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Control Charts for Monitoring the Zero-Inflated Generalized Poisson Processes

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Abstract : This paper developed the *c-Chart* based on the Zero-Inflated Generalized Poisson (*ZIGP*) processes. We called the *c-Chart* based on *ZIGP* distribution the $c_G - Chart$. We first develop the control limits of the $c_G - Chart$ by using the expected and variance of *ZIGP* distribution; namely $c_{ZG} - Chart$. We then develop an approximated *ZIGP* distribution by a geometric distribution with parameter p. The p estimated the fit for *ZIGP* distribution used in calculating the expected skewness and variance of geometric distribution for constructing the control limits of $c_G - Chart$; namely $c_{Gg} - Chart$, $c_{Gk} - Chart$ and also to study the effects of the cumulative count of conforming items chart (*CCC-Chart*) which is used for monitoring a *ZIGP* process we call $CCC_g - Chart$. For $c_{Gg} - Chart$, we developed $c_G - Chart$ by using the expected and variance of the geometric distribution. For $c_{Gk} - Chart$, the skewness and variance were used for constructing the control limits. The $CCC_g - Chart$ developed control limits of *CCC-Chart* from the p estimation of geometric distribution. The performance considered the Average Run Length and Average Coverage Probability. We found that for an in-control process, the $CCC_g - Chart$ is superior for all levels of the mean (μ), proportion zero(ω), mean shift(ρ) and over dispersion (φ). For an out-of-control process, the $c_{Gg} - Chart$ is the best for $\mu = 1$ at low ω for all ρ and φ. The $c_{Gk} - Chart$ is the best for $\mu = 2$ at all parameters and for $\mu = 3$, 4 at high ω for all ρ and φ. The $c_{ZG} - Chart$ is the best for $\mu = 3$ at low ω and $\mu = 4$ at high ω for all ρ and φ.

Keywords : average coverage probability; average run length; cumulative count of conforming items chart; zero-inflated generalized poisson distribution.

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1 Introduction

In manufacturing processes of industries, the traditional Shewhart control chart of nonconformities (*c-Chart*) based on the Poisson distribution can be used to monitor the number of nonconformities per unit of product, when the sample sizes are constant. In a situation where there is an occurrence of a large number of zeros nonconformities in the product processes, then the *c-Chart* is unsuitable for processes control. That is the reason that the Poisson distribution is an inappropriate model to fit data. For an excess zeros nonconformities in processes, the ratio of estimated variance to the estimated mean is greater than 1 (He et al. [1]), this is called over dispersion and the *c-Chart* based on the Poisson distribution has underestimated control limits, which directly leads to a large number of false alarms rates (Xie and Goh [2], Sim and Lim [3]).

The generalized Poisson distribution (*GPD*) is one alternative to fit model apart from the *ZIP* model (Lambert [4]) for excess zeros nonconformities in processes. The *GPD* is a generalization of the discrete Poisson distribution which was developed by Consul and Jain [5]. The Poisson distribution is different from the *GPD*. While the Poisson distribution has a single parameter (λ), a *GPD* has two parameters (λ, φ), which are as follows: λ is the mean of nonconformities in a sample unit and φ is the ratio of the variance and means. He et al. [1] constructed the cumulative count of conforming items chart (*CCC-Chart*) and the attributive chart with exact probability (*ACEP*) for sensitive analysis of the two monitoring procedures together with the *GPD*. The *CCC-Chart* for low defective rate processes is used for the sensitive analysis of λ and the *ACEP* for the sensitive analysis of φ, because using a single chart to monitor the processes for λ and φ make it difficult to tell which parameter has the shift.

In this paper, we focus on another alternative of distribution to monitoring for fit data, namely a zero-inflated generalized Poisson (*ZIGP*) distribution. The *ZIGP* was introduced by Famoye and Singh [6] and authors (see Gupta et al. [7]). The characteristic function of the *ZIGP* is a further extension of the *GPD* with mixture of a distribution specifically at zeros. The *ZIGP* has three parameters λ, φ and ω, where ω is a proportion of zero nonconformity. The *ZIGP* reduces to Poisson distribution when $\varphi = 1$ and $\omega = 0$, reduces to *GPD* when $\omega = 0$ and reduces to *ZIP* when $\varphi = 1$ (Khamkong [8]). Famoye and Singh [6] and Gupta et al. [7] studied score tests for testing over-dispersion in *ZIGP* regression model. Famoye and Singh [9] study applied a *ZIGP* regression model for domestic violence data and investigated the maximum likelihood method of parameters. They found that the *ZIGP* model sufficiently fit the domestic violence data for a large number of zeros.

The result of the Poisson distribution is an inopportune model for the *c-Chart* when there are a large number of zeros nonconformities in processes. The performance of the *c-Chart* might perform well for the average run length (ARL), but not well for the average coverage probability (ACP).

In this paper, we called the *c-Chart* based on the *ZIGP* distribution the $c_G - Chart$. The aims of the present study are to develop modified control limits of a $c_G - Chart$ that executes satisfactorily for a range of parameters of the *ZIGP* and also to study the influence of the *CCC-Chart* to monitoring the excess zeros nonconformities in processes. The framework of the paper is as follows. We first develop the $c_G - Chart$ by constructing the control limits based on the expected and variances of *ZIGP* distribution which we called $c_{ZG} - Chart$. After that we develop an approximation for the distribution of the *ZIGP* as a geometric distribution with parameter p (p_g), and we examine how the value of p_g varies as the parameter of the *ZIGP* is changed. We then use the p_g estimated for calculating the expected skewness and variance of geometric distribution for modifying the control limits of $c_G - Chart$ by two difference methods, which we called $c_{Gg} - Chart$ and $c_{Gk} - Chart$. In the $c_{Gg} - Chart$, the expected and variance of geometric distribution are used in the control limits of the $c_G - Chart$. In the $c_{Gk} - Chart$, we constructed the control limits with the skewness and variance of geometric distribution. We also use the p_g estimated to replace the p for modified control limits of the *CCC-Chart*; namely *CCCg-Chart*. The performance of these control charts is then compared with the performance of the c_G-Chart and $c_{ZG} - Chart$. We compare the performance of these charts for both the ARL and the ACP.

2 Materials and Methods

The Zero-Inflated Generalized Poisson *(ZIGP)* distribution

The probability function is given by: (Famoye and Singh [6])

$$P(Y=y) = \begin{cases} \omega + (1-\omega)exp(-\lambda\varphi) & , \quad y=0 \\ (1-\omega)exp(-\frac{1}{\varphi}(\lambda + y(\varphi-1)))\frac{\lambda(\lambda+y(\varphi-1))^{y-1}}{\varphi^y y!} & , \quad y>0, \end{cases} \tag{2.1}$$

where Y = the random variables of nonconformities in a sample unit,
λ = the mean of nonconformities in a sample unit based on the *ZIGP* distribution,
φ = the ratios of variance and means of Y,
ω = is a measure of the extra proportion of zero nonconformity in a sample unit, and

$$E(Y) = (1-\omega)\lambda \text{ and } V(Y) = (1-\omega)\lambda(\varphi^2 + \lambda\omega). \tag{2.2}$$

The Geometric distribution

The probability function is given by: (Krishnamoorthy [10])

$$P(Y=k) = (1-p)^k p \quad , \quad k = 0, 1, 2, ..., \tag{2.3}$$

where Y = the random variables of the number of failures until the first success to occur,

p = the probability of success on each trial, $(p = p_g)$
$E(Y) = \frac{1-p_g}{p_g}$, $V(Y) = \frac{1-p_g}{p_g^2}$, $skewness = \frac{2-p_g}{\sqrt{1-p_g}}$ *and when the* p_g *is unknown,* however $\hat{p_g}$ is estimated as $\hat{p_g} = (1 + \frac{1}{n}\sum_{i=1}^{n} k_i)^{-1}$. (2.4)

The Shewhart control chart of nonconformities (*c-Chart*)

The shewhart control chart based on Poisson distribution (*c-Chart*) is used to monitoring the number of nonconformities in processes. The control limits are given by: (Montgomery [11])

$$\begin{aligned} UCL &= c + 3\sqrt{c} \\ CL &= c \\ LCL &= c - 3\sqrt{c}, \end{aligned} \tag{2.5}$$

where c = is assumed to be the mean number of nonconformities if the mean of the probability distribution is known, otherwise c is estimated as the mean of the number of nonconformities in a sample of observed product units ($\bar{c}$).

The cumulative count of conforming items chart (*CCC-Chart*)

The *CCC-Chart* is constructed by the cumulative distribution function of k, where k follows the geometric distribution with parameter p . The control limits are given by: (Goh [12])

$$\begin{aligned} UCL &= \frac{ln(\alpha/2)}{ln(1-p)} - 1 \\ LCL &= \frac{ln(1-\alpha/2)}{ln(1-p)} - 1, \end{aligned} \tag{2.6}$$

where p = the probability of success on each trial, and
α = the probability of false alarm, meaning the processes is an out of control when in fact the process is still in control.

Development of the control charts of nonconformities based on *ZIGP* distribution

1. The $c_G - Chart$ is a modified control limits of a one-sided of *c-Chart* for the number of nonconformities based on *ZIGP* distribution that we called $c_G - Chart$. The control limits are given by:

$$\begin{aligned} UCL &= c_G + 3\sqrt{c_G} \\ LCL &= 0, \end{aligned} \tag{2.7}$$

where c_G = is the population mean of the number of nonconformities based on *ZIGP* distribution if c_G unknown, c_G is estimated as the sample mean of nonconformities in a sample of observed product units ($\bar{c}_G$).

2. The $c_{ZG} - Chart$ is a modified control limits of $c_G - Chart$ obtained by using the expected and variance of *ZIGP* distribution that we called $c_{ZG} - Chart$,

the control limits are given by:

$$UCL = E(Y) + 3\sqrt{V(Y)}$$
$$LCL = 0, \tag{2.8}$$

where $E(Y) = (1-\hat{\omega})\bar{c}_G$, $V(Y) = (1-\hat{\omega})\bar{c}_G(\hat{\varphi}^2 + \bar{c}_G\hat{\omega})$ and (2.9)
$\bar{c}_G$ = the sample mean of the number of nonconformities in a sample units,
$\hat{\omega}$= the mean of the number zeros of nonconformities in a sample units,
$\hat{\varphi}$= the ratios of the variance and mean in a sample units.

For a given *ZIGP* distribution and Y are geometric distribution, we first obtain an approximate geometric distribution with parameter p (p_g) by using the Kolmogorov-Smirnov test [13]. The charts for a *ZIGP* distribution are then defined as follows.

3. The $c_{Gg} - Chart$ is a modified control limits of the $c_G - Chart$ obtained by using the $\hat{p}_g$ of the geometric distribution that fit for *ZIGP* distribution for calculated the expected and variance of geometric distribution for constructed the control limits of a one-sided $c_G - Chart$. Therefore the control limit of $c_{Gg} - Chart$ is given by:

$$UCL = E(Y) + 3\sqrt{V(Y)}$$
$$LCL = 0, \tag{2.10}$$

where $E(Y), V(Y)$ and $\hat{p}_g$ caculated from (2.4).

4. The $c_{Gk} - Chart$ is a modified control limits of the $c_G - Chart$ obtained by using the $\hat{p}_g$ of the geometric distribution that fit for *ZIGP* distribution for calculated the skewness (K) and variance, after that replacing the sample mean of nonconformities in control limits with K and variance of geometric distribution for constructed the control limits of a one-sided $c_G - Chart$. Therefore the control limit of $c_{Gk} - Chart$ is given by:

$$UCL = K + 3\sqrt{V(Y)}$$
$$LCL = 0, \tag{2.11}$$

where $K, V(Y)$ and $\hat{p}_g$ caculated from (2.4).

5. The $CCC_g - Chart$ is a developed control limits of the *CCC-Chart* obtained by replacing p value with $\hat{p}_g$ of the geometric distribution that fit for *ZIGP*. Therefore the control limit of $CCC_g - Chart$ is given by:

$$UCL = \frac{ln(\alpha/2)}{ln(1-\hat{p}_g)} - 1$$
$$LCL = \frac{ln(1-\alpha/2)}{ln(1-\hat{p}_g)} - 1, \tag{2.12}$$

where $\hat{p}_g$ caculated from (2.4).

3 Simulation Results

In this section we have shown the results of tests of the charts from a simulation study. For the simulations, we assume the following ranges of parameter values. The means for the in-control process are: $(\mu_0) = 1.0(1.0)4.0$. The means for the out-of-control process are: $(\mu_1 = \mu_0 + \rho)$ where the mean shifts are: $(\rho) = 0.00$, 0.40, 0.80 and 1.20. The proportions of zero nonconformity are: $(\omega) = 0.30(0.10)0.90$. Finally, the value for the over-dispersion $(\varphi) = 1.2$ and 1.4.

The evaluation of the performance of the control charts was conducted as follows:

1. The R program was used to simulate the number of nonconforming items for a *ZIGP* with values for the parameters $(n, \mu_0, \varphi, \omega)$ chosen from the set of values given above.

2. The value of the parameter p_g which gives a best fit between the *ZIGP* from step 1 and geometric distribution.

3. The Kolmogorov-Smirnov test was used to test the hypothesis that a geometric distribution with the p_g value from step 2 could give a reasonable fit to the distribution of data obtained in step 1. Based on simulations with 20,000 replications, the results of the test showed that the hypothesis was satisfied for at least 95% of the replications. For the p_g fit values with a *ZIGP*, we used the number of failures until the first success to occur of a geometric distribution for calculating the $\hat{p}_g$ from (2.4).

4. The values of the average $\hat{p}_g$ in step 3 based on 100,000 replications were then used for calculating the expected, skewness and variance of geometric distribution for constructing the control limits of the $c_{Gg} - Chart$, $c_{Gk} - Chart$ and for calculating the control limits of the $CCCg - Chart$.

5. The number of nonconforming of *ZIGP* in step 1 based on 100,000 replications were then used for calculating the $\bar{c}_G$, expected and variance of *ZIGP* distribution for constructing the control limits of the $c_G - Chart$ and $c_{ZG} - Chart$.

6. Based on a new set of 100,000 replications, the control limits calculated in steps 4 and 5 were then used to compute the average run length (ARL) and the average coverage probability *(ACP)* for each chart.

7. Steps 1 to 6 were then repeated for a new set of values for parameters $(n, \mu_0, \varphi, \omega)$.

4 Results

In this section a summary is given of some of the results that were obtained from the simulations.

Table 1 shows the values of $\hat{p}_g$ for the geometric distribution that gives the best fit between the geometric and the *ZIGP* distribution for the range of ω, φ and μ values. It can be seen that as the values of $\mu = 1.0$, the values of $\hat{p}_g$ for all of ω, φ are a constant value (0.53) and when the values of $\mu = 2.0 - 4.0$, as the values of ω are increased, the values of $\hat{p}_g$ vary depending on the φ and μ values.

The process in the in-control state ($\rho = 0.00$)

The results for the in-control case are shown in table 2. Table 2 shows a comparison of the average run length (ARL_0) and the average coverage probability (ACP) values for the $c_G - Chart(c_G), c_{ZG} - Chart(c_{ZG}), c_{Gk} - Chart(c_{Gk}), c_{Gg} - Chart(c_{Gg})$ and $CCCg - Chart(CCCg)$. Also a comparison of ARL_0 values for the charts is given in Fig. 1. It can be seen that for all levels of μ_0, ω and φ, the $CCCg - Chart$ returns the highest ARL_0 values. Therefore, for ARL_0 criteria the $CCCg - Chart$ is accepted as the preferred control chart.

Fig. 2 shows the absolute values of the differences between the ACP values and the confidence level of 0.9973, which we call the *ACP-DIFF* value. It can be seen that for levels of $\mu_0 = 1.0$, at all levels of ω and φ, the $c_{Gk} - Chart$, $c_{Gg} - Chart$ and $CCCg - Chart$ have similar low *ACP-DIFF* values. That is, these charts give ACP values close to the target level of 0.9973. However, when $\mu_0 = 2.0 - 4.0$, only the $CCCg - Chart$ gives ACP values close to the target.

When both ARL_0 and ACP values are considered, the $CCCg - Chart$ will be the preferred control chart for all levels of μ_0, ω and φ.

The process in an out-of-control state ($\rho > 0.00$)

Results for this case are shown in Figs. 3 and 4. Fig. 3 gives a comparison of ARL_1 values for a range of values of $\mu_1 = \mu_0 + \rho$, φ and ω. It can be seen that when $\mu_0 = 1.0$, $\rho = 0.4$ and $\varphi = 1.2$ for $\omega = 0.3 - 0.6$, the $c_G - Chart$, $c_{ZG} - Chart$ and $c_{Gg} - Chart$ return similar low values of ARL_1 (as $\rho = 0.8, 1.2$ and $\varphi = 1.4$ return similar results). That is, these three charts are able to detect shifts faster than the other charts. However, only the $c_G - Chart$ and $c_{ZG} - Chart$ return low values of ARL_1 for $\omega = 0.7$-0.9. When $\mu_0 = 2.0$, $\rho = 0.4$ and $\varphi = 1.4$, the $c_G - Chart$, $c_{ZG} - Chart$, $c_{Gk} - Chart$ and $c_{Gg} - Chart$ return similar low values of ARL_1 for all levels of ω with the same results as $\rho = 0.8, 1.2$ and $\varphi = 1.2$. When $\mu_0 = 3.0$,$\rho = 1.2$ and $\varphi = 1.2$, the $c_G - Chart$, $c_{ZG} - Chart$, $c_{Gk} - Chart$ and $c_{Gg} - Chart$ return similar low values of ARL_1 for all levels of ω (similar results as $\rho = 0.4, 0.8$ and $\varphi = 1.4$). When $\mu_0 = 4.0$, $\rho = 1.2$ and $\varphi = 1.4$ for all levels of ω, the $c_G - Chart$, $c_{Gk} - Chart$ and $c_{Gg} - Chart$ give the lowest values of ARL_1 (the same results as the other values of ρ and φ). However, for $\omega = 0.7 - 0.9$, the $c_{ZG} - Chart$ gives the ARL_1 values close to these three charts. Fig. 3 also shows that all control charts can detect a shift more decrease for values of ω increase from 0.8-0.9.

Fig. 4 gives a comparison of the *ACP-DIFF* values only for the preferred charts that considered the ARL_1 values. It show that for $\mu_0 = 1.0$, $\rho = 0.4$ and $\varphi = 1.2$, the $c_{Gk} - Chart$ and $c_{Gg} - Chart$ returns the lowest *ACP-DIFF* values for all values of ω with the same results as $\rho = 0.8, 1.2$ and $\varphi = 1.4$. When $\mu_0 = 2.0$, $\rho = 0.4$ and $\varphi = 1.4$, the $c_{Gk} - Chart$ returns the lowest *ACP-DIFF* values for all levels of ω (similar results as $\rho = 0.8, 1.2$ and $\varphi = 1.2$). For $\mu_0 = 3.0$, $\rho = 1.2$ and $\varphi = 1.2$ when $\omega = 0.3 - 0.5$, the $c_{ZG} - Chart$ gives the minimum *ACP-DIFF* values (the same results as other values of ρ and φ). However, for $\omega = 0.6 - 0.9$, the $c_{Gk} - Chart$ returns the lowest *ACP-DIFF* value. When $\mu_0 = 4.0$, $\rho = 1.2$ and $\varphi = 1.4$ for $\omega = 0.3 - 0.6$, the $c_{ZG} - Chart$ returns the lowest *ACP-DIFF* values (as $\rho = 0.4, 0.8$ and $\varphi = 1.2$ return similar results). For $\omega = 0.7, 0.8$, the $c_{Gk} - Chart$ and $c_{ZG} - Chart$ return the lowest *ACP-DIFF* values. However, for $\omega = 0.9$, only the $c_{Gk} - Chart$ gives the minimum values of *ACP-DIFF*.

When both ARL_1 and ACP values are considered, in the case of $\mu_0 = 1.0$, for all levels of ρ and φ when $\omega = 0.3 - 0.6$, the $c_{Gg} - Chart$ will be the preferred control chart. However, no control charts are preferred for $\omega = 0.7 - 0.9$. When $\mu_0 = 2.0$, for all levels of ρ, φ and ω, the $c_{Gk} - Chart$ will be the preferred control chart. When $\mu_0 = 3.0$, for all levels of ρ and φ when $\omega = 0.3 - 0.5$, the $c_{ZG} - Chart$ will be the preferred control chart. However, for $\omega = 0.6 - 0.9$, the $c_{Gk} - Chart$ will be the superior control chart. At $\mu_0 = 4.0$, for all levels of ρ and φ when $\omega = 0.3 - 0.6$, there is no suitable control chart, but for $\omega = 0.7 - 0.8$, the $c_{Gk} - Chart$ and $c_{ZG} - Chart$ are the optimal charts. However, for $\omega = 0.9$, only the $c_{Gk} - Chart$ is the superior chart.

Table 1: The $\hat{p}_g$ values for the geometric that give the best fit to the distribution of the *ZIGP* model for a range of ω, φ and μ.

ω	φ	μ			
		1.0	2.0	3.0	4.0
0.30	1.2	0.53	0.48	0.38	0.36
	1.4	0.53	0.40	0.34	0.30
0.40	1.2	0.53	0.50	0.40	0.38
	1.4	0.53	0.44	0.38	0.32
0.50	1.2	0.53	0.53	0.44	0.40
	1.4	0.53	0.46	0.40	0.34
0.60	1.2	0.53	0.53	0.46	0.44
	1.4	0.53	0.49	0.44	0.36
0.70	1.2	0.53	0.53	0.50	0.48
	1.4	0.53	0.53	0.46	0.38
0.80	1.2	0.53	0.53	0.52	0.51
	1.4	0.53	0.53	0.51	0.52
0.90	1.2	0.53	0.53	0.52	0.51
	1.4	0.53	0.53	0.51	0.52

Table 2: Comparison of ARL_0 and ACP values of the $c_G - Chart$, $c_{ZG} - Chart$, $c_{Gk} - Chart$, $c_{Gg} - Chart$ and $CCCg - Chart$ for a range of μ_0, ω and φ values.

μ_0	ω	φ	ARL_0					ACP				
			c_G	c_{ZG}	c_{Gk}	c_{Gg}	$CCCg$	c_G	c_{ZG}	c_{Gk}	c_{Gg}	$CCCg$
	0.30	1.2	15.8	78.3	79.1	34.6	633.2	0.9405	0.9902	0.9874	0.9727	0.9989
		1.4	12.5	74.0	72.4	40.3	558.9	0.9257	0.9878	0.9864	0.9764	0.9987
	0.40	1.2	18.9	40.9	91.4	18.8	424.8	0.9501	0.9807	0.9895	0.9513	0.9982
		1.4	14.8	47.4	48.5	48.1	431.5	0.9364	0.9781	0.9791	0.9802	0.9979
	0.5	1.2	10.7	49.1	49.3	22.8	248.0	0.9194	0.9691	0.9792	0.9576	0.9958
		1.4	9.9	32.2	57.3	31.9	316.1	0.9108	0.9661	0.9841	0.9685	0.9972
2.0	0.60	1.2	13.7	28.5	62.2	28.3	301.4	0.9352	0.9523	0.9836	0.9671	0.9966
		1.4	12.5	22.4	72.1	40.1	229.5	0.9272	0.9509	0.9869	0.9778	0.9951
	0.70	1.2	9.4	18.8	82.4	38.5	381.9	0.9058	0.9391	0.9875	0.9741	0.9976
		1.4	9.7	31.0	54.2	30.3	172.4	0.9073	0.9414	0.9827	0.9693	0.9954
	0.80	1.2	14.6	14.5	123.4	58.6	500.4	0.9353	0.9386	0.9921	0.9832	0.9985
		1.4	18.8	26.4	81.7	46.0	255.0	0.9338	0.9427	0.9881	0.9782	0.9963
	0.90	1.2	17.5	30.1	246.7	118.0	693.9	0.9452	0.9538	0.9959	0.9915	0.9992
		1.4	18.9	31.2	163.4	93.5	444.1	0.9497	0.9568	0.9935	0.9890	0.9979

μ_0	ω	φ	ARL_0					ACP				
			c_G	c_{ZG}	c_{Gk}	c_{Gg}	$CCCg$	c_G	c_{ZG}	c_{Gk}	c_{Gg}	$CCCg$
	0.30	1.2	15.8	391.2	29.5	15.9	604.7	0.9421	0.9975	0.9666	0.9396	0.9990
		1.4	12.0	344.9	30.7	49.3	758.8	0.9199	0.9974	0.9690	0.9805	0.9996
	0.40	1.2	10.5	246.4	18.7	18.8	434.3	0.9149	0.9968	0.9485	0.9502	0.9982
		1.4	9.0	253.9	36.2	35.7	676.8	0.8998	0.9965	0.9724	0.9717	0.9990
	0.50	1.2	7.4	151.3	22.6	12.8	293.5	0.8955	0.9937	0.9573	0.9266	0.9962
		1.4	7.0	113.5	27.4	27.1	438.3	0.8903	0.9929	0.9647	0.9648	0.9980
4.0	0.60	1.2	9.5	98.4	16.1	16.3	189.4	0.8931	0.9847	0.9413	0.9426	0.9942
		1.4	9.0	88.4	34.1	21.4	358.7	0.8905	0.9929	0.9711	0.9552	0.9980
	0.70	1.2	8.0	38.3	22.1	12.9	132.5	0.8887	0.9672	0.9571	0.9281	0.9929
		1.4	8.3	45.9	29.0	29.4	303.0	0.8901	0.9814	0.9672	0.9662	0.9970
	0.80	1.2	8.3	19.9	20.0	12.5	105.7	0.8913	0.9499	0.9524	0.9272	0.9905
		1.4	8.9	29.0	18.7	12.8	100.6	0.8972	0.9691	0.9504	0.9278	0.9873
	0.90	1.2	13.2	26.2	40.9	26.5	210.4	0.9293	0.9530	0.9769	0.9631	0.9953
		1.4	14.1	26.5	39.1	26.6	140.2	0.9316	0.9515	0.9758	0.9633	0.9927

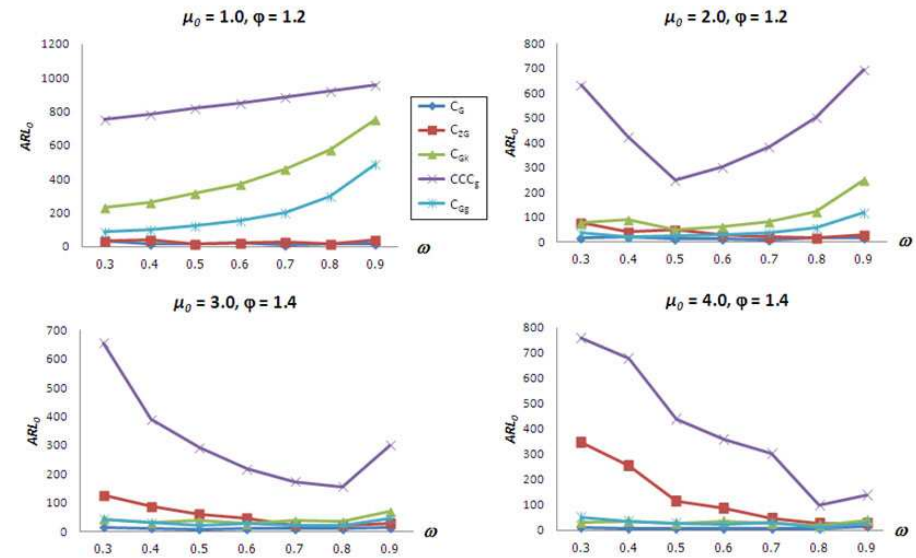

Figure 1: Comparison of ARL_0 values of the Charts for a range of μ_0, φ and ω values in the case of in-control state.

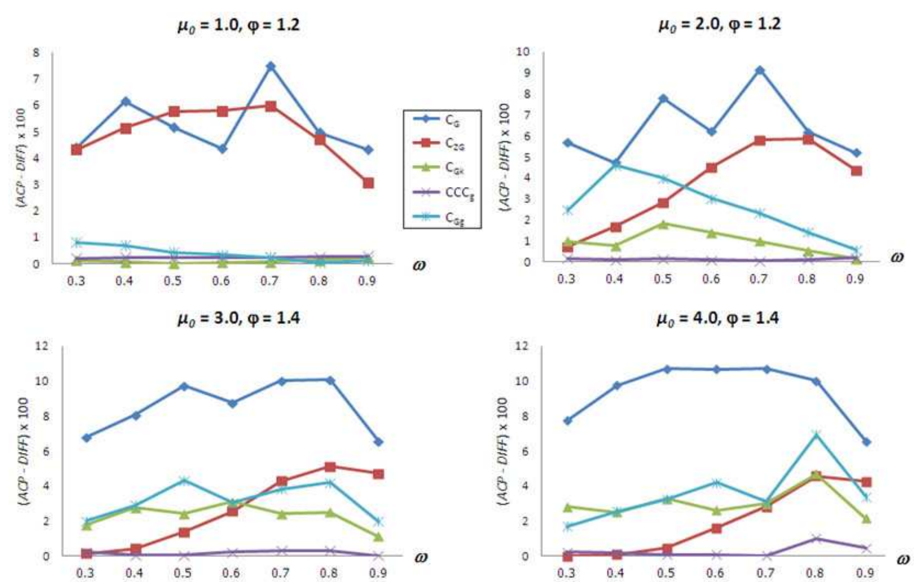

Figure 2: Comparison of $ACP\text{-}DIFF$ values of the Charts for a range of μ_0, φ and ω values in the case of in-control state.

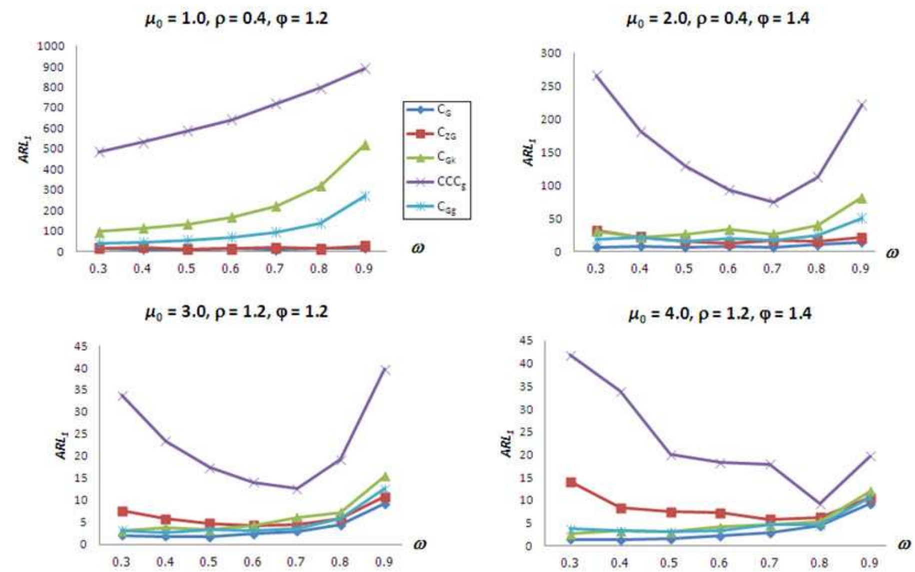

Figure 3: Comparison of ARL_1 values of the Charts for a range of $\mu_1 = \mu_0 + \rho$, φ and ω values in the case of out-of-control state.

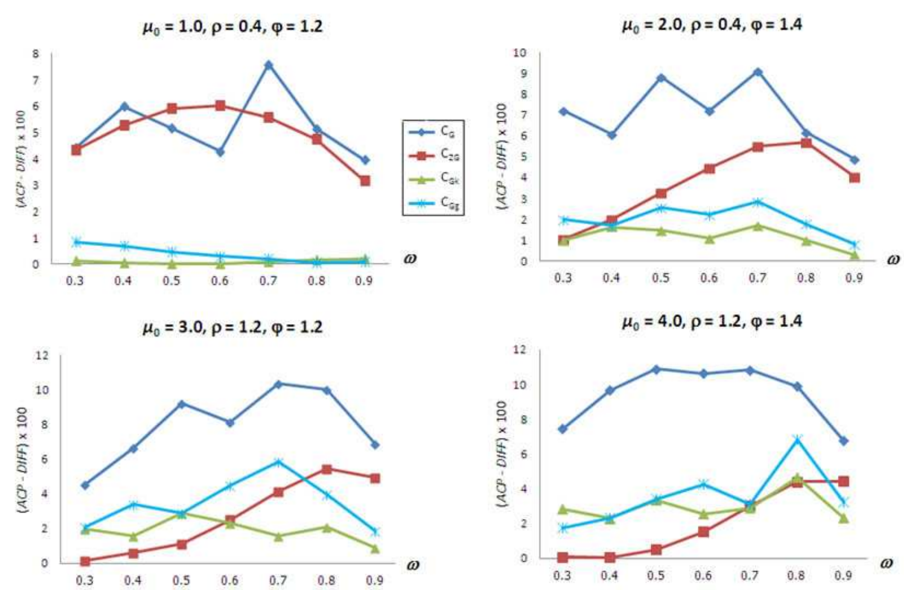

Figure 4: Comparison of *ACP-DIFF* values of the Charts for a range of $\mu_1 = \mu_0 + \rho$, φ and ω values in the case of out-of-control state.

Table 3: Summary of preferred control charts.

Mean of process (μ_0/μ_1)	Proportion of zero (ω)	Mean shift of process	Preferred control charts: For ARL values	For ACP values	For both ARL and ACP values
1.0	0.3 - 0.9	In-control (all levels of φ)	$CCCg-Chart$	$c_{Gk}-Chart$,$c_{Gg}-Chart$,$CCCg-Chart$	$CCCg-Chart$
2.0 - 4.0	0.3 - 0.9		$CCCg-Chart$	$CCCg-Chart$	$CCCg-Chart$
1.0	0.3 - 0.6	Out-of-control(all levels of ρ and φ)	$c_G-Chart$,$c_{ZG}-Chart$, $c_{Gg}-Chart$	$c_{Gk}-Chart$,$c_{Gg}-Chart$	$c_{Gg}-Chart$
	0.7 - 0.9		$c_G-Chart$, $c_{ZG}-Chart$	$c_{Gk}-Chart$,$c_{Gg}-Chart$	-
2.0	0.3 - 0.9		$c_G-Chart$,$c_{ZG}-Chart$ $c_{Gk}-Chart$,$c_{Gg}-Chart$	$c_{Gk}-Chart$	$c_{Gk}-Chart$
3.0	0.3 - 0.5		$c_G-Chart$,$c_{ZG}-Chart$ $c_{Gk}-Chart$,$c_{Gg}-Chart$	$c_{ZG}-Chart$	$c_{ZG}-Chart$
	0.6 - 0.9		$c_G-Chart$,$c_{ZG}-Chart$ $c_{Gk}-Chart$,$c_{Gg}-Chart$	$c_{Gk}-Chart$	$c_{Gk}-Chart$
4.0	0.3 - 0.6		$c_G-Chart$,$c_{Gk}-Chart$, $c_{Gg}-Chart$	$c_{ZG}-Chart$	-
	0.7 - 0.8		$c_G-Chart$,$c_{ZG}-Chart$ $c_{Gk}-Chart$, $c_{Gg}-Chart$	$c_{Gk}-Chart$,$c_{ZG}-Chart$	$c_{Gk}-Chart$, $c_{ZG}-Chart$
	0.9		$c_G-Chart$,$c_{ZG}-Chart$ $c_{Gk}-Chart$,$c_{Gg}-Chart$	$c_{Gk}-Chart$	$c_{Gk}-Chart$

5 Conclusion

The purpose of this paper is to develop the *c-Chart* based on a *ZIGP* processes. We called the control limits of *c-Chart* based on a *ZIGP* distribution the $c_G-Chart$. In developing these charts, we first develop the control limits of $c_G-Chart$ by using the expected and variance of *ZIGP* distribution for constructing the control limits namely $c_{ZG}-Chart$. We then develop the number of nonconformities based on a geometric distribution with parameter p_g, where p_g estimated gives the best fit between the geometric and *ZIGP* distributions used in calculating the expected, skewness and variance of geometric distribution for modifying the $c_G-Chart$ and moreover the $CCC-Chart$ is used in monitoring the *ZIGP* processes as well. The three different methods for the charts are called the $c_{Gk}-Chart$, $c_{Gg}-Chart$ and $CCCg-Chart$. In the $c_{Gg}-Chart$, the control limits of the chart are constructed by expected and variance of geometric distribution to be used in the control limits of the $c_G-Chart$. In the $c_{Gk}-Chart$, the skewness and variance of geometric distribution are calculated for constructing the control limits of the $c_G-Chart$. In the $CCCg-Chart$, we replace the p with p_g estimated for use in the control limits of $CCC-Chart$.

Furthermore, simulations have been carried out to compare the performances of the three control charts with the performances of $c_G-Chart$ and $c_{ZG}-Chart$. We compared these charts by using the average run length (ARL) and average coverage probability (ACP). The results of the comparisons are summarized in table 3 which gives a list of preferred control charts for both in-control and out-of control states for a range of values of *ZIGP* parameters.

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