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Some s-Orthogonal Matrices Constructed by Strong Kronecker Multiplication

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Abstract : Strong Kronecker multiplication of two matrices is useful for constructing new s-orthogonal matrices from those known. These results are particularly important as they allow small matrices to be combined to from larger matrices, but of smaller order than the straight-forward Kronecker product would permit.

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1 Introduction and Basic Definitions

The study of secondary symmetric, skew-symmetric and orthogonal matrices was initiated by Lee [1, 2] and the concept of some orthogonal designs constructed by Kronecker and Hadamard products was introduced by Seberry [3, 4]. In this paper we extend the results concerning orthogonal matrices to a secondary orthogonal matrices.

Throughout this paper we use the following notation:

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Notation 1.1. *Write* $\varepsilon = \{1, -1, i, -i\}, X = \{x_1, ..., x_u, 0\}, Y = \{y_1, ..., y_v, 0\}, Z = \{xy/x \in X, y \in Y\}$ *where* $x_1, ..., x_u, y_1, ..., y_v$ *are real commuting variables, in the otherwords, the complex conjugate of* $x_i(y_i)$ *is* $x_i(y_i)$. *Let* $\mathcal{R} = \{\alpha x/\alpha \in \varepsilon, x \in X\}, \mathcal{T} = \{\beta y/\beta \in \varepsilon, y \in Y\}, \mathcal{S} = \{\gamma xy/\gamma \in \varepsilon, x \in X, y \in Y\}$. *Further we write* $\phi = \sum_{j=1}^{u} p_j x_j^2, \psi = \sum_{j=1}^{v} q_j y_j^2$, *where* p_j *and* q_j *are positive integers.*

Notation 1.2. *The secondary transpose (conjugate secondary transpose) of* A *is defined by* $A^s = VA^TV(A^\Theta = VA^*V)$, *where "*$V$*" is the fixed disjoint permutation matrix with units in its secondary diagonal.*

Definition 1.3 ([5])**.** A matrix $A \in \mathbb{C}_{n\times n}$ is called *secondary orthogonal (s-orthogonal)*, if $AA^s = A^sA = I$, that is $A^s = A^{-1}$.

Definition 1.4. Let C be a $(1, -1, i, -i, 0)$ matrix of order c, satisfying $CC^\Theta = rI$, where $C^\Theta(C^\Theta = \overline{C}^s)$ is the conjugate secondary transpose of C. We call C a complex weighing matrix order c and weight r, denoted by $CW(c, r)$. In particular, if C is a real matrix, we call C a weighing matrix denoted by $W(c, r)$. $CW(c, c)$ is called a complex Hadamard matrix of order c.

From [6] any complex Hadamard matrix has order 1 or order divisible by 2. Let $C = X + iY$, where X, Y consist of $1, -1, 0$ and $X \bigcirc Y = 0$, where $\bigcirc$ is the Hadamard product. Clearly if C is a $CW(c, r)$ then $XX^s + YY^s = rI$, $XY^s = YX^s$.

Definition 1.5. A Complex Secondary Orthogonal Design (CSOD) of order n and type $(p_1, ..., p_u)$ denoted by $CSOD(m; p_1, ..., p_u)$ on the commuting variables $x_1, ..., x_u$ is a matrix of order n, say A, with elements from $\mathcal{R}$, satisfying

$$AA^\Theta = \phi I_n.$$

In particular, if A has elements from only X, the complex s-orthogonal will be called an s-orthogonal design by $SOD(m; p_1, ..., p_u)$.

Let M be a matrix of order tm. Then M can be expressed as

$$M = \begin{bmatrix} M_{11} & M_{12} & \dots & M_{1t} \\ M_{21} & M_{22} & \dots & M_{2t} \\ \dots & \dots & \dots & \dots \\ M_{t1} & M_{t2} & \dots & M_{tt} \end{bmatrix},$$

where M_{ij} is of order $m(i, j = 1, 2, ..., t)$. Let N be a matrix of order tn. Then, write

$$N = \begin{bmatrix} N_{11} & N_{12} & \dots & N_{1t} \\ N_{21} & N_{22} & \dots & N_{2t} \\ \dots & \dots & \dots & \dots \\ N_{t1} & N_{t2} & \dots & N_{tt} \end{bmatrix},$$

where N_{ij} is of order $n(i, j = 1, 2, ..., t)$. We now define the operation $\otimes$ as the following

$$M \otimes N = \begin{bmatrix} L_{11} & L_{12} & \dots & L_{1t} \\ L_{21} & L_{22} & \dots & L_{2t} \\ \dots & \dots & \dots & \dots \\ L_{t1} & L_{t2} & \dots & L_{tt} \end{bmatrix},$$

where M_{ij}, N_{ij} and L_{ij} are of order m, n, and mn, respectively and

$$L_{ij} = M_{i1} \times N_{1j} + M_{i2} \times N_{2j} + \cdots + M_{it} \times N_{tj},$$

where $\times$ is the Kronecker product, $i, j = 1, 2, ..., t$. We call this the strong Kronecker multiplication of two matrices.

The aim is to construct new s-orthogonal designs from those known previously. The most popular method has been the Kronecker product, so that if there exist Hadamard matrices of order $4h$ and $4n$ then there exists an Hadamard matrix of order $16hn$.

In this paper, we systematically study constructions for various s-orthogonal matrices with special properties, including CSODs, SODs, CWs and weighing matrices, by using strong Kronecker multiplication.

2 Strong Kronecker Product

Theorem 2.1 (Strong Kronecker Product Lemma). *Let $A = (A_{ij})$ satisfy $AA^s = \phi I_{tm}$, where A_{ij} have order m and $B = (B_{ij})$ satisfy $BB^s = \psi I_{tn}$, where B_{ij} have order n then*

$$(A \otimes B)(A \otimes B)^s = \phi\psi I_{tmn}.$$

Proof. Since $AA^s = \phi I_{tm}, BB^s = \psi I_{tn}$,

$$\begin{aligned}(A \otimes B)(A \otimes B)^s &= (A \otimes B)(A^s \otimes B^s) \\ &= AA^s \otimes BB^s \\ &= \phi I_{tm} \otimes \psi I_{tn} \\ &= \phi\psi I_{tmn}.\end{aligned}$$

□

Remark 2.2. *If A and B are s-orthogonal designs $A \otimes B$ is not an s-orthogonal design but an s-orthogonal matrix.*

Theorem 2.3. *Let $A = (A_{ij})$ with elements from $\mathcal{R}$ satisfy $AA^s = \phi I_{tm}$, where A_{ij} have order m and $B = (B_{ij})$ with elements from $\mathcal{T}$ satisfy $BB^s = \psi I_{tn}$, where B_{ij} have order n. Then if $C = A \otimes B$,*

$$CC^{\Theta} = (A \otimes B)(A \otimes B)^{\Theta} = \phi\psi I_{tmn}.$$

Remark 2.4. *$C = A \otimes B$ is not a complex s-orthogonal design but a complex s-orthogonal matrix.*

Corollary 2.5. *Let $A = CW(tm, p)$, and $B = CW(tn, q)$. Then, writing $C = A \otimes B$, $CC^{\Theta} = pqI_{tmn}$.*

The strong Kronecker multiplication has the potential to yield still construction for new orthogonal matrices as has been shown by [7]. We now extend those to s-orthogonal matrix.

3 Conferred Amicability Theorem

Lemma 3.1 (Structure Lemma)**.** *Let $A = (A_{kj}), C = (C_{kj})$ be matrices of order tm with elements from $\mathcal{T}$, where A_{kj}, C_{kj} are of order m and $B = (B_{kj}), D = (D_{kj})$ be matrices of order tn with elements from $\mathcal{R}$, where B_{kj}, D_{kj} are of order n. Write $(A \otimes B)(C \otimes D)^{\Theta} = (L_{ab})$, where $a, b = 1, ..., t$ then*

$$L_{ab} = \sum_{r=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t} A_{aj}C_{bk}^{\Theta} \times B_{jr}D_{kr}^{\Theta}.$$

In particular, if $C = A$ and $D = B$

$$L_{ab} = \sum_{r=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t} A_{aj}A_{bk}^{\Theta} \times B_{jr}B_{kr}^{\Theta}.$$

Now if B is s-orthogonal with $BB^{\Theta} = \psi I_{tn}$, where ψ is defined in (1.1), then

$$L_{ab} = \left(\sum_{j=1}^{t} A_{aj}A_{bj}^{\Theta}\right) \times \psi I_{mn}.$$

Further if A is s-orthogonal with $AA^{\Theta} = \phi I_{tn}$, where ϕ is defined in (1.1), then $L_{ab} = 0$, for $a \neq b$ and $L_{aa} = \phi\psi I_{mn}$.

Proof. It is easy to calculate

$$\begin{aligned} L_{ab} &= \sum_{r=1}^{t}(A_{a1} \times B_{1r} + \cdots + A_{at} \times B_{tr})(C_{a1}^{\Theta} \times D_{1r}^{\Theta} + \cdots + C_{at}^{\Theta} \times D_{tr}^{\Theta}) \\ &= \sum_{r=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t}(A_{aj} \times B_{jr})(C_{bk}^{\Theta} \times D_{kr}^{\Theta}). \end{aligned}$$

Obviously, if $C = A$ and $D = B$

$$L_{ab} = \sum_{r=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t} A_{aj}A_{bk}^{\Theta} \times B_{jr}B_{kr}^{\Theta}.$$

Further if B is s-orthogonal, $\sum_{r=1}^{t} B_{jr}B_{kr}^{\Theta} = 0, forj \neq k$. So

$$L_{ab} = \sum_{j=1}^{t}\sum_{k=1}^{t} A_{aj}A_{bk}^{\Theta} \times \left(\sum_{r=1}^{t} B_{jr}B_{kr}^{\Theta}\right) = \sum_{r=1}^{t} A_{aj}A_{bk}^{\Theta} \times \psi I_n.$$

So, $L_{ab} = 0, a \neq b$ and $L_{aa} = \phi\psi I_{mn}$. ∎

Theorem 3.2 (Conferred Amicability Theorem)**.** *Suppose $A = (A_{kj})$ is a matrix of order tm with elements from $\mathcal{R}$, where A_{kj} is of order m and $B = (B_{kj})$ and $C = (C_{kj})$ are matrices of order tn with elements from $\mathcal{T}$, where B_{kj} and C_{kj} are of order n. Write $P = A \otimes B$ and $Q = A \otimes C$. Suppose $BC^\Theta = CB^\Theta$. Then $PQ^\Theta = QP^\Theta$.*

Proof. Let $PQ^\Theta = (L_{ab})$ and $QP^\Theta = (R_{ab})$, where $a, b = 1, ..., t$. By the Structure Lemma,

$$\begin{aligned} L_{ab} &= \sum_{r=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t} A_{aj}A_{bk}^\Theta \times B_{jr}C_{kr}^\Theta \\ &= \sum_{j=1}^{t}\sum_{k=1}^{t} A_{aj}A_{bk}^\Theta \times \left(\sum_{r=1}^{t} B_{jr}C_{kr}^\Theta\right). \end{aligned}$$

Similarly,

$$\begin{aligned} R_{ab} &= \sum_{r=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t} A_{aj}A_{bk}^\Theta \times C_{jr}B_{kr}^\Theta \\ &= \sum_{j=1}^{t}\sum_{k=1}^{t} A_{aj}A_{bk}^\Theta \times \left(\sum_{r=1}^{t} C_{jr}B_{kr}^\Theta\right). \end{aligned}$$

Note $BC^\Theta = CB^\Theta$ implies $\sum_{r=1}^{t} B_{jr}C_{kr}^\Theta = \sum_{r=1}^{t} C_{jr}B_{kr}^\Theta, j, k = 1, ..., t$. So $L_{ab} = R_{ab}$ and $PQ^\Theta = QP^\Theta$. □

Corollary 3.3. *Suppose $A = (A_{kj})$ is a matrix of order tm with elements from $\mathcal{R}$, where A_{kj} is of order m and $B = (B_{kj})$ and $C = (C_{kj})$ are matrices of order tn with elements from $\mathcal{T}$, where B_{kj} and C_{kj} are of order n. Write $P = A \otimes B$ and $Q = A \otimes C$. Suppose $BC^\Theta = 0$. Then $PQ^\Theta = 0$.*

Proof. Let $PQ^\Theta = (L_{ab})$ and $QP^\Theta = (R_{ab})$, where $a, b = 1, ..., t$. By the Structure Lemma,

$$\begin{aligned} L_{ab} &= \sum_{r=1}^{t}\sum_{j=1}^{t}\sum_{k=1}^{t} A_{aj}A_{bk}^\Theta \times B_{jr}C_{kr}^\Theta \\ &= \sum_{j=1}^{t}\sum_{k=1}^{t} A_{aj}A_{bk}^\Theta \times \left(\sum_{j=1}^{t} B_{jr}C_{kr}^\Theta\right). \end{aligned}$$

Note $BC^\Theta = 0 \Rightarrow \sum_{r=1}^{t} B_{jr}C_{kr}^\Theta = 0$, $j, k = 1, ..., t$. So, $L_{ab} = 0$ and then $PQ^\Theta = 0$, also $QP^\Theta = 0$. □

4 Using CSOD$(2n; p_1, ..., p_u)$

Theorem 4.1. *Let A be a CSOD $(2a; p_1, ..., p_u)$ with elements from $\mathcal{R}$ and B be a CSOD $(2b; q_1, ..., q_v)$ with elements from $\mathcal{T}$. If $A = (A_{ij})$ with blocks of order a has the additional property that $A_{ij} \bigcirc A_{ik} = 0$ or $(A_{ij})^s \bigcirc (A_{ik})^s = 0[i.e., A_{a-j+1,a-i+1} \bigcirc A_{a-k+1,a-i+1} = 0], j \neq k, i = 1, 2$ then there exist four matrices with elements from $\mathcal{S}$ of order $2ab, P, Q, U, V$ satisfying*

1. $PQ^\Theta = QP^\Theta, PP^\Theta = QQ^\Theta = \phi\psi I_{2ab}$.
2. $UU^\Theta + VV^\Theta = \phi\psi I_{2ab}, U \bigcirc V = 0, UV^\Theta = VU^\Theta = 0, U + V = P, U - V = Q$.

Corollary 4.2. *Suppose there exist a CSOD$(2n; p_1, ..., p_u)$ with elements from $\mathcal{R}$ and a $W(2h, r) = A = (A_{ij})$ with blocks of order h which has the additional property that$A_{ij} \bigcirc A_{ik} = 0$ or $(A_{ij})^s \bigcirc (A_{ik})^s = 0, j \neq k, i = 1, 2$ then there exist*

1. *Two CSOD$(2hn; rp_1, ..., rp_u)$, P and Q, satisfying $PQ^\Theta = QP^\Theta$.*
2. *Two matrices with elements from $\mathcal{R}$, of order $2hn$, U and V, satisfying $UU^\Theta + VV^\Theta = r\phi I_{2hn}, U \bigcirc V = 0, UV^\Theta = VU^\Theta = 0, U + V = P, U - V = Q$.*

Corollary 4.3. *Suppose there exist a Hadamard matrix of order $2c$ and a $CW(2h, e) = A = (A_{ij})$ with blocks of order h which satisfy $A_{ij} \bigcirc A_{ik} = 0$ or $(A_{ij})^s \bigcirc (A_{ik})^s = 0, j \neq k, i = 1, 2$ then there exist*

1. *Two $CW(2ch, 2ce)$, P and Q, satisfying $PQ^\Theta = QP^\Theta$.*
2. *Two $(1, -1, 0)$ matrices U and V of order $2ch$, satisfying $UU^\Theta + VV^\Theta = 2ceI_{2ch}, U \bigcirc V = 0, UV^\Theta = VU^\Theta = 0, U + V = P, U - V = Q$.*

Corollary 4.4. *If there exist a $W(2n, e)$and a $W(2h, t) = A = (A_{ij})$ with blocks of order h which satisfy $A_{ij} \bigcirc A_{ik} = 0$ or $(A_{ij})^s \bigcirc (A_{ik})^s = 0, j \neq k, i = 1, 2$ then there exist*

1. *Two $W(2hn, et)$, P and Q, satisfying $PQ^s = QP^s$.*
2. *Two $(1, -1, 0)$ matrices U and V of order $2hn$, satisfying $UU^s + VV^s = etI_{2hn}, U \bigcirc V = 0, UV^s = VU^s = 0, U + V = P, U - V = Q$.*

Corollary 4.5. *Let there exist a CSOD $(2n; p_1, ..., p_u)$ with elements from $\mathcal{R}$ and an Hadamard matrix of order $4h$ then there exist*

1. *Two CSOD$(4hn; 2hp_1, ..., 2hp_u)$, P and Q, satisfying $PQ^\Theta = QP^\Theta$.*
2. *Two matrices with elements from $\mathcal{R}$, of order $4hn$, U and V, satisfying $UU^\Theta + VV^\Theta = 2h\phi I_{4hn}, U \bigcirc V = 0, UV^\Theta = VU^\Theta = 0, U + V = P, U - V = Q$.*

Corollary 4.6. *If there exist a $CW(2n, k)$ and an Hadamard matrix of order $4h$ then there exist*

1. *Two $CW(4hn, 2hk)$, P and Q, satisfying $PQ^\Theta = QP^\Theta$.*

2. *Two matrices with elements from* $\{\pm 1, \pm i, 0\}$, U *and* V *of order* $4hn$, *satisfying* $UU^{\Theta} + VV^{\Theta} = 2kh\phi I_{4hn}, U \bigcirc V = 0, UV^{\Theta} = VU^{\Theta} = 0, U + V = P, U - V = Q$.

Corollary 4.7. *If there exist a complex Hadamard matrix of order* $2c$ *and an Hadamard matrix of order* $4h$ *then there exist*

1. *Two complex Hadamard matrices of order* $4hc$, P *and* Q, *satisfying* $PQ^{\Theta} = QP^{\Theta}$.
2. *Two matrices with elements from* $\{\pm 1, \pm i, 0\}$, U *and* V *of order* $4hc$, *satisfying* $UU^{\Theta} + VV^{\Theta} = 4hc\phi I_{4hc}, U \bigcirc V = 0, UV^{\Theta} = VU^{\Theta} = 0, U + V = P, U - V = Q$.

Corollary 4.8. *If there exist a* $W(2n, k)$ *and an Hadamard matrix of order* $4h$ *then there exist*

1. *Two* $W(4hn, 2hk)$, P *and* Q, *satisfying* $PQ^s = QP^s$.
2. *Two* $\{1, -1, 0\}$ *matrices* U *and* V *of order* $4hn$, *satisfying* $UU^s + VV^s = 2hk\phi I_{4hn}, U \bigcirc V = 0, UV^s = VU^s = 0, U + V = P, U - V = Q$.

Corollary 4.9. *If there exist a CSOD* $(m; p_1, ..., p_u)$ *with elements from* $\mathcal{R}$ *and a CSOD* $(2n; q_1, ..., q_v)$ *with elements from* $\mathcal{T}$ *then there exist four matrices with elements from* $\mathcal{S}$, *of order* $2mn, P, Q, U, V$ *satisfying*

1. $PQ^{\Theta} = QP^{\Theta}, PP^{\Theta} = QQ^{\Theta} = \phi\psi I_{2mn}$.
2. $UU^{\Theta} + VV^{\Theta} = \phi\psi I_{2mn}, U \bigcirc V = 0, UV^{\Theta} = VU^{\Theta} = 0, U + V = P, U - V = Q$.

Corollary 4.10. *If there exist a CSOD* $(m; p_1, ..., p_u)$ *with elements from* $\mathcal{R}$ *and a* $W(2n, k)$ *then there exist*

1. *Two* $SOD(2n; kp_1, ..., kp_u)$ *with elements from* X, P *and* Q, *satisfying* $PQ^s = QP^s$.
2. *Two matrices* U *and* V *with elements from* X, *of order* $2mn$, *satisfying* $UU^s + VV^s = k\phi I_{2mn}, U \bigcirc V = 0, UV^s = VU^s = 0, U + V = P, U - V = Q$.

Corollary 4.11. *If there exist a* $SOD(2n; p_1, ..., p_u)$ *with elements from* X *and a* $CW(c, r)$ *then there exist*

1. *Two* $SOD(2cn; rp_1, ..., rp_u)$ *with elements from* X, P *and* Q, *satisfying* $PQ^s = QP^s$.
2. *Two matrices* U *and* V *with elements from* X, *of order* $2cn$, *satisfying* $UU^s + VV^s = r\phi I_{2cn}, U \bigcirc V = 0, UV^s = VU^s = 0, U + V = P, U - V = Q$.

Corollary 4.12. *If there exist a* $CW(c, r)$ *and a* $W(2n, k)$ *then there exist*

1. *Two* $W(2cn, 2rk)$, P *and* Q, *satisfying* $PQ^s = QP^s$.
2. *Two* $(1, -1, 0)$ *matrices* U *and* V *of order* $2cn$, *satisfying* $UU^s + VV^s = rk\phi I_{2cn}, U \bigcirc V = 0, UV^s = VU^s = 0, U + V = P, U - V = Q$.

5 Using CSOD$(4n; p_1, ..., p_u)$

Theorem 5.1. *Let A be a CSOD $(4a; p_1, ..., p_u)$ with elements from $\mathcal{R}$ and B be a CSOD $(4b; q_1, ..., q_v)$ with elements from $\mathcal{T}$. If $A = (A_{ij})$ with blocks of order a has the additional property that (i) $A_{ij} \bigcirc A_{ik} = 0$ or (ii) $(A_{ij})^s \bigcirc (A_{ik})^s = 0, (j,k) = (1,2), (j,k) = (3,4), i = 1,2,3,4$ then there exist four matrices U_1, U_2, U_3, U_4 with elements from $\mathcal{S}$ of order $4hb$, satisfying*

1. $U_1U_1^{\Theta} + U_2U_2^{\Theta} + U_3U_3^{\Theta} + U_4U_4^{\Theta} = \phi\psi I_{4ab}$.
2. $U_iU_j^{\Theta} = 0$ *for* $i \neq j$.
3. $U_1 \bigcirc U_2 = 0, U_3 \bigcirc U_4 = 0$.

Corollary 5.2. *Let A be a CSOD $(4a; p_1, ..., p_u)$ with elements from $\mathcal{R}$ and B be a CSOD $(4b; q_1, ..., q_v)$ with elements from $\mathcal{T}$. If $A = (A_{ij})$ with blocks of order a has the additional property that (i) $A_{ij} \bigcirc A_{ik} = 0$ or (ii)$(A_{ij})^s \bigcirc (A_{ik})^s = 0, (j,k) = (1,2), (j,k) = (3,4), i = 1,2,3,4$ then there exist two matrices E and F with elements from $\mathcal{S}$ of order $4ab$, satisfying $EF^{\Theta} = FE^{\Theta} = 0, EE^{\Theta} + FF^{\Theta} = \phi\psi I_{4ab}$.*

Corollary 5.3. *If there exist a CSOD $(4a; p_1, ..., p_u)$ with elements from $\mathcal{R}$ and an Hadamard matrix of order $4h$ then there exist four matrices U_1, U_2, U_3, U_4 with elements from $\mathcal{R}$ of order $4hn$, satisfying*

1. $U_1U_1^{\Theta} + U_2U_2^{\Theta} + U_3U_3^{\Theta} + U_4U_4^{\Theta} = 2\phi\psi I_{4hn}$.
2. $U_iU_j^{\Theta} = 0$ *for* $i \neq j$.
3. $U_1 \bigcirc U_2 = 0, U_3 \bigcirc U_4 = 0$.

Corollary 5.4. *If there exist CSOD $(4n; p_1, ..., p_u)$ with elements from $\mathcal{R}$ and an Hadamard matrix of order $4h$ then there exist two matrices E and F with elements from $\mathcal{S}$ of order $4hn$, satisfying $EF^{\Theta} = FE^{\Theta} = 0, EE^{\Theta} + FF^{\Theta} = 2h\phi I_{4hn}$ also we have a CSOD $(8hn; 2hp_1, ..., 2hp_u)$.*

6 Conclusion

We have constructeted the some orthogonal matrices by Strong Kronecker multiplication. These results are particularly important as they allow small matrices to be combined to from larger matrices, but of smaller order than the straight forward Kronecker product would permit.

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