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# Comparison of the Rate of Convergence of Various Iterative Methods for the Class of Weak Contractions in Banach Spaces

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**Abstract :** In this article, we propose a new iterative method for finding fixed points of a class of weak contractions in arbitrary Banach spaces. Comparison of the rate of convergence between Mann, Ishikawa, Noor iterations, and the proposed iterative method are also discussed. It is shown that the proposed iterative method converges faster than the others. Moreover, we give a numerical example for comparing the rate of convergence of those methods.

**Keywords :** fixed point; rate of convergence; weak contractions; Banach spaces.
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## 1 Introduction

Let $C$ be a nonempty convex subset of a Banach space $X$, and $T : C \to C$ be a mapping. A point $x \in C$ is a *fixed point* of $T$ if $Tx = x$. The set of all fixed points of $T$ is denoted by $F(T)$. The *Mann iteration* is defined by $u_1 \in C$ and

$$u_{n+1} = (1 - \alpha_n)u_n + \alpha_n T u_n, \text{ for all } n \in \mathbb{N}, \tag{1.1}$$

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where $\{\alpha_n\}$ is a sequence in $[0,1]$. For $\alpha_n = \lambda$ (constant), the iteration (1.1) reduces to the so-called *Krasnoselskij iteration*. The *Ishikawa iteration* is defined by $s_1 \in C$ and

$$\begin{cases} r_n = (1-\beta_n)s_n + \beta_n T s_n, \\ s_{n+1} = (1-\alpha_n)s_n + \alpha_n T r_n, \text{ for all } n \in \mathbb{N}, \end{cases} \tag{1.2}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in $[0,1]$. The *Noor iteration* is defined by $w_1 \in C$ and

$$\begin{cases} h_n = (1-\gamma_n)w_n + \gamma_n T w_n, \\ q_n = (1-\beta_n)w_n + \beta_n T h_n, \\ w_{n+1} = (1-\alpha_n)w_n + \alpha_n T q_n, \text{ for all } n \in \mathbb{N}, \end{cases} \tag{1.3}$$

where $\{\alpha_n\}$, $\{\beta_n\}$, and $\{\gamma_n\}$ are sequences in $[0,1]$. Clearly, Mann and Ishikawa iterations are special cases of Noor iteration.

There are many papers have been concentrated on the iteration methods for approximation of fixed points of nonlinear mappings, see for instance [1, 2, 3, 4]. However, there are only a few paper concerning comparison of those iteration methods in order to establish which one converges faster. In 2004, Berinde [5] provided the following concept to compare the rate of convergence of the iteration methods.

**Definition 1.1.** Let $\{a_n\}$ and $\{b_n\}$ be two sequences of positive numbers that converge to $a$, $b$, respectively. Assume there exists

$$l = \lim_{n\to\infty} \frac{|a_n - a|}{|b_n - b|}.$$

If $l = 0$, then it is said that the sequence $\{a_n\}$ converges to *a faster* than the sequence $\{b_n\}$ to $b$. If $0 < l < \infty$, then we say that the sequence $\{a_n\}$ and $\{b_n\}$ *have that same rate of convergence*.

Suppose that for two fixed point iterations $\{x_n\}$ and $\{y_n\}$ converging to the same fixed point $z$ of a mapping $T$, the error estimates $\|x_n - z\| \le a_n$ and $\|y_n - z\| \le b_n$, $n \in \mathbb{N}$, are available, where $\{a_n\}$ and $\{b_n\}$ are two sequences of positive real numbers (converging to zero). Then, in view of Definition 1.1, the following concept appears to be very natural.

**Definition 1.2.** If $\{a_n\}$ converges faster than $\{b_n\}$, then we say that the fixed point iteration $\{x_n\}$ *converges faster* than the fixed point iteration $\{y_n\}$ to $z$.

We observe that the comparison of the rate of convergence in above definition depends on the choice of sequences $\{a_n\}$ and $\{b_n\}$ which are error bounds of $\{x_n\}$ and $\{y_n\}$, respectively. It seem not to be clear if we use above definition for comparing the rate of convergence. So, we modify above definition for comparing the rate of convergence as follows:

**Definition 1.3.** Suppose that $\{x_n\}$ and $\{y_n\}$ are two iteration methods converging to the same fixed point $z$ of a mapping $T$. We say that $\{x_n\}$ *converges faster* than $\{y_n\}$ to $z$ if

$$\lim_{n\to\infty}\frac{\|x_n - z\|}{\|y_n - z\|} = 0.$$

Berinde [5] compared the rate of convergence of the Picard and Mann iterations for a class of Zamfirescu operators in arbitrary Banach spaces. In 2006, Babu and Prasad [6] showed that the Mann iteration converges faster than the Ishikawa iteration for this class of operators. Two year later, Qing and Rhoades [7] provided an example to show that the claim of Babu and Prasad [6] is false. Later, Xue [8] compared convergence speed of the Picard, Krasnoselskij, Mann and Ishikawa iterations for the class of Zamfirescu operators. He proved that the Picard iteration converges faster than the Mann iteration and the Krasnoselskij iteration converges faster than the Ishikawa iteration under some appropriate conditions. In 2010, Rhoades and Xue [9] provided sufficient conditions for Picard iteration to converge faster than Krasnoselskij, Mann, Ishikawa, or Noor iteration for quasi-contractive operators, and also compared the rate of convergence between Krasnoselskij and Mann iterations for Zamfirescu operators.

The following class of operators is more general than that of Zamfirescu operators.

**Definition 1.4.** Let $C$ be a nonempty subset of a Banach space $X$. A mapping $T : C \to C$ is called *weak contraction* if there exist a constant $\delta \in (0, 1)$ and some $L \geq 0$ such that

$$\|Tx - Ty\| \leq \delta\|x - y\| + L\|y - Tx\| \text{ for all } x, y \in C.$$

This mapping is also partially cover the quasi-contraction operator. Many researcher studied the existence and convergence theorems for finding fixed points of the class of weak contractions, Zamfirescu operators, quasi-contraction operators, see for instance [5, 6, 8, 9, 10, 11]. The following weak contraction theorem obtained by Berinde in 2004.

**Proposition 1.5** ([11])**.** *Let $C$ be a nonempty closed convex subset of a Banach space $X$ and $T : C \to C$ be a weak contraction. Then $F(T) \neq \emptyset$. Moreover, the Picard iteration $\{x_n\}$ defined by $x_{n+1} = Tx_n$ for all $n \in \mathbb{N}$, converges to a fixed point of $T$.*

In this article, we propose a new iterative method as follows:

$$\begin{cases} z_n = (1-\gamma_n)x_n + \gamma_n T x_n, \\ y_n = (1-\beta_n)z_n + \beta_n T z_n, \\ x_{n+1} = (1-\alpha_n-\lambda_n)y_n + \alpha_n T y_n + \lambda_n T z_n, \text{ for all } n \in \mathbb{N}, \end{cases} \tag{1.4}$$

where $x_1 \in C$, $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\lambda_n\}$, and $\{\alpha_n + \lambda_n\}$ are sequences in $[0, 1]$. Then, we prove the strong convergence of the proposed iterative method to a fixed point of a weak contraction, and also compare the rate of convergence of this iterative method with Mann, Ishikawa and Noor iterations.

## 2 Main Results

Firstly, we prove the following convergence theorems to a fixed point of a weak contraction in Banach spaces.

**Theorem 2.1.** *Let $C$ be a nonempty closed convex subset of a Banach space $X$ and $T : C \to C$ be a weak contraction with the following condition: there exist a constant $\delta' \in (0, 1)$ and some $L' \geq 0$ such that*

$$\|Tx - Ty\| \leq \delta'\|x - y\| + L'\|y - Ty\| \text{ for all } x, y \in C. \tag{2.1}$$

*Suppose the sequence $\{x_n\}$ generated by (1.4) and $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\lambda_n\}$, $\{\alpha_n + \lambda_n\}$ are sequences in $[0, 1]$ which satisfy one of the following conditions:*

*(i)* $\sum_{n=1}^{\infty} \alpha_n = \infty$; *(ii)* $\sum_{n=1}^{\infty} \beta_n = \infty$; *(iii)* $\sum_{n=1}^{\infty} \gamma_n = \infty$.

*Then $\{x_n\}$ converges strongly to a unique fixed point of $T$.*

*Proof.* Proposition 1.5 guarantees that $T$ has a fixed point. Obviously, by condition (2.1), the fixed point of $T$ is unique, say $p$. By condition (2.1) again, we have $\|Tx_n - p\| \leq \delta'\|x_n - p\|$, $\|Ty_n - p\| \leq \delta'\|y_n - p\|$ and $\|Tz_n - p\| \leq \delta'\|z_n - p\|$. Thus, it follows by (1.4) that

$$\begin{aligned}
\|x_{n+1} - p\| &\leq (1 - \alpha_n - \lambda_n)\|y_n - p\| + \alpha_n\|Ty_n - p\| + \lambda_n\|Tz_n - p\| \\
&\leq (1 - \alpha_n(1 - \delta') - \lambda_n)\|y_n - p\| + \lambda_n\delta'\|z_n - p\| \\
&\leq ((1 - \alpha_n(1 - \delta') - \lambda_n)(1 - \beta_n(1 - \delta')) + \lambda_n\delta')\|z_n - p\| \\
&\leq ((1 - \alpha_n(1 - \delta') - \lambda_n)(1 - \beta_n(1 - \delta')) + \lambda_n\delta')(1 - \gamma_n(1 - \delta')))\|x_n - p\| \\
&= (((1 - \alpha_n(1 - \delta'))(1 - \beta_n(1 - \delta')) \\
&\qquad - \lambda_n(1 - \beta_n)(1 - \delta')))(1 - \gamma_n(1 - \delta'))\|x_n - p\| \\
&\leq ((1 - \alpha_n(1 - \delta'))(1 - \beta_n(1 - \delta'))(1 - \gamma_n(1 - \delta'))\|x_n - p\| \\
&\;\;\vdots \\
&\leq \prod_{k=1}^{n}((1 - \alpha_k(1 - \delta'))(1 - \beta_k(1 - \delta'))(1 - \gamma_k(1 - \delta'))\|x_1 - p\|.
\end{aligned} \tag{2.2}$$

By the assumption, we can conclude that $\{x_n\}$ converges to $p$. □

**Theorem 2.2.** *Assume $X$, $C$, $T$ are as in Theorem 2.1 and $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$ are sequences in $[0, 1]$ such that $\sum_{n=1}^{\infty} \alpha_n = \infty$. Suppose $\{u_n\}$, $\{s_n\}$, $\{w_n\}$ are sequences generated by Mann iteration (1.1), Ishikawa iteration (1.2), and Noor iteration (1.3), respectively. Then $\{u_n\}$, $\{s_n\}$, $\{w_n\}$ converge strongly to a unique fixed point of $T$.*

*Proof.* By Proposition 1.5 and condition (2.1), $T$ has a unique fixed point, say $p$. Using (1.3) and condition (2.1), we have

$$\begin{aligned}
\|w_{n+1}-p\| &\le (1-\alpha_n)\|w_n-p\| + \alpha_n\delta'\|q_n-p\| \\
&\le (1-\alpha_n+\alpha_n\delta'(1-\beta_n))\|w_n-p\| + \alpha_n\beta_n\delta'^2\|h_n-p\| \\
&\le (1-\alpha_n(1-\delta'(1-\beta_n)-\beta_n\delta'^2(1-\gamma_n(1-\delta'))))\|w_n-p\| \\
&= (1-\alpha_n(1-\delta'+\beta_n\delta'(1-\delta'(1-\gamma_n(1-\delta')))))\|w_n-p\| \qquad (2.3)\\
&\le (1-\alpha_n(1-\delta'))\|w_n-p\| \\
&\ \ \vdots \\
&\le \prod_{k=1}^{n}(1-\alpha_k(1-\delta'))\|w_1-p\|.
\end{aligned}$$

This implies by $\sum_{n=1}^{\infty}\alpha_n=\infty$ that $\{w_n\}$ converges to $p$.

Since Mann and Ishikawa iterations are special cases of Noor iteration, we can conclude that $\{s_n\}$ and $\{u_n\}$ converge to a unique fixed point of $T$. □

**Remark 2.3.** *It is known that only weak contraction does not guarantee the uniqueness of fixed point of $T$. But if $T$ also satisfies the condition (2.1), its fixed point must be unique.*

The following results, we consider the rate of convergence of the proposed iterative method (1.4) and the well-known iterative methods.

**Theorem 2.4.** *Let $C$ be a nonempty closed convex subset of a Banach space $X$ and $T : C \to C$ be a weak contraction with condition (2.1). Suppose $\{u_n\}$, $\{s_n\}$, $\{w_n\}$ are sequences generated by Mann iteration (1.1), Ishikawa iteration (1.2) and Noor iteration (1.3), respectively, and $\{\alpha_n\}$, $\{\beta_n\}$, $\{\gamma_n\}$, $\{\lambda_n\}$, $\{\alpha_n+\lambda_n\}$ are sequences in $[0,1]$ which satisfy the conditions:*

*(C1)* $0<\eta\le\beta_n<1$ *or* $0<\eta\le\gamma_n<1$;

*(C2)* $0<\alpha_n<\frac{1}{1+\delta'}$, $\sum_{n=1}^{\infty}\alpha_n=\infty$ *and* $\lim_{n\to\infty}\alpha_n=0$.

*Then the sequence $\{x_n\}$ generated by (1.4) converges faster than Mann, Ishikawa and Noor iterations to a unique fixed point of $T$, provided that $x_1=u_1=s_1=w_1\in C$.*

*Proof.* By Theorems 2.1 and 2.2, the sequence $\{x_n\}$ generated by (1.4), Mann iteration $\{u_n\}$, Ishikawa iteartion $\{s_n\}$ and Noor iteration $\{w_n\}$ converge to a unique fixed point of $T$, say $p$. Firstly, by Noor iteration (1.3),

$$\begin{aligned}\|w_{n+1}-p\| &\geq (1-\alpha_n)\|w_n-p\|-\alpha_n\|Tq_n-p\|\\ &\geq (1-\alpha_n)\|w_n-p\|-\alpha_n\delta'\|q_n-p\|\\ &\geq (1-\alpha_n)\|w_n-p\|-\alpha_n\delta'((1-\beta_n)\|w_n-p\|+\beta_n\delta'\|h_n-p\|)\\ &\geq (1-\alpha_n-\alpha_n\delta'(1-\beta_n))\|w_n-p\|-\alpha_n\beta_n\delta'^2(1-\gamma_n(1-\delta'))\|w_n-p\|\\ &= (1-\alpha_n(1+\delta'(1-\beta_n(1-\delta'(1-\gamma_n(1-\delta'))))))\|w_n-p\|\\ &\geq (1-\alpha_n(1+\delta'))\|w_n-p\|\\ &\ \ \vdots\\ &\geq \prod_{k=1}^{n}(1-\alpha_k(1+\delta'))\|w_1-p\|. \end{aligned} \tag{2.4}$$

It follows by (C1), (2.2) and (2.4) that

$$\frac{\|x_{n+1}-p\|}{\|w_{n+1}-p\|}\leq \frac{(1-\eta(1-\delta'))^n}{\prod_{k=1}^{n}(1-\alpha_k(1+\delta'))}.$$

Put $\theta_n=\frac{(1-\eta(1-\delta'))^n}{\prod_{k=1}^{n}(1-\alpha_k(1+\delta'))}$. By the assumption $0<\alpha_n<\frac{1}{1+\delta'}$ and $\lim_{n\to\infty}\alpha_n=0$, we obtain

$$\begin{aligned}\lim_{n\to\infty}\frac{\theta_{n+1}}{\theta_n} &= \lim_{n\to\infty}\frac{(1-\eta(1-\delta'))^{n+1}}{\prod_{k=1}^{n+1}(1-\alpha_k(1+\delta'))}\frac{\prod_{k=1}^{n}(1-\alpha_k(1+\delta'))}{(1-\eta(1-\delta'))^n}\\ &= \lim_{n\to\infty}\frac{1-\eta(1-\delta')}{1-\alpha_{n+1}(1+\delta')}\\ &= 1-\eta(1-\delta')<1.\end{aligned}$$

By the ratio test, we conclude that $\sum_{n=1}^{\infty}\theta_n<\infty$. So, $\lim_{n\to\infty}\theta_n=0$. This implies that the sequence generated by (1.4) converges faster than Noor iteration.

Secondly, by Ishikawa iteration (1.2),

$$\begin{aligned}\|s_{n+1}-p\| &\geq (1-\alpha_n)\|s_n-p\|-\alpha_n\|Tr_n-p\|\\ &\geq (1-\alpha_n)\|s_n-p\|-\alpha_n\delta'\|r_n-p\|\\ &\geq (1-\alpha_n)\|s_n-p\|-\alpha_n\delta'(1-\beta_n+\beta_n\delta')\|s_n-p\|\\ &= (1-\alpha_n(1+\delta'(1-\beta_n(1-\delta'))))\|s_n-p\|\\ &\geq (1-\alpha_n(1+\delta'))\|s_n-p\|\\ &\ \ \vdots\\ &\geq \prod_{k=1}^{n}(1-\alpha_k(1+\delta'))\|s_1-p\|. \end{aligned} \tag{2.5}$$

It follows by (C1), (2.2) and (2.5) that

$$\frac{\|x_{n+1}-p\|}{\|s_{n+1}-p\|} \leq \frac{(1-\eta(1-\delta'))^n}{\prod_{k=1}^{n}(1-\alpha_k(1+\delta'))}.$$

By the assumption and same argument as above, we can show that the sequence generated by (1.4) converges faster than Ishikawa iteration.

Finally, by Mann iteration (1.1),

$$\begin{aligned}\|u_{n+1}-p\| &\geq (1-\alpha_n(1+\delta'))\|u_n-p\| \\ &\geq \prod_{k=1}^{n}(1-\alpha_k(1+\delta'))\|u_1-p\|. \end{aligned} \tag{2.6}$$

It follows by (C1), (2.2) and (2.6) that

$$\frac{\|x_{n+1}-p\|}{\|u_{n+1}-p\|} \leq \frac{(1-\eta(1-\delta'))^n}{\prod_{k=1}^{n}(1-\alpha_k(1+\delta'))}.$$

By the assumption and same argument as above, we can show that the sequence generated by (1.4) converges faster than Mann iteration. □

Set $\gamma_n = 0$ for all $n \in \mathbb{N}$ in (1.4), we get the following two-step iterative method:

$$\begin{cases} y_n = (1-\beta_n)x_n + \beta_n T x_n, \\ x_{n+1} = (1-\alpha_n-\lambda_n)y_n + \alpha_n T y_n + \lambda_n T x_n, \quad \text{for all } n \in \mathbb{N}, \end{cases} \tag{2.7}$$

where $x_1 \in C$, $\{\alpha_n\}$, $\{\beta_n\}$, $\{\lambda_n\}$, $\{\alpha_n+\lambda_n\}$ are sequences in $[0,1]$. By Theorem 2.1, we obtain the following convergence theorem of the iterative scheme (2.7) and by Theorem 2.4, we obtain the following result for comparing the rate of convergence of two-step iteration (2.7) with Mann, Ishikawa and Noor iterations.

**Theorem 2.5.** *Assume $X$, $C$, $T$ are as in Theorem 2.1 and $\{\alpha_n\}$, $\{\beta_n\}$, $\{\lambda_n\}$, $\{\alpha_n+\lambda_n\}$ are sequences in $[0,1]$ such that $\sum_{n=1}^{\infty}\alpha_n = \infty$ or $\sum_{n=1}^{\infty}\beta_n = \infty$. Then the sequence $\{x_n\}$ generated by two-step iteration (2.7) converges strongly to a unique fixed point of $T$.*

**Theorem 2.6.** *Assume $X$, $C$, $T$, $\{u_n\}$, $\{s_n\}$, $\{w_n\}$ are as in Theorem 2.4 and $\{\alpha_n\}$, $\{\beta_n\}$, $\{\lambda_n\}$, $\{\alpha_n+\lambda_n\}$ are sequences in $[0,1]$ which satisfy the conditions:*

*(C1)* $0 < \eta \leq \beta_n < 1$;

*(C2)* $0 < \alpha_n < \frac{1}{1+\delta'}$, $\sum_{n=1}^{\infty}\alpha_n = \infty$ *and* $\lim_{n\to\infty}\alpha_n = 0$.

*Then the sequence $\{x_n\}$ generated by two-step iteration (2.7) converges faster than Mann, Ishikawa and Noor iterations to a unique fixed point of $T$.*

It is worth to observe that above theorem shows that the two-step iteration (2.7) convergence faster than three-step iteration (1.3) under some suitable control conditions.

**Remark 2.7.**

(i) *If a mapping $T$ satisfies only the condition (2.1) with $F(T) \neq \emptyset$, under the same control conditions, Theorems 2.1, 2.2, 2.4, 2.5 and 2.6 hold true for this class of mappings.*

(ii) *Any Zamfirescu operators, quasi-contraction operators with $h \in (0, 1/2)$ are weak contractions and also satisfy condition (2.1). Therefore, Theorems 2.1, 2.2, 2.4, 2.5 and 2.6 are obtained with these mappings.*

The following example shows that our iterative method converges faster than Mann, Ishikawa and Noor iterations.

**Example 2.8.** *Let $T : [0, 1] \to [0, 1]$ be defined by*

$$Tx = \begin{cases} \dfrac{x}{3}, & \text{if } x \in \left[0, \frac{2}{5}\right) \\ \dfrac{2x}{5}, & \text{if } x \in \left[\frac{2}{5}, 1\right]. \end{cases}$$

*Obviously, if $x, y \in [0, \frac{2}{5})$ or $x, y \in [\frac{2}{5}, 1]$, then $T$ is satisfies the weak contraction condition and condition (2.1). For the case $x \in [0, \frac{2}{5})$ and $y \in [\frac{2}{5}, 1]$, or $x \in [\frac{2}{5}, 1]$ and $y \in [0, \frac{2}{5})$, it follows that*

$$|Tx - Ty| \leq \frac{1}{3}|x - y| + \frac{1}{9}|y - Ty|,$$

*and*

$$|Tx - Ty| \leq \frac{1}{3}|x - y| + |y - Tx|.$$

*Then $T$ is a weak contraction and satisfies condition (2.1). The comparison of the convergence of the sequence $\{x_n\}$ generated by (1.4) and (2.7), Mann, Ishikawa and Noor iterations to the exact fixed point $p = 0$ are given in the following table, with the initial point $x_1 = u_1 = s_1 = w_1 = 1$, and the control conditions $\alpha_n = \lambda_n = \gamma_n = \frac{1}{n^{0.2}+1}$ and $\beta_n = \frac{n}{5n+1}$. It is clear that these control conditions satisfy those in Theorems 2.4 and 2.6.*

| | Mann iteration | Ishikawa iteration | Noor iteration | iteration (2.7) | iteration (1.4) | |
|---|---|---|---|---|---|---|
| $n$ | $u_n$ | $s_n$ | $w_n$ | $x_n$ | $x_n$ | $\|Tx_n - x_n\|$ |
| 5 | 0.18965816 | 0.16545924 | 0.16121130 | 0.00782504 | 0.00136267 | 9.0844E-04 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| 14 | 0.01275348 | 0.00903830 | 0.00856897 | 0.00000442 | 0.00000005 | 3.4494E-08 |
| 15 | 0.00962621 | 0.00667622 | 0.00631260 | 0.00000203 | 0.00000002 | 1.1942E-08 |
| 16 | 0.00728501 | 0.00494558 | 0.00466392 | 0.00000094 | 0.00000001 | 4.1749E-09 |

Table 2.1: Comparison of the rate of convergence of the iterative methods (1.4), (2.7), Mann, Ishikawa and Noor iterations for the mapping given in Example 2.8.

From Table 2.1, we see that the iterative methods (1.4) and (2.7) converge faster than Mann, Ishikawa and Noor iterations.

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