Thai Journal of Mathematics
Volume 11 (2013) Number 2 : 275–283

http://thaijmath.in.cmu.ac.th
ISSN 1686-0209

Existence and Stability Properties of Positive Weak Solutions for a Class of Dirichlet Equations Involving Indefinite Weight Functions Driven by a $(p_1, \ldots, p_n)$-Laplacian Operator

G.A. Afrouzi[1], S. Shakeri and A. Hadjian

Department of Mathematics, Faculty of Mathematical Sciences
University of Mazandaran, Babolsar, Iran
e-mail : afrouzi@umz.ac.ir (G.A. Afrouzi)
s.shakeri@umz.ac.ir (S. Shakeri)
a.hadjian@umz.ac.ir (A. Hadjian)

Abstract : In this note, we prove the existence and stability properties of positive weak solutions to a class of nonlinear equations driven by a $(p_1, \ldots, p_n)$-Laplacian operator and indefinite weight functions. First by using the method of sub-super solution we study the existence of positive weak solution. Next we study the stability properties of positive weak solution.

Keywords : $(p_1, \ldots, p_n)$-Laplacian; sub-super solution; linearized stability.
2010 Mathematics Subject Classification : 35J60; 35B30; 35B35.

1 Introduction and Preliminaries

In this work, we study the existence and stability properties of positive weak solutions to the nonlinear elliptic system

$$\begin{cases} -\Delta_{p_i} u_i = \lambda a_i(x) \prod_{j=1}^{n} u_j^{\alpha_j^i} - c_i & x \in \Omega, \\ u_i(x) = 0 & x \in \partial\Omega \end{cases} \tag{1.1}$$

[1]Corresponding author.

for $1 \leq i \leq n$, where Ω is a bounded domain in $\mathbb{R}^N (N \geq 1)$ with C^2-boundary $\partial\Omega$, $p_i > 1$, $\Delta_{p_i} u_i := \text{div}(|\nabla u_i|^{p_i-2}\nabla u_i)$ is the p_i-Laplacian operator, λ, c_i and α_j^i are positive parameters for $1 \leq i, j \leq n$, and the weight functions a_i satisfies $a_i \in C(\Omega)$ and $a_i(x) > a_0^i > 0$ for all $x \in \Omega$ for $1 \leq i \leq n$. First by using the method of sub-super solution we study the existence of positive weak solution. Next we study the stability properties of positive weak solution directly by analyzing the linearized system.

Problems involving the p-Laplacian arise from many branches of pure mathematics as in the theory of quasiregular and quasiconformal mapping (see [1]) as well as from various problems in mathematical physics notably the flow of non-Newtonian fluids.

Systems of the form

$$\begin{cases} -\Delta_p u = \lambda a(x) v^\alpha & x \in \Omega, \\ -\Delta_q v = \lambda b(x) u^\beta & x \in \Omega, \\ u(x) = 0 = v(x) & x \in \partial\Omega, \end{cases} \tag{1.2}$$

and

$$\begin{cases} -\Delta_p u = \lambda u^\alpha v^\gamma & x \in \Omega, \\ -\Delta_q v = \lambda u^\delta v^\beta & x \in \Omega, \\ u(x) = 0 = v(x) & x \in \partial\Omega \end{cases} \tag{1.3}$$

arise in several context in biology and engineering (see [2, 3]). These systems provide simple models to describe, for instance, the interaction of three diffusing biological species. See [4] for details on the physical models involving more general reaction-diffusion system. Semipositone problems have been of great interest during the past two decades, and they continue to pose mathematically difficult problems in the study of positive solutions (see [5–10]). We refer to [11, 12] for additional results in nonlinear elliptic systems.

Throughout this paper, we let X be the Cartesian product of n spaces $W_0^{1,p_i}(\Omega)$ for $1 \leq i \leq n$, i.e., $X = W_0^{1,p_1}(\Omega) \times \cdots \times W_0^{1,p_n}(\Omega)$. We give the definition of weak solution and sub-super solution of (1.1) as follows.

Definition 1.1. We say that $u = (u_1, \ldots, u_n) \in X$ is a *weak solution* of the system (1.1), if we have

$$\int_\Omega \sum_{i=1}^n |\nabla u_i(x)|^{p_i-2}\nabla u_i(x)\nabla w_i(x)dx - \int_\Omega \sum_{i=1}^n \Big(\lambda a_i(x) \prod_{j=1}^n u_j^{\alpha_j^i}(x) - c_i\Big) w_i(x)dx = 0$$

for all $w = (w_1, \ldots, w_n) \in X$.

Definition 1.2. We say that $\psi = (\psi_1, \ldots, \psi_n)$ and $z = (z_1, \ldots, z_n)$ in X are a *subsolution* and a *supersolution* of the system (1.1), if we have

$$\int_\Omega \sum_{i=1}^n |\nabla \psi_i(x)|^{p_i-2}\nabla \psi_i(x)\nabla w_i(x)dx \leq \int_\Omega \sum_{i=1}^n \Big(\lambda a_i(x) \prod_{j=1}^n \psi_j^{\alpha_j^i}(x) - c_i\Big) w_i(x)dx$$

and

$$\int_{\Omega}\sum_{i=1}^{n}|\nabla z_i(x)|^{p_i-2}\nabla z_i(x)\nabla w_i(x)dx \geq \int_{\Omega}\sum_{i=1}^{n}\Big(\lambda a_i(x)\prod_{j=1}^{n} z_j^{\alpha_j^i}(x)-c_i\Big)w_i(x)dx,$$

respectively, for all $w=(w_1,\ldots,w_n)\in X$.

Now if there exist a subsolution $\psi = (\psi_1,\ldots,\psi_n)$ and a supersolution $z=(z_1,\ldots,z_n)$ such that $0\leq\psi_i(x)\leq z_i(x)$ for all $x\in\Omega$ for $1\leq i\leq n$, then the system (1.1) has a positive solution $u=(u_1,\ldots,u_n)\in X$ such that $\psi_i(x)\leq u_i(x)\leq z_i(x)$ for all $x\in\Omega$ for $1\leq i\leq n$ (see [12]).

2 Existence Results

In this section, we shall prove that if $0<\alpha_j^i<1$ for $1\leq i,j\leq n$, then there exist positive constants c_0 and λ^* such that the system (1.1) has a positive solution when $c_i\leq c_0$ for $1\leq i\leq n$ and $\lambda\geq\lambda^*$. We will obtain the existence of positive weak solution to the system (1.1) by constructing a positive subsolution $\psi=(\psi_1,\ldots,\psi_n)$ and a positive supersolution $z=(z_1,\ldots,z_n)$.

To precisely state our theorem, for $1\leq i\leq n$ we first consider the eigenvalue problem

$$\begin{cases} -\Delta_{p_i}\phi_i=\lambda_i|\phi_i|^{p_i-2}\phi_i & x\in\Omega,\\ \phi_i=0 & x\in\partial\Omega.\end{cases} \tag{2.1}$$

Let λ_{1,p_i} be the respective first eigenvalue of Δ_{p_i} with Dirichlet boundary condition and ϕ_{1,p_i} the corresponding eigenfunction with

$$\phi_{1,p_i}>0,\ \ \|\phi_{1,p_i}\|_\infty=1,\quad \text{for} 1\leq i\leq n.$$

It can be shown that $|\nabla\phi_{1,p_i}|\neq 0$ on $\partial\Omega$ for $1\leq i\leq n$, and hence, depending on Ω, there exist positive constants k,η and μ such that

$$\begin{cases} |\nabla\phi_{1,p_i}|^{p_i}-\lambda_{1,p_i}\phi_{1,p_i}^{p_i}\geq k & \text{in } \overline{\Omega}_\eta,\\ \phi_{1,p_i}\geq\mu & \text{in } \Omega\setminus\overline{\Omega}_\eta\end{cases} \tag{2.2}$$

where $\overline{\Omega}_\eta=\{x\in\Omega|d(x,\partial\Omega)\leq\eta\}$.

For $1\leq i\leq n$, we will also consider the unique solution, $\zeta_i\in C^1(\overline{\Omega})$ of the boundary value problem

$$\begin{cases} -\Delta_{p_i}\zeta_i=1 & x\in\Omega,\\ \zeta_i=0\quad \text{for } 1\leq i\leq n & x\in\partial\Omega.\end{cases}$$

To discuss our existence result, it is known that $\zeta_i>0$ in Ω and $\partial\zeta_i/\partial n<0$ on $\partial\Omega$ where n denotes the outward unit normal to $\partial\Omega$ for $1\leq i\leq n$ (see [13]). Now we state our main result as follows.

Theorem 2.1. *Let* $0 < \alpha_j^i < 1$ *for* $1 \le i, j \le n$. *Then there exist positive constants* c_0 *and* λ^* *such that the system (1.1) has a positive solution for* $c_i \le c_0$ $(1 \le i \le n)$ *and* $\lambda \ge \lambda^*$.

Proof. To obtain the existence of positive weak solution to the system (1.1), we shall construct a positive subsolution $\psi = (\psi_1, \ldots, \psi_n)$ and a supersolution $z = (z_1, \ldots, z_n)$ of the system (1.1). First, we construct a positive subsolution. For this, we shall verify that $(\psi_1, \ldots, \psi_n)$ with $\psi_i = \frac{p_i - 1}{p_i} \phi_{1,p_i}^{\frac{p_i}{p_i-1}}$ for $1 \le i \le n$ is a subsolution of the system (1.1). Let the test function $w = (w_1, \ldots, w_n) \in X$. Then

$$\begin{aligned}
\int_\Omega |\nabla \psi_i|^{p_i - 2} \nabla \psi_i \nabla w_i dx &= \int_\Omega \phi_{1,p_i} |\nabla \phi_{1,p_i}|^{p_i-2} \nabla \phi_{1,p_i} \nabla w_i dx \\
&= \int_\Omega |\nabla \phi_{1,p_i}|^{p_i-2} \nabla \phi_{1,p_i} \nabla(\phi_{1,p_i} w_i) dx \\
&\quad - \int_\Omega |\nabla \phi_{1,p_i}|^{p_i} w_i dx \\
&= \int_\Omega \lambda_{1,p_i} |\phi_{1,p_i}|^{p_i-2} \phi_{1,p_i} (\phi_{1,p_i} w_i) dx \\
&\quad - \int_\Omega |\nabla \phi_{1,p_i}|^{p_i} w_i dx. \\
&= \int_\Omega (\lambda_{1,p_i} |\phi_{1,p_i}|^{p_i} - |\nabla \phi_{1,p_i}|^{p_i}) w_i dx.
\end{aligned}$$

Thus $(\psi_1, \ldots, \psi_n)$ is a subsolution if

$$\lambda_{1,p_i} \phi_{1,p_i}^{p_i} - |\nabla \phi_{1,p_i}|^{p_i} \le \lambda a_i(x) \prod_{j=1}^n \psi_j^{\alpha_j^i} - c_i, \quad \text{for } 1 \le i \le n.$$

This inequality holds, because we have from (2.2)

$$\lambda_{1,p_i} \phi_{1,p_i}^{p_i} - |\nabla \phi_{1,p_i}|^{p_i} \le -k, \quad \text{in } \overline{\Omega}_\eta \ \text{ for } 1 \le i \le n,$$

and therefore if $c_i \le c_0 := k$ for $1 \le i \le n$, then

$$\lambda_{1,p_i} \phi_{1,p_i}^{p_i} - |\nabla \phi_{1,p_i}|^{p_i} \le -k = -c_0 \le \lambda a_i(x) \prod_{j=1}^n \psi_j^{\alpha_j^i} - c_i$$

for $1 \le i \le n$, since

$$\lambda a_i(x) \prod_{j=1}^n \psi_j^{\alpha_j^i} \ge 0.$$

On the other hand, in $\Omega \setminus \overline{\Omega}_\eta$ we have $\phi_{1,p_i} \ge \mu > 0$ for $1 \le i \le n$. Thus in $\Omega \setminus \overline{\Omega}_\eta$ we have

$$\lambda_{1,p_i} \phi_{1,p_i}^{p_i} - |\nabla \phi_{1,p_i}|^{p_i} \le \lambda_{1,p_i} \le \lambda a_i(x) \prod_{j=1}^n \psi_j^{\alpha_j^i} - c_i \ \text{ for } 1 \le i \le n,$$

if

$$\lambda \geq \hat{\lambda_i} := \frac{(\lambda_{1,p_i} + k) \prod_{j=1}^{n} (\frac{p_j}{p_j - 1})^{\alpha_j^i}}{a_0^i \prod_{j=1}^{n} \mu^{\frac{\alpha_j^i p_j}{p_j - 1}}} \quad \text{for } 1 \leq i \leq n.$$

Therefore, $\psi = (\psi_1, \ldots, \psi_n)$ is a subsolution of the system (1.1) for $c_i \leq c_0$ $(1 \leq i \leq n)$ and

$$\lambda \geq \lambda^* := \max\{\hat{\lambda_1}, \ldots, \hat{\lambda_n}\}.$$

Next we construct a supersolution $z = (z_1, \ldots, z_n)$ of the system (1.1) such that $0 < \psi_i(x) \leq z_i(x)$ for $x \in \Omega$ and $1 \leq i \leq n$. We denote

$$(z_1, \ldots, z_n) = (A_1\zeta_1, \ldots, A_n\zeta_n),$$

where the constants $A_1, \ldots, A_n > 0$ are large and to be chosen later. We shall verify that $z = (z_1, \ldots, z_n)$ is a supersolution of the system (1.1). To this end, letting $w = (w_1, \ldots, w_n) \in X$, we have

$$\begin{aligned} \int_\Omega |\nabla z_i(x)|^{p_i-2} \nabla z_i(x) \nabla w_i(x) dx &= A_i^{p_i-1} \int_\Omega |\nabla \zeta_i(x)|^{p_i-2} \nabla \zeta_i(x) \nabla w_i(x) dx \\ &= A_i^{p_i-1} \int w_i(x) dx. \end{aligned}$$

Let $l_i = \|\zeta_i\|_\infty$ for $1 \leq i \leq n$. Bearing in mind that $0 < \alpha_j^i < 1$ for $1 \leq i, j \leq n$, it is easy to prove that there exist positive large constants $A_1, \ldots, A_n$ such that

$$A_i \geq \Big[\lambda \|a_i\|_\infty \prod_{j=1}^{n} (A_j l_j) \alpha_j^i\Big]^{\frac{1}{p_i - 1}}$$

for $1 \leq i \leq n$, and then

$$\begin{aligned} A_i^{p_i-1} \geq \lambda \|a_i\|_\infty \prod_{j=1}^{n} (A_j l_j)^{\alpha_j^i} &\geq \lambda a_i(x) \prod_{j=1}^{n} (A_j l_j)^{\alpha_j^i} - c_i \\ &\geq \lambda a_i(x) \prod_{j=1}^{n} (A_j \zeta_j)^{\alpha_j^i} - c_i \\ &= \lambda a_i(x) \prod_{j=1}^{n} z_j^{\alpha_j^i} - c_i \end{aligned}$$

for $1 \leq i \leq n$. Therefore

$$\begin{aligned} \int_\Omega |\nabla z_i(x)|^{p_i-2} \nabla z_i(x) \nabla w_i(x) dx &= A_i^{p_i-1} \int_\Omega w_i(x) dx \\ &\geq \int_\Omega \big(\lambda a_i(x) \prod_{j=1}^{n} z_j^{\alpha_j^i}(x) - c_i\big) w_i(x) dx, \end{aligned}$$

i.e., $z = (z_1, \ldots, z_n)$ is a supersolution of the system (1.1) with $z_i \geq \psi_i$ in Ω for large A_i for $1 \leq i \leq n$. Then the system (1.1) has a positive solution $u = (u_1, \ldots, u_n) \in X$ such that $\psi_i \leq u_i \leq z_i$ for $1 \leq i \leq n$. Hence, Theorem 2.1 is proven. □

3 Stability Results

Here, we would establish stability of positive solution $u = (u_1, \ldots, u_n) \in X$ to the system (1.1) directly by showing that the principle eigenvalue η_1 of its linearization is positive.

We recall that, if $u = (u_1, \ldots, u_n)$ be any positive solution to the system

$$\begin{cases} -\Delta_{p_i} u_i = \lambda f^i(x, u_1, \ldots, u_n) & x \in \Omega, \\ u_i(x) = 0 & x \in \partial\Omega \end{cases}$$

for $1 \leq i \leq n$, then the linearized equation about $u = (u_1, \ldots, u_n)$ is

$$\begin{cases} -(p_i - 1)\operatorname{div}(|\nabla u_i|^{p_i-2}\nabla w_i) \\ \qquad\qquad -\lambda \sum_{j=1}^{n} f^i_{u_j}(x, u_1, \ldots, u_n) w_j = \eta\omega_i & x \in \Omega, \\ w_i(x) = 0 & x \in \partial\Omega \end{cases} \tag{3.1}$$

for $1 \leq i \leq n$, where $f^i_{u_j}(x, u_1, \ldots, u_n)$ denotes the partial derivative of $f^i(x, u_1, \ldots, u_n)$ with respect to u_j for $1 \leq i \leq n$. Equation (3.1) obtained from the formal derivative of the operator Δ_{p_i} (see [13]).

Definition 3.1. Let η_1 denote the first eigenvalue of (3.1). We say that $u = (u_1, \ldots, u_n)$ is *linearly stable*, if all eigenvalues of (3.1) are strictly positive, which can be inferred if the principal eigenvalue $\eta_1 > 0$. Otherwise $u = (u_1, \ldots, u_n)$ is linearly unstable.

Let $u = (u_1, \ldots, u_n)$ be any positive solution of the system (1.1). Then from (3.1) the linearized equation about $u = (u_1, \ldots, u_n)$ is

$$\begin{cases} -(p_i - 1)\operatorname{div}(|\nabla u_i|^{p_i-2}\nabla w_i) - \lambda a_i(x)\alpha_1^i u_1^{\alpha_1^i - 1} u_2^{\alpha_2^i} \ldots u_n^{\alpha_n^i} w_1 \\ \qquad\qquad - \cdots - \lambda a_i(x)\alpha_n^i u_1^{\alpha_1^i} u_2^{\alpha_2^i} \ldots u_n^{\alpha_n^i - 1} w_n = \eta\omega_i & x \in \Omega, \\ w_i(x) = 0 & x \in \partial\Omega \end{cases} \tag{3.2}$$

for $1 \leq i \leq n$.

Let η_1 be the principal eigenvalue and $\psi = (\psi_1, \ldots, \psi_n)$ be the corresponding eigenfunction. We make take ψ_i such that $\psi_i > 0$ in Ω and $\|\psi_i\|_\infty = 1$ for $1 \leq i \leq n$ (see [14]). Finally, we state our stability result as follows.

Theorem 3.2. *Suppose $c_i \leq c_0$ for $1 \leq i \leq n$, and $\lambda \geq \lambda^*$. Let $u = (u_1, \ldots, u_n)$ be the solution of the system (1.1) obtained in Theorem 1. Moreover, let α_j^i , c_i*

for $1 \leq i, j \leq n$, and λ be such that

$$\sum_{i=1}^{n} c_i(p_i-1)\psi_i(x) + \lambda \sum_{i=1}^{n} a_i(x)u_i\big(\alpha_1^i u_1^{\alpha_1^i-1}u_2^{\alpha_2^i}\cdots u_n^{\alpha_n^i}\psi_1 + \cdots + \alpha_n^i u_1^{\alpha_1^i}u_2^{\alpha_2^i}\cdots u_n^{\alpha_n^i-1}\psi_n\big)$$
$$< \lambda \sum_{i=1}^{n}(p_i-1)a_i(x)u_1^{\alpha_1^i}u_2^{\alpha_2^i}\cdots u_n^{\alpha_n^i}\psi_i$$

for all $x \in \Omega$. Then $u = (u_1, \ldots, u_n)$ is linearly stable.

Proof. We give from equations (1.1) and (3.2) that

$$\begin{aligned}
&\int_\Omega \sum_{i=1}^{n}(p_i-1)\big(u_i(x)\mathrm{div}(|\nabla u_i(x)|^{p_i-2}\nabla\psi_i(x)) - \psi_i(x)\mathrm{div}(|\nabla u_i(x)|^{p_i-2}\nabla u_i(x))\big)dx \\
&\quad + \lambda\int_\Omega \sum_{i=1}^{n} a_i(x)u_i(x)\big(\alpha_1^i u_1^{\alpha_1^i-1}u_2^{\alpha_2^i}\ldots u_n^{\alpha_n^i}\psi_i + \cdots + \alpha_n^i u_1^{\alpha_1^i}u_2^{\alpha_2^i}\cdots u_n^{\alpha_n^i-1}\psi_n\big)dx \\
&\quad - \lambda\int_\Omega \sum_{i=1}^{n}(p_i-1)\psi_i(x)\big(a_i(x)u_1^{\alpha_1^i}\cdots u_n^{\alpha_n^i}\big)dx + \int_\Omega \sum_{i=1}^{n} c_i(p_i-1)\psi_i(x)dx \\
&\quad = -\eta_1\int_\Omega \sum_{i=1}^{n} u_i(x)\psi_i(x)dx. \qquad (3.3)
\end{aligned}$$

But by using the Green's first identity, for $1 \leq i \leq n$ we obtain

$$\begin{aligned}
\int_\Omega u_i(x)\,\mathrm{div}(|\nabla u_i(x)|^{p_i-2}\nabla\psi_i(x))dx &= -\int_\Omega |\nabla u_i(x)|^{p_i-2}\nabla u_i(x)\nabla\psi_i(x)dx \\
&\quad + \int_{\partial\Omega} u_i(x)|\nabla u_i(x)|^{p_i-2}\Big(\frac{\partial\psi_i}{\partial n}\Big)\,dS \\
&= -\int_\Omega |\nabla u_i(x)|^{p_i-2}\nabla u_i(x)\nabla\psi_i(x)dx \qquad (3.4)
\end{aligned}$$

and

$$\begin{aligned}
\int_\Omega \psi_i(x)\,\mathrm{div}(|\nabla u_i(x)|^{p_i-2}\nabla u_i(x))dx &= -\int_\Omega |\nabla u_i(x)|^{p_i-2}\nabla u_i(x)\nabla\psi_i(x)dx \\
&\quad + \int_{\partial\Omega} \psi_i(x)|\nabla u_i(x)|^{p_i-2}\Big(\frac{\partial u_i}{\partial n}\Big)\,dS \\
&= -\int_\Omega |\nabla u_i(x)|^{p_i-2}\nabla u_i(x)\nabla\psi_i(x)dx. \qquad (3.5)
\end{aligned}$$

By using (3.4) and (3.5) in (3.3) and from the hypotheses we get

$$\begin{aligned}
&-\eta_1 \int_\Omega \sum_{i=1}^n u_i(x)\psi_i(x) \\
&= \lambda \int_\Omega \sum_{i=1}^n a_i(x)u_i(x)\big(\alpha_1^i u_1^{\alpha_1^i-1}u_2^{\alpha_2^i}\cdots u_n^{\alpha_n^i}\psi_i + \cdots + \alpha_n^i u_1^{\alpha_1^i}u_2^{\alpha_2^i}\cdots u_n^{\alpha_n^i-1}\psi_n\big)dx \\
&\quad - \lambda \int_\Omega \sum_{i=1}^n \big((p_i-1)\psi_i(x)\big(a_i(x)u_1^{\alpha_1^i}\cdots u_n^{\alpha_n^i}\big)dx + \int_\Omega \sum_{i=1}^n c_i(p_i-1)\psi_i(x)dx < 0.
\end{aligned}$$

But $\psi_1, \ldots, \psi_n > 0$ and $u_1, \ldots, u_n > 0$ in Ω, and hence $\eta_1 > 0$. This completes the proof. □

Acknowledgement : The authors would like to thank the referee for his/her comments and suggestions on the manuscript.

References

[1] P. Tolksdorf, Regularity for a more general class of quasilinear elliptic equations, J. Differential Equations 51 (1984) 126–150.

[2] C. Chen, On positive weak solutions for a class of quasilinear elliptic systems, Nonlinear Anal. 62 (2005) 751–756.

[3] A. Leung, Systems of nonlinear partial differential equations, applications to biology and engineering, Math. Appl., Kluwer Academic Publishers, Dordrecht, 1989.

[4] C.V. Pao, *Nonlinear Parabolic and Elliptic Equations*, Plenum Press, New York, 1992.

[5] G.A. Afrouzi, M. Baghery, S.H. Rasouli, On positive weak solutions for a class of semipositone equations, World J. Mod. Sim. 3 (4) (2007) 286–288.

[6] G.A. Afrouzi, S.H. Rasouli, On positive solutions for some nonlinear semipositone elliptic boundary value problems, Nonlinear Anal. : Model. Cont. 4 (11) (2006) 323–329.

[7] G.A. Afrouzi, J. Vahidi, S.H. Rasouli, On critical exponent for exsistence of positive solutions for some semipositone problems involving the weight function, Int. J. Math. Anal. 2 (20) (2008) 987–991.

[8] V. Anuradha, D.D. Hai, R. Shivaji, Existence results for superlinear semipositone boundary value problems, Proc. Amer. Math. Soc. 124 (3) (1996) 757–763.

[9] A. Castro, S. Gadam, R. Shivaji, Evolution of Positive Solution Curves in Semipositone Problems with Concave Nonlinearities, J. Math. Anal. Appl. 245 (2000) 282–293.

[10] S. Oruganti, R. Shivaji, Existence results for classes of p-Laplacian semipositone equations, Boundary Value Problems (2006) 1–7.

[11] G.A. Afrouzi, S.H. Rasouli, A remark on the existence of multiple solutions to a multiparameter nonlinear elliptic system, Nonlinear Anal. 71 (2009) 445–455.

[12] P. Drábek, J. Hernandez, Existence and uniqueness of positive solutions for some quasilinear elliptic problem, Nonlinear Anal. 44 (2001) 189–204.

[13] P. Drábek, P. Kerjci, P. Takac, *Nonlinear Differential Equations*, Chapman and Hall, London, 1999.

[14] C.V. Coffman, M.M. Marcus, V.J. Mizel, On Green's function and eigenvalues of nonuniformly elliptic boundary value problems, Math. Z. 182 (1983) 321–326.

(Received 24 July 2011)
(Accepted 11 April 2012)