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Common Fixed Point of Generalized Rational Type Contraction Mappings in Partially Ordered Metric Spaces

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Abstract : Some common fixed point results for generalized weak contractive condition satisfying rational type expressions in the framework of partially ordered metric spaces are obtained. The proved results generalize and extend some known results in the literature.

Keywords : common fixed point; weakly compatible mappings; ordered metric spaces.
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1 Introduction and Preliminaries

The Banach Contraction Mapping Principle is one of the cornerstone results of nonlinear functional analysis. It is very essential and crucial tool for qualitative nonlinear sciences such as biology, chemistry, physics, various branches of mathematics etc. In particular, solving existence and uniqueness problems in many different fields of mathematics. Due to its importance and applications potential, The Banach Contraction Mapping Principle has been investigated heavily

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by many authors. Consequently, a number of generalizations of this celebrated principle have appeared in the literature (see [1–28]).

One of the interesting characterization of Banach fixed point in partially ordered sets is done by Ran and Reurings [19]. In this interesting paper, authors [19] state some applications of this new fixed point theorem to linear and nonlinear matrix equations. Meanwhile Nieto and Rodŕiguez-López [18] extend the result of Ran and Reurings and apply their main theorems to obtain a unique solution for a first order ordinary differential equation with periodic boundary conditions. After these initial papers, a number of papers have appeared in this direction (see e.g. [2, 5, 11, 12, 14, 16, 29]). On the other hand, Bhaskar and Lakshmikantham [5] introduce the notion of coupled fixed point via mixed monotone mappings. In this crucial paper, the authors [5] obtain some coupled fixed point results and apply their theorems to solve a first order differential equation with periodic boundary conditions.

The concept of weakly contractive mappings, a generalization of Boyd and Wong [6] type contraction, is introduced by Alber and Guerre-Delabriere [1]. In this papaer, authors [1] proved the existence of fixed points for single-valued weakly contractive mappings in the context of Hilbert spaces. The notion of weak contractive type mappings and related fixed point problems have been studied heavily by many authors (see e.g. [21]). The purpose of this paper is to establish some common fixed point results satisfying a generalized rational type weak contraction mappings in partially ordered metric spaces.

First, we recall some necessary definitions.

Khan et al. [15] initiated the use of a control function that alters distance between two points in a metric space. Such mappings are called an altering distance function in the literature: A function $\mu : [0,\infty) \to [0,\infty)$ is called an *altering distance function* if the following properties are satisfied:

- (i) μ is monotone increasing and continuous;
- (ii) $\mu(t) = 0$ if and only if $t = 0$.

Let M be a nonempty subset of a metric space (X, d). A point $x \in M$ is called a *common fixed (coincidence) point* of f and T if $x = fx = Tx$ ($fx = Tx$). The set of fixed points (respectively, coincidence points) of f and T is denoted by $F(f, T)$ (respectively, $C(f, T)$). The mappings $T, f : M \to M$ are called *commuting* if $Tfx = fTx$ for all $x \in M$; *compatible* if $\lim_{n\to\infty} d(Tfx_n, fTx_n) = 0$ whenever $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} fx_n = t$ for some t in M; *weakly compatible* if they commute at their coincidence points, i.e., if $fTx = Tfx$ whenever $fx = Tx$.

Suppose that $(X, \leq)$ is a partially ordered set and $T, f : X \to X$ are self-mappings. A mapping T is said to be *monotone f-nondecreasing* if for all $x, y \in X$,

$$fx \leq fy \text{ implies } Tx \leq Ty. \tag{1.1}$$

If f is identity mapping, then T is called a *monotone nondecreasing mapping.*

A subset W of a partially ordered set X is said to be *well ordered* if every two elements of W are comparable.

2 Main Results

We start to this section with our main theorem.

Theorem 2.1. *Let $(X, \leq)$ be a partially ordered set and suppose that there exists a metric d on X such that (X, d) is a complete metric space. Suppose that T and f are continuous self mappings on X, $T(X) \subseteq f(X)$, T is monotone f-nondecreasing mapping and*

$$d(Tx, Ty) \leq M(x, y) - \psi(M(x, y)) \tag{2.1}$$

for all $x, y \in X$ with $f(x)$ and $f(y)$ are comparable, $\psi : [0, \infty) \to [0, \infty)$ is a lower semi-continuous mapping such that $\psi(t) = 0$ if and only if $t = 0$,

$$M(x, y) = \max\left\{\frac{d(fx, Tx)d(fy, Ty)}{d(fx, fy)}, d(fx, fy)\right\}.$$

If there exists $x_0 \in X$ such that $f(x_0) \leq T(x_0)$ and T and f are compatible, then T and f have a coincidence point.

Proof. Let $x_0 \in X$ such that $f(x_0) \leq T(x_0)$. Since $T(X) \subseteq f(X)$, we can choose $x_1 \in X$ so that $fx_1 = Tx_0$. Since $Tx_1 \in f(X)$, there exists $x_2 \in X$ such that $fx_2 = Tx_1$. By induction, we can construct a sequence $\{x_n\}$ in X such that $fx_{n+1} = Tx_n$, for every $n \geq 0$.

Since $f(x_0) \leq T(x_0) = f(x_1)$, T is monotone f-nondecreasing mapping, $T(x_0) \leq T(x_1)$. Similarly, since $f(x_1) \leq f(x_2)$, we have $T(x_1) \leq T(x_2)$, and $f(x_2) \leq f(x_3)$. Continuing, we obtain

$$T(x_0) \leq T(x_1) \leq T(x_2) \leq \cdots \leq T(x_n) \leq T(x_{n+1}) \leq \cdots.$$

We suppose that $d(T(x_n), T(x_{n+1})) > 0$ for all n. If not then $T(x_{n+1}) = T(x_n)$ for some n, $T(x_{n+1}) = f(x_{n+1})$, i.e. T and f have a coincidence point x_{n+1}, and so we have the result.

Consider

$$\begin{aligned} d(Tx_{n+1}, Tx_n) &\leq \max\left\{\frac{d(fx_{n+1}, Tx_{n+1})d(fx_n, Tx_n)}{d(fx_{n+1}, fx_n)}, d(fx_{n+1}, fx_n)\right\} \\ &\quad - \psi\left(\max\left\{\frac{d(fx_{n+1}, Tx_{n+1})d(fx_n, Tx_n)}{d(fx_{n+1}, fx_n)}, d(fx_{n+1}, fx_n)\right\}\right) \\ &= \max\{d(Tx_n, Tx_{n+1}), d(Tx_n, Tx_{n-1})\} \\ &\quad - \psi(\max\{d(Tx_n, Tx_{n+1}), d(Tx_n, Tx_{n-1})\}). \end{aligned} \tag{2.2}$$

Suppose there exists m such that $d(Tx_{m+1}, Tx_m) > d(Tx_m, Tx_{m-1})$, from (2.2), we have

$$\begin{aligned} d(Tx_{m+1}, Tx_m) &\leq \max\{d(Tx_{m+1}, Tx_m), d(Tx_m, Tx_{m-1})\} \\ &\quad - \psi(\max\{d(Tx_{m+1}, Tx_m), d(Tx_m, Tx_{m-1})\}) \\ &= d(Tx_{m+1}, Tx_m) - \psi(d(x_{m+1}, Tx_m)) \\ &< d(Tx_{m+1}, Tx_m) \end{aligned}$$

which is a contradiction. Hence, $d(Tx_{n+1}, Tx_n) \leq d(Tx_n, Tx_{n-1})$ for all $n \geq 1$. Thus $\{d(Tx_{n+1}, Tx_n)\}$ is a monotone decreasing sequence of non-negative real numbers and hence is convergent. Hence there exists $r \geq 0$ such that $d(Tx_{n+1}, Tx_n) \to r$.

Now, we shall show that $r = 0$. Suppose, to the contrary, that $r > 0$. Taking the upper limit as $n \to \infty$ in (2.2) and using the properties of the function ψ, we get

$$\begin{aligned} r &\leq r - \lim_{n\to\infty} \inf\{\psi(\max\{d(Tx_n, Tx_{n+1}), d(Tx_n, Tx_{n-1})\})\} \\ &\leq r - \psi(r) < r, \end{aligned}$$

which is a contradiction. Therefore, $r = 0$, that is,

$$d(Tx_{n+1}, Tx_n) \to 0. \tag{2.3}$$

Now, we shall prove that $\{Tx_n\}$ is a Cauchy sequence. If otherwise, then there exists $\epsilon > 0$ for which we can find subsequences $\{Tx_{m(k)}\}$ and $\{Tx_{n(k)}\}$ of $\{Tx_n\}$ with $m(k) > n(k) \geq k$ such that for every k, $d(Tx_{m(k)}, Tx_{n(k)}) \geq \epsilon$, $d(Tx_{m(k)}, Tx_{n(k)-1}) < \epsilon$. So, we have

$$\begin{aligned} \epsilon &\leq d(Tx_{m(k)}, Tx_{n(k)}) \\ &\leq d(Tx_{m(k)}, Tx_{n(k)-1}) + d(Tx_{n(k)-1}, Tx_{n(k)}) \\ &< \epsilon + d(Tx_{n(k)-1}, Tx_{n(k)}). \end{aligned}$$

Letting $k \to \infty$ and using $d(Tx_{n+1}, Tx_n) \to 0$, we have

$$\lim_{n\to\infty} d(Tx_{m(k)}, Tx_{n(k)}) = \epsilon = \lim_{n\to\infty} d(Tx_{m(k)}, Tx_{n(k)-1}). \tag{2.4}$$

Again,

$$\begin{aligned} d(Tx_{m(k)}, Tx_{n(k)-1}) \leq d(Tx_{m(k)}, Tx_{m(k)-1}) &+ d(Tx_{m(k)-1}, Tx_{n(k)}) \\ &+ d(Tx_{n(k)}, Tx_{n(k)-1}), \end{aligned}$$

and

$$d(Tx_{m(k)-1}, Tx_{n(k)}) \leq d(Tx_{m(k)-1}, Tx_{m(k)}) + d(Tx_{m(k)}, Tx_{n(k)}).$$

Letting $k \to \infty$ in the above two inequalities and using (2.4) we get,

$$\lim_{k\to\infty} d(Tx_{m(k)-1}, Tx_{n(k)}) = \epsilon.$$

Also, we have

$$\begin{aligned}
&d(Tx_{m(k)}, Tx_{n(k)})\\
&\leq \max\left\{\frac{d(fx_{m(k)}, Tx_{m(k)})d(fx_{n(k)}, Tx_{n(k)})}{d(fx_{m(k)}, fx_{n(k)})}, d(fx_{m(k)}, fx_{n(k)})\right\}\\
&\quad - \psi\left(\max\left\{\frac{d(fx_{m(k)}, Tx_{m(k)})d(fx_{n(k)}, Tx_{n(k)})}{d(fx_{m(k)}, fx_{n(k)})}, d(fx_{m(k)}, fx_{n(k)})\right\}\right)\\
&= \max\left\{\frac{d(Tx_{m(k)-1}, Tx_{m(k)})d(Tx_{n(k)-1}, Tx_{n(k)})}{d(Tx_{m(k)-1}, Tx_{n(k)-1})}, d(Tx_{m(k)-1}, Tx_{n(k)-1})\right\}\\
&\quad - \psi\left(\max\left\{\frac{d(Tx_{m(k)-1}, Tx_{m(k)})d(Tx_{n(k)-1}, Tx_{n(k)})}{d(Tx_{m(k)-1}, Tx_{n(k)-1})}, d(Tx_{m(k)-1}, Tx_{n(k)-1})\right\}\right).
\end{aligned}$$

Taking $k \to \infty$, and using the lower semi-continuity of ψ, we have $\epsilon \leq \max\{0, \epsilon\} - \psi(\max\{0, \epsilon\}) = \epsilon - \psi(\epsilon) < \epsilon$, which is contradiction since $\epsilon > 0$. Thus $\{Tx_n\}$ is a Cauchy sequence in a complete metric space X. Therefore there exits $u \in X$ such that $\lim_{n\to\infty} Tx_n = u$. By the continuity of T, we have $\lim_{n\to\infty} T(Tx_n) = Tu$. Since $fx_{n+1} = Tx_n \to u$ and the pair (T, f) is compatible, we have

$$\lim_{n\to\infty} d(f(Tx_n), T(fx_n)) = 0.$$

By the triangular inequality, we have

$$d(Tu, fu) \leq d(Tu, T(fx_n)) + d(T(fx_n), f(Tx_n)) + d(f(Tx_n), fu).$$

Letting $n \to \infty$, and using the fact T and f are continuous, we get $d(Tu, fu) = 0$, i.e. $Tu = fu$ and u is a coincidence point of T and f. □

Theorem 2.2. *Let $(X, \leq)$ be a partially ordered set and suppose that there exists a metric d on X such that (X, d) is a metric space. Suppose that T and f are self-mappings on X, $T(X) \subseteq f(X)$, T is monotone f-nondecreasing mapping and*

$$d(Tx, Ty) \leq M(x, y) - \psi(M(x, y)) \tag{2.5}$$

for all $x, y \in X$ with $f(x)$ and $f(y)$ are comparable, $\psi : [0, \infty) \to [0, \infty)$ is a lower semi-continuous mapping such that $\psi(t) = 0$ if and only if $t = 0$,

$$M(x, y) = \max\left\{\frac{d(fx, Tx)d(fy, Ty)}{d(fx, fy)}, d(fx, fy)\right\}.$$

Assume that there exists $x_0 \in X$ such that $f(x_0) \leq T(x_0)$ and $\{fx_n\}$ is a non-decreasing sequence in X such that $fx_n \to fx$, then $fx = \sup\{fx_n\}$. If fX is a complete subspace of X, then T and f have a coincidence point.

Further, if T and f are weakly compatible, then T and f have a common fixed point. Moreover, the set of common fixed points of T and f is well ordered if and only if T and f have one and only one common fixed point.

Proof. Following the proof of Theorem 2.1, we have $\{Tx_n\}$ is a Cauchy sequence. As $fx_{n+1} = Tx_n$, so $\{fx_n\}$ is a Cauchy sequence in $(f(X), d)$. Since $f(X)$ is complete, there is $fv \in f(X)$ such that $\lim_{n\to\infty} fx_n = fv = u$. Since $\{fx_n\}$ is a non-decreasing sequence in X such that $fx_n \to fv = u$, then $u = fv = \sup\{fx_n\}$. Particularly, $fx_n \leq fu$ for all $n \in \mathbb{N}$. Since T is monotone f-nondecreasing mapping $Tx_n \leq Tu$, for all $n \in \mathbb{N}$ or, equivalently, $fx_{n+1} \leq Tu$, for all $n \in \mathbb{N}$. Moreover, as $fx_n < fx_{n+1} \leq Tu$ and $u = fv = \sup\{fx_n\}$, we get $u \leq Tu$.

Construct a sequence $\{fy_n\}$ as $fy_0 = fx$, $fy_{n+1} = T(fy_n)$, for all $n \geq 0$. Since $fy_0 \leq T(fy_0)$, arguing like above part, we obtain that $\{fy_n\}$ is a non-decreasing sequence and $\lim_{n\to\infty} fy_n = fy$ for certain $fy \in f(X)$, so we have $fy = \sup\{fy_n\}$. Since $fx_n < fx = fy_0 \leq T(fx) = T(fy_0) \leq fy_n \leq fy$, for all n, using (2.5), we have

$$\begin{aligned}
&d(fx_{n+1}, fy_{n+1})\\
&\quad = d(Tx_n, T(fy_n))\\
&\quad \leq \max\left(\frac{d(fx_n, Tx_n)d(f(fy_n), T(fy_n))}{d(f(fy_n), Tx_n)}, d(f(fy_n), Tx_n)\right)\\
&\qquad - \psi\left(\max\left(\frac{d(fx_n, Tx_n)d(f(fy_n), T(fy_n))}{d(f(fy_n), Tx_n)}, d(f(fy_n), Tx_n)\right)\right) \qquad (2.6)\\
&\quad = \max\left(\frac{d(fx_n, fx_{n+1})d(f(fy_n), fy_{n+1})}{d(f(fy_n), fx_{n+1})}, d(f(fy_n), fx_{n+1})\right)\\
&\qquad - \psi\left(\max\left(\frac{d(fx_n, fx_{n+1})d(f(fy_n), fy_{n+1})}{d(f(fy_n), fx_{n+1})}, d(f(fy_n), fx_{n+1})\right)\right)
\end{aligned}$$

letting $n \to \infty$, we have $d(fv, fy) \leq \max\{0, d(fv, fy)\} - \psi(\max\{0, d(fv, fy)\}) < d(fv, fy)$, which implies that $d(fv, fy) = 0$. Particularly, $u = fv = fy = \sup\{fy_n\}$, and consequently, $u \leq Tu \leq u$, which is a contradiction. Hence we conclude that fv is a coincidence point of T and f.

Now suppose that T and f are weakly compatible. Let $w = T(z) = f(z)$. Then $T(w) = T(f(z)) = f(T(z)) = f(w)$. Consider

$$\begin{aligned}
d(T(z), T(w)) &\leq \max\left\{\frac{d(fz, Tz)d(fw, Tw)}{d(fz, fw)}, d(fz, fw)\right\}\\
&\qquad - \psi\left(\max\left\{\frac{d(fz, Tz)d(fw, Tw)}{d(fz, fw)}, d(fz, fw)\right\}\right)\\
&\leq \max\{0, d(Tz, Tw)\} - \psi(0, d(Tz, Tw)).
\end{aligned}$$

Using the lower semi-continuity of ψ, we have $d(Tz, Tw) = 0$. Therefore, $T(w) = f(w) = w$.

Now suppose that the set of common fixed points of T and f is well ordered. We claim that common fixed points of T and f is unique. Assume on contrary that, $Tu = fu = u$ and $Tv = fv = v$ but $u \neq v$. Consider

$$\begin{aligned}
\mu(d(u,v)) &= \mu(d(Tu,Tv)) \\
&\leq \max\left\{\frac{d(fu,Tu)d(fv,Tv)}{d(fu,fv)}, d(fu,fv)\right\} \\
&\quad - \psi\left(\max\left\{\frac{d(fu,Tu)d(fv,Tv)}{d(fu,fv)}, d(fu,fv)\right\}\right) \\
&\leq \max\{0, d(u,v)\} - \psi(\max\{0, d(u,v)\}).
\end{aligned}$$

This implies that $d(u,v) = 0$. Conversely, if T and f have only one common fixed point then the set of common fixed point of f and T being singleton is well ordered. □

Corollary 2.3. *Let $(X, \leq)$ be a partially ordered set and suppose that there exists a metric d on X such that (X,d) is a metric space. Suppose that T and f are self-mappings on X, $T(X) \subseteq f(X)$, T is monotone f-nondecreasing mapping and*

$$d(Tx,Ty) \leq k \max\left\{\frac{d(fx,Tx)d(fy,Ty)}{d(fx,fy)}, d(fx,fy)\right\} \tag{2.7}$$

for all $x, y \in X$ with $f(x)$ and $f(y)$ are comparable, $\psi : [0,\infty) \to [0,\infty)$ is a lower semi-continuous mapping such that $\psi(t) = 0$ if and only if $t = 0$, and $k \in (0,1)$. Assume that there exists $x_0 \in X$ such that $f(x_0) \leq T(x_0)$ and $\{fx_n\}$ is a non-decreasing sequence in X such that $fx_n \to fx$, then $fx = \sup\{fx_n\}$. If fX is a complete subspace of X, then T and f have a coincidence point.

Further, if T and f are weakly compatible, then T and f have a common fixed point. Moreover, the set of common fixed points of T and f is well ordered if and only if T and f have one and only one common fixed point.

Proof. By taking $\psi(t) = (1-k)t$ in Theorem 2.2, we get the result. □

Remark 2.4. *If $f = I$ (identity mapping) in the Theorems 2.1 and 2.2, then we have the Theorem 2.1 of Luong and Thuan [16].*

Other consequences of our results are the following for the mappings involving contractions of integral type.

Denote by Λ the set of functions $\mu : [0,\infty) \to [0,\infty)$ satisfying the following hypotheses:

(h1) μ is a Lebesgue-integrable mapping on each compact subset of $[0,\infty)$;

(h2) for any $\epsilon > 0$, we have $\int_0^\epsilon \mu(t)dt > 0$.

Corollary 2.5. *Let $(X, \leq)$ be a partially ordered set and suppose that there exists a metric d on X such that (X,d) is a complete metric space. Suppose that T and f*

are continuous self-mappings on X, $T(X) \subseteq f(X)$, T is monotone f-nondecreasing mapping and

$$\int_0^{d(Tx,Ty)} \alpha(t)dt \leq \int_0^{\max\{\frac{d(fx,Tx)d(fy,Ty)}{d(fx,fy)},d(fx,fy)\}} \alpha(t)dt - \int_0^{\psi(\max\{\frac{d(fx,Tx)d(fy,Ty)}{d(fx,fy)},d(fx,fy)\})} \beta(t)dt$$

for all $x, y \in X$ with $f(x)$ and $f(y)$ are comparable, $\alpha, \beta \in \Lambda$.

If there exists $x_0 \in X$ such that $f(x_0) \leq T(x_0)$ and T and f are compatible, then T and f have a coincidence point.

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