\mathcal{I}$-convergence in 2-normed spaces.

**Example 2.3** Let $\mathcal{I} = \mathcal{I}_\delta$. Define the $(x_n)$ in 2-normed space $(X, \|.,.\|)$ by

$$x_n = \begin{cases} (0, n), & n = k^2, k \in \mathbb{N}, \\ (0, 0), & \text{otherwise.} \end{cases}$$

and let $\xi = (0, 0)$ and $z = (z_1, z_2)$. Then for every $\varepsilon > 0$ and $z \in X$

$$\{n \in \mathbb{N} : \|x_n - \xi, z\| \geq \varepsilon\} \subset \{1, 4, 9, 16, \ldots, n^2, \ldots\}.$$

We have that

$$\delta(\{n \in \mathbb{N} : \|x_n - \xi, z\| \geq \varepsilon\}) = 0,$$

for every $\varepsilon > 0$ and nonzero $z \in X$. This implies that $st - \lim_{n\to\infty} \|x_n, z\| = \|\xi, z\|$. But, the sequence $(x_n)$ is not convergent to $\xi$.

**Definition 2.4** Let $\mathcal{I} \subset 2^{\mathbb{N}}$ be an admissible ideal and $x = (x_n)_{n\in\mathbb{N}}$ be a sequence in linear 2-normed space $(X, \|.,.\|)$.

(i) A number $\xi$ is called to be an $\mathcal{I}$-limit point of $x$ provided that there is set $M = \{m_1 < m_2 < \ldots\} \subset \mathbb{N}$ such that $M \notin \mathcal{I}$ and $\lim_{k\to\infty} \|x_{m_k} - \xi, z\| = 0$ for each nonzero $z$ in $X$. The set of all $\mathcal{I}$-limit points of $x$ is denoted by $\mathcal{I}(\wedge_x^2)$.

(ii) A number $\xi$ is said to be an $\mathcal{I}$-cluster point of $x$ provided that $\{n \in \mathbb{N} : \|x_n - \xi, z\| < \varepsilon\} \notin \mathcal{I}$ for each $\varepsilon > 0$ and nonzero $z \in X$. The set of all $\mathcal{I}$-cluster points of $x$ is denoted by $\mathcal{I}(\Gamma_x^2)$.

**Proposition 2.5** *Let $\mathcal{I} \subset 2^{\mathbb{N}}$ be an admissible ideal. Then for each sequence $x = (x_n)_{n\in\mathbb{N}}$ of $X$ we have $\mathcal{I}(\wedge_x^2) \subset \mathcal{I}(\Gamma_x^2)$ and the set $\mathcal{I}(\Gamma_x^2)$ is a closed set.*

**Proof.** Let $\xi \in \mathcal{I}\left(\wedge_x^2\right)$. Then there exists a set $M = \{m_1 < m_2 < \ldots\} \notin \mathcal{I}$ such that

$$\lim_{k\to\infty} \|x_{m_k} - \xi, z\| = 0 \tag{2.1}$$

for each nonzero $z$ in $X$. Take $\delta > 0$. According to (2.1) there exists $k_0 \in \mathbb{N}$ such that for $k > k_0$ and each nonzero $z \in X$ we have $\|x_{m_k} - \xi, z\| < \delta$. Hence

$$\{n \in \mathbb{N} : \|x_n - \xi, z\| < \delta\} \supset M \backslash \{m_1, \ldots, m_{k_0}\}$$

and so

$$\{n \in \mathbb{N} : \|x_n - \xi, z\| < \delta\} \notin \mathcal{I},$$

which means that $\xi \in \mathcal{I}\left(\Gamma_x^2\right)$.

Let $y \in \overline{\mathcal{I}\left(\Gamma_x^2\right)}$. Take $\varepsilon > 0$. There exists $\xi_0 \in \mathcal{I}\left(\Gamma_x^2\right) \cap B_u(y, \varepsilon)$. Choose $\delta > 0$ such that $B_u(\xi_0, \delta) \subset B_u(y, \varepsilon)$. We obviously have

$$\{n \in \mathbb{N} : \|y - x_n, z\| < \varepsilon\} \supset \{n \in \mathbb{N} : \|\xi_0 - x_n, z\| < \delta\}.$$

Hence $\{n \in \mathbb{N} : \|y - x_n, z\| < \varepsilon\} \notin \mathcal{I}$ and $y \in \mathcal{I}\left(\Gamma_x^2\right)$. □

**Definition 2.6** Let $\mathcal{I} \subset 2^{\mathbb{N}}$ be an admissible ideal and $x = (x_n)_{n\in\mathbb{N}}$ be a sequence in linear 2-normed space $(X, \|.,.\|)$.

If $K = \{k_1 < k_2 < \ldots\} \in \mathcal{I}$, then the subsequence $x_k = (x_k)_n \in \mathbb{N}$ is called *$\mathcal{I}$-thin subsequence of the sequence $x$.*

If $M = \{m_1 < m_2 < \ldots\} \notin \mathcal{I}$, then the sequence $x_M = (x_m)_{n\in\mathbb{N}}$ is called *$\mathcal{I}$-nonthin subsequence* of $x$.

It is clear that if $\xi$ is a $\mathcal{I}$-limit point of $x$, then there is a $\mathcal{I}$-nonthin subsequence $x_M$ that converges to $\xi$.

Let $L_x^2$ be the set of all ordinary limit points of sequence $x$. It is obvious $\mathcal{I}\left(\wedge_x^2\right) \subseteq L_x^2$ : Take $\xi \notin L_x^2$, then there is $\varepsilon' > 0$ such that the interval $(\xi - \varepsilon', \xi + \varepsilon')$ contains only a finite number of elements of $x$. Then $\{n \in \mathbb{N} : \|x_n - \xi, z\| < \varepsilon'\} \in \mathcal{I}$, but it contradicts to $\xi \in \mathcal{I}\left(\Gamma_x^2\right)$. Hence $x \in \mathcal{I}\left(\Gamma_x^2\right)$. Hence $x \in L_x^2$, so $\mathcal{I}\left(\Gamma_x^2\right) \subseteq L_x^2$.

**Lemma 2.7** *Let $\mathcal{I} \subset 2^{\mathbb{N}}$ be an admissible ideal and $x = (x_n)_{n\in\mathbb{N}}$ be a sequence in linear 2-normed space $(X, \|.,.\|)$. If $x$ is $\mathcal{I}$-convergent in 2-normed space, then $\mathcal{I}\left(\wedge_x^2\right)$ and $\left(\Gamma_x^2\right)$ are both equal to the singleton set $\left\{\mathcal{I}\text{-}\lim_{n\to\infty} \|x_n, z\|\right\}$ for each nonzero $z$ in $X$.*

**Proof.** Let $\mathcal{I}\text{-}\lim_{n\to\infty} \|x_n, z\|$ for each nonzero $z$ in $X$. Show that $\xi \in \mathcal{I}\left(\Gamma_x^2\right)$.By definition of $\mathcal{I}$-convergence we have

$$A(\varepsilon) = \left\{n \in \mathbb{N} : \|x_n - \xi, z\| \geq \varepsilon\right\} \in \mathcal{I}$$

for each $\varepsilon > 0$ and nonzero $z$ in $X$. Since $\mathcal{I}$ is an admissible ideal we can choose the set $M = \{n_1 < n_2 < \ldots\} \subset \mathbb{N}$ such that $n_k \notin A\left(\frac{1}{k}\right)$ and $\|x_{n_k} - \xi, z\| < \frac{1}{k}$ for all $k \in \mathbb{N}$ and nonzero $z$ in $X$. That is $\lim_{n\to\infty} \|x_{n_k} - \xi, z\| = 0$. Suppose $M \in \mathcal{I}$. Since $M \subset \{n \in \mathbb{N} : \|x_n - \xi, z\| < 1\}$ for each nonzero $z$ in $X$, then $(\mathbb{N}\backslash M) \cap \{n \in \mathbb{N} : \|x_n - \xi, z\| < 1\} = \emptyset$, but $\mathbb{N}\backslash M \in \mathcal{F}(\mathcal{I})$ and $\{n \in \mathbb{N} : \|x_n - \xi, z\| < 1\} \in \mathcal{F}(\mathcal{I})$ for each nonzero $z$ in $X$. This contradiction gives $M \notin \mathcal{I}$. Hence we get $M = \{m_1 < m_2 < \ldots\} \subset \mathbb{N}$ and $M \notin \mathcal{I}$ such that $\lim_{n\to\infty} \|x_{n_k} - \xi, z\| = 0$. i.e. $\xi \in \mathcal{I}\left(\wedge_x^2\right)$. Since $\mathcal{I}\left(\wedge_x^2\right) \subset \mathcal{I}\left(\Gamma_x^2\right)$, then $\xi \in \mathcal{I}\left(\Gamma_x^2\right)$.

Now suppose there is $\eta \in \mathcal{I}\left(\Gamma_x^2\right)$ such that $\eta \neq \xi$. It is clear that

$$\begin{aligned} A &= \left\{n \in \mathbb{N} : \|x_n - \xi, z\| \geq \frac{|\eta - \xi|}{2}\right\} \in \mathcal{I} \text{ and} \\ B &= \left\{n \in \mathbb{N} : \|x_n - \xi\| < \frac{|\eta - \xi|}{2}\right\} \notin \mathcal{I} \end{aligned}$$

for each nonzero $z$ in $X$. On the other hand, since

$$\|x_n - \xi, z\| \geq \| |x_n - \eta| - |\eta - \xi|, z\| > \frac{|\eta - \xi|}{2}$$

for each $n \in B$ nonzero $z$ in $X$, we have $B \subset A \in \mathcal{I}$. This contradiction shows $\mathcal{I}\left(\Gamma_x^2\right) = \{\xi\}$ we have $\mathcal{I}\left(\wedge_x^2\right) = \mathcal{I}\left(\Gamma_x^2\right) = \{\xi\}$. The Lemma 1 is proved. □

**Theorem 2.8** *Let* $\mathcal{I} \subset 2^{\mathbb{N}}$ *be an admissible ideal and* $x = (x_n)_{n\in\mathbb{N}}$, $y = (y_n)_{n\in\mathbb{N}}$ *are sequences in linear* 2*-normed space* $(X, \|.,.\|)$ *such that*

$$M = \{n \in \mathbb{N} : x_n \neq y_n \} \in \mathcal{I}.$$

*Then* $\mathcal{I}\left(\wedge_x^2\right) = \mathcal{I}\left(\wedge_y^2\right)$ *and* $\mathcal{I}\left(\Gamma_x^2\right) = \mathcal{I}\left(\Gamma_y^2\right)$.

**Proof.** Let $M = \{n \in \mathbb{N} : x_n \neq y_n\} \in \mathcal{I}$. If $\xi \in \mathcal{I}\left(\wedge_x^2\right)$, then there is a set $K = \{k_1 < k_2 < \ldots\} \notin \mathcal{I}$ such that $\mathcal{I}\text{-}\lim_{n\to\infty} \|x_{k_n} - \xi, z\| = 0$. Since

$$K_1 = \{n \in \mathbb{N} : n \in K \wedge x_n \neq y_n\} \subset M \in \mathcal{I},$$

then $K_2 = \{n \in \mathbb{N} : n \in K \wedge x_n = y_n\} \notin \mathcal{I}$ (indeed, if $K_2 \in \mathcal{I}$, then $K = K_1 \cup K_2 \in \mathcal{I}$, but $K \notin \mathcal{I}$. Hence the sequence $y_{K_2} = (y_n)_{n\in K_2}$ is a $\mathcal{I}$-nonthin subsequence of $y = (y_n)_{n\in\mathbb{N}}$ and $y_{K_2}$ converges to $\xi$ in 2-normed space, i.e. $\xi \in \mathcal{I}\left(\wedge_y^2\right)$.

Now let $\xi \in \mathcal{I}\left(\Gamma_x^2\right)$. Then $K_3 = \{n \in \mathbb{N} : \|x_n - \xi, z\| < \varepsilon\} \notin \mathcal{I}$ for each $\varepsilon > 0$ and nonzero $z$ in $X$ and $K_4 = \{n \in \mathbb{N} : n \in K_3 \wedge x_n = y_n\} \notin \mathcal{I}$. Therefore, $K_4 \subset \{n \in \mathbb{N} : \|y_n - \xi, z\| < \varepsilon\}$ for each nonzero $z$ in $X$. It shows that, for each $\varepsilon > 0$ and nonzero $z$ in $X$, $\{n \in \mathbb{N} : \|y_n - \xi, z\| < \varepsilon\} \notin \mathcal{I}$ i.e. $\xi \in \mathcal{I}\left(\Gamma_y^2\right)$. Theorem 1 is proved. □

The next theorem proves a strong connection between $\mathcal{I}$-cluster and ordinary limit points of a given sequence.

**Theorem 2.9** *Let $\mathcal{I} \subset 2^{\mathbb{N}}$ be an admissible ideal with property $(AP)$ and $x = (x_n)_{n\in\mathbb{N}}$ be a sequence in linear 2-normed space $(X, \|.,.\|)$. Then there is a sequence $y = (y_n)_{n\in\mathbb{N}}$ such that $L_y^2 = \mathcal{I}\left(\Gamma_x^2\right)$, and $\{n \in \mathbb{N} : x_n \neq y_n\} \in \mathcal{I}$, where $L_y^2$ is ordinary limit points set of the sequence $y = (y_n)_{n\in\mathbb{N}}$. Moreover $\{y_n : n \in \mathbb{N}\} \subset \{x_n : n \in \mathbb{N}\}$.*

**Proof.** If $\mathcal{I}\left(\Gamma_x^2\right) = L_x^2$, then $y = x$ and this case is trivial. Let $\mathcal{I}\left(\Gamma_x^2\right)$ is a proper subset of $L_x^2 : \mathcal{I}\left(\Gamma_x^2\right) \subset L_x^2$. Then $L_x^2 \backslash \mathcal{I}\left(\Gamma_x^2\right) \neq \emptyset$ and for each $\xi \in L_x^2 \backslash \mathcal{I}\left(\Gamma_x^2\right)$ there is an open interval $E_\xi = (\xi - \delta, \xi + \delta)$ such that $\mathcal{I}\text{-}\lim_{n\to\infty} \|x_{j_k} - \xi, z\| = 0$. Hence, there is an open interval $E_\xi = (\xi - \delta, \xi + \delta)$ such that

$$\left\{k \in \mathbb{N} : x_k \in E_\xi\right\} \in \mathcal{I}.$$

It is obvious that the collection of all intervals $E_\xi$ is an open cover of $L_x^2 \backslash \mathcal{I}\left(\Gamma_x^2\right)$,so by Covering Theorem there is a countable and mutually disjoint subcover $\{E_\xi\}_{j=1}^{\infty}$ such that each $E_j$ contains an $\mathcal{I}$-thin subsequence of $(x_n)_{n\in\mathbb{N}}$.

Now let $A_j = \{n \in \mathbb{N} : x_n \in E_j,\ j\varepsilon\mathbb{N}\}$. It is clear that $A_j \in \mathcal{I}$ $(j = 1, 2, ..)$ and $A_i \cap A_j = \emptyset$. Then by $(AP)$ property of $\mathcal{I}$ there is a countable collection $\{B_j\}_{j=1}^{\infty}$ of subsets of $\mathbb{N}$ such that $B = \cup_{j=1}^{\infty} B_j$ and $A_j \backslash B$ is a finite set for each $j\varepsilon\mathbb{N}$.

Let $M = \mathbb{N}\backslash B = \{m_1 < m_2 < \ldots\} \subset \mathbb{N}$. Now the sequence $y = (y_k)$ is defined by $y = y_k$ if $k \in B$ and $y_k = x_k$ if $k \in M$. Obviously, $\{k \in \mathbb{N} : x_k \neq y_k\} \subset B \in \mathcal{I}$, so by Theorem 1 $\mathcal{I}(\Gamma_y) = \mathcal{I}(\Gamma_x)$ Since $A_j \backslash B$ is a finite set then the subsequence $y_B = (y_k)_{k\varepsilon B}$ has no limit point that is not also an $\mathcal{I}$-limit point of $y$ i.e. $L_y^2 = \mathcal{I}\left(\Gamma_y^2\right)$. Therefore, we have proved $L_y^2 = \mathcal{I}\left(\Gamma_x^2\right)$. Moreover, the construction of the sequence $y = (y_n)_{n\in\mathbb{N}}$ shows

$$\{y_n : n \in \mathbb{N}\} \subset \{x_n : n \in \mathbb{N}\}$$

Theorem 2.9 is proved. □

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