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Testing for Jumps in the Presence of Market Microstructure Noise[1]

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Abstract : Discontinuous price changes called jumps are an essential component of financial asset price dynamics. As it was shown by Andersen, Bollerslev [1], Andersen et al. [2] and Lahaye et al. [3], jump occurrence in prices of various financial instruments is strongly correlated with macroeconomic announcements. Simulation researches show that the tests for jumps generally are sufficiently powerful, provided that high frequency data was used. Unfortunately, high frequency data is usually polluted by market microstructure noise (nonsynchronous trading, bid-ask bounce, discreteness etc.). In this paper we present results of test for jumps in the levels of three European stock indexes. We use two alternative approaches to testing of jump occurence by the assuption of presence of market microstructure noise: by Barndorff-Nielsen and Shephard [4] with, introduced by Andersen et al. [2], staggered bi- and tripower variation as estimators of integrated volatility and quarticity, and analytically modified form of swap variance tests introduced by Jiang and Oomen [5].

Keywords : autocorrelation function; jump diffusion process; test for jumps.

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1 Introduction

Towards the end of the 20th century Black and Scholes [6] introduced first option pricing models which assumed that the underlying price process may be described by a simple diffusion model. In the following years an application of models with continuous time to modeling and forecasting financial instruments became important and the model has been gradually developed.

The most modern derivative pricing models assume that the underlying is described by jump diffusion models − the continuous time models which take into account uncontinuous price changes called jumps. The models can also explain the excess kurtosis which is typical feature of most financial time series. An application of jumps diffusion models requires identification of jumps in studying processes. Testing for jumps allows to assess connection between the jumps occurrence and various macroeconomical announcements. Empirical studies of Barndorff-Nielsen and Shephard [7], Jiang Oomen [5] show that all tests mentioned in the following paper give satisfying results only if there is high frequency data used to estimating multipower variations, swap variances and realized variances. An important complication that makes the estimating difficult is the presence of market microstructure noise − the pollution of observed data connected with some of typical features of financial markets like nonsynchronous trading, bid-ask spread etc. (see [8] for more details). The market microstructure effects have been being studied for over 40 years. Niederhoffer and Osborne [9] showed that the existence of a bid-ask spread introduces the first order autocorellation in observed returns. Over the last ten years the interest in market microstructure rised. One of important problems to be solved is finding market microstructure noise-corrected estimators of integrated variance. Ait-Sahalia et al. [10], [11] and Zhang et al. [12] studied properties of estimators of the integrated volatility based on data frequencies high enough for that noise to be dominant consideration. Moreover, they proposed noise-resistant integrated volatility estimators. Zhang [13] introduced multi-scale realized volatility estimator which converge very fast to the true volatility. Another bias-corrected integrated variance estimators were proposed by and Hansen and Lunde [14] and Christensen et al. [15]. An essential issue is also testing for jumps in the prices contaminated with the market microstructure noise. Fan and Wang [16] introduced methods to estimate both integrated volatility and jump variation. Andersen et al. [2] proposed an application of Barndorff-Nielsen and Shephard [7] and [4] statistics with staggered version of realized bi- and tripower variations. Huang and Tauchen [17] showed empirically that using logarithmic statistic with this type of integrated variance and quarticity estimators gives surprisingly good results. Jiang and Oomen [5] proposed modified swap variance test, called noise adjusted swap variance test, which retains power for noisy high frequency data. In the following paper we test jump occurrence in stock index levels of three european markets. We use high frequency 5-minute data. The main purpose of our empirical studies is the assesment of the influence scale of market microstructure noise on the number of detected jumps. Our goal is to check whether the influence is big enough to justify using noise-corrected tests for jumps with typical 5-minute sampling applied.

2 Two Alternalive Approaches to Testing Jumps in Logarithmic Price Process

Assume that $X(t)$ is a logarithmic price process. Barndorff-Nielsen and Shephard [7] introduced the realized bipower variation process. The realized bipower variation in the period $(t, t+1)$ is defined in normalized form as

$$BV_{(t,t+1)}(\delta) = \mu_1^{-2} \sum_{j=1}^{[1/\delta]-1} |y_{t+j\delta,\delta}| \left|y_{t+(j+1)\delta,\delta}\right|,$$

where $y_{t,\delta} = X(t) - X(t-\delta)$, $\mu_r = E|u|^r$ for $r > 0$ and $u \backsim N(0,1)$. Moreover, it can be shown that $\mu_r = \frac{2^{r/2}}{\sqrt{\pi}}\Gamma\left(\frac{1}{2}(r+1)\right)$.

The authors showed that realized bipower variation process introduced by them tends (as $\delta \to 0$) to the integrated variance defined by

$$IV_{(t,t+1)} = \int_t^{t+1} \sigma^2(s)ds$$

independently of jumps occurrence in processes of logarithmic prices of financial instruments. Moreover, the realized variance introduced by Andersen and Bolerslev [1], which is defined as

$$RV_{(t,t+1)}(\delta) = \sum_{j=1}^{[1/\delta]} y^2_{t+j\delta,\delta},$$

converges to integrated variance as $\delta \to 0$ in the case of absence of jumps in the logarithmic price process. Otherwise, it converges to sum of integrated variance and squared jumps. Therefore we have

$$\operatorname*{plim}_{\delta\to 0}\left[RV_{(t,t+1)}(\delta) - BV_{(t,t+1)}(\delta)\right] = \begin{cases} 0 \text{ if there is no jumps in } (t, t+1) \\ \sum\limits_{t\le i\le t+1}^{[1/\delta]} c^2(i) \text{ otherwise,} \end{cases} \tag{2.1}$$

where $c(i)$ is size of the jump. This difference became the starting point for construction of tests for jumps by Barndorff-Nielsen and Shephard [4]. They showed that

$$G_{BNS} = \delta^{-1/2}(RV_{(t,t+1)}(\delta) - BV_{(t,t+1)}(\delta)) \Big/ \sqrt{Q_{(t,t+1)}} \xrightarrow{d} N(0,\vartheta)$$

and

$$H_{BNS} = \frac{\delta^{-1/2}(RV_{(t,t+1)}(\delta) - BV_{(t,t+1)}(\delta))}{RV_{(t,t+1)}(\delta)} IV_{(t,t+1)} \Big/ \sqrt{Q_{(t,t+1)}} \xrightarrow{d} N(0,\vartheta),$$

where $\vartheta = \frac{\pi^2}{4} + \pi - 5$ and $Q_{(t,t+1)}$ is the quarticity and

$$Q_{(t,t+1)} = \int_t^{t+1} \sigma^4(s)ds.$$

The G_{BNS} and H_{BNS} statistics test the null hypothesis that the asset prices have continuous sample paths in $(t, t+1)$ against the alternative hypothesis that there are some jumps in asset price process in $(t, t+1)$.The unobserved quarticity $\int_t^{t+1} \sigma^4(s)ds$ can be estimated by the realized tripower or fourpower variation defined in normalized form respectively as

$$TQ_{(t,t+1)}(\delta) = \mu_{4/3}^{-3} \delta^{-1} \sum_{j=1}^{[1/\delta]-2} \left|y_{t+j\delta,\delta}\right|^{4/3} \left|y_{t+(j+1)\delta,\delta}\right|^{4/3} \left|y_{t+(j+2)\delta,\delta}\right|^{4/3},$$

and

$$QQ_{(t,t+1)}(\delta) = \mu^{-4} \delta^{-1} \sum_{j=1}^{[1/\delta]-2} \left|y_{t+j\delta,\delta}\right| \left|y_{t+(j+1)\delta,\delta}\right| \left|y_{t+(j+2)\delta,\delta}\right| \left|y_{t+(j+3)\delta,\delta}\right|.$$

After normalization, the rough version, of statistics G_{BNS}, H_{BNS} can be written in the form

$$\widehat{G}_{BNS} = \delta^{-1/2} (RV_{(t,t+1)}(\delta) - BV_{(t,t+1)}(\delta)) \Big/ \sqrt{\vartheta TQ_{(t,t+1)}(\delta)}$$

and

$$\widehat{H}_{BNS} = \frac{\delta^{-1/2} (RV_{(t,t+1)}(\delta) - BV_{(t,t+1)}(\delta))}{RV_{(t,t+1)}(\delta)} \left[BV_{(t,t+1)}(\delta)\right] \Big/ \sqrt{\vartheta TQ_{(t,t+1)}(\delta)^2}\,.$$

Huang and Tauchen [17] suggested that the linear statistics can overstate number of detected jumps. They advise to use the normalized logarithmic statistic obtained by using delta method, which is given by the formula

$$\widehat{J}_{BNS} = \delta^{-1/2} \left[\left(\log RV_{(t,t+1)}(\delta)\right) - \left(\log BV_{(t,t+1)}(\delta)\right)\right] \Big/ \sqrt{\vartheta TQ_{(t,t+1)}(\delta)}\,.$$

In subsequent literature $\widehat{G}_{BNS}$, $\widehat{H}_{BNS}$ and $\widehat{J}_{BNS}$ are called bipower variation test statistics. Moreover, to avoid understating of realized bi- and tripower variation, Huang and Tauchen [17] recommended to multiply them by scaling factors. The scaled realized bi-, tripower variation are given by the formulas

$$BV^s_{(t,t+1)}(\delta) = \frac{1/\delta}{(1/\delta) - 1} BV_{(t,t+1)}(\delta)$$

and

$$TQ^s_{(t,t+1)}(\delta) = \frac{1/\delta}{(1/\delta) - 2} TQ_{(t,t+1)}(\delta).$$

Furthermore, in the case of rejection of H_0 hypothesis of no jumps in the period $(t, t+1)$ and by the assumption that in this period at the most one jump can occur, we can approximate the jump value from the following equation

$$|c_j| \approx \sqrt{RV_{(t,t+1)}(\delta) - BV_{(t,t+1)}(\delta)}.$$

Jiang and Oomen [5] proposed an alternative approach to tests for jumps. First, they showed that by the suitable assumptions on the logarithmic price process the following relation holds

$$2\int_t^{t+1} (dS(s)/S(s) - dX(s)) = \int_t^{t+1} \sigma^2(s)ds + 2\int_t^{t+1} (\exp c(s) - c(s) - 1)dq(s), \quad (2.2)$$

where $S(t) = \exp(X(t))$ is the price process and $dq(t)$ is a counting process equal to 1 if the jump happens in time t and 0 otherwise. The left-hand side of equation 2.2, which can be interpreted as the cumulative delta-hedged gains of two short log contracts (Neuberger [18]), converges to integrated variance in the case of absence of jumps in $(t, t+1)$. Otherwise, it tends to sum of integrated variance and the expression

$$2\int_t^{t+1} (\exp c(s) - c(s) - 1)dq(s).$$

The discretized version of this expression is called realized swap variance and it is the accumulated difference between simple and log returns

$$SwV_{(t,t+1)}(\delta) = 2\sum_{j=1}^{[1/\delta]} (Y_{t+j\delta,\delta} - y_{t+j\delta,\delta}),$$

where $Y_{t+j\delta,\delta} = (S_t - S_{t-\delta})/\, S_{t-\delta}$ and $y_{t+j\delta,\delta}$ are given as above. Therefore, the following relationship holds

$$\operatorname*{plim}_{\delta\to 0} \left[SwV_{(t,t+1)}(\delta) - RV_{(t,t+1)}(\delta)\right] = \begin{cases} 0 \text{ if there is no jumps in } (t, t+1) \\ 2\int_t^{t+1} (\exp c(s) - \frac{1}{2}c^2(s) - c(s) - 1)dq(s) \\ \hfill \text{otherwise.} \end{cases}$$

This difference was used in the construction of a new class of tests for jumps by Jiang and Oomen [5] called swap variance tests. Jiang and Oomen [5] showed the following test statistics

$$\widehat{G}_{JO} = 3(SwV_{(t,t+1)}(\delta) - RV_{(t,t+1)}(\delta)) \Big/ \delta\sqrt{\mu_6 S_{(t,t+1)}}\ ,$$

$$\widehat{H}_{JO} = \frac{3(SwV_{(t,t+1)}(\delta) - RV_{(t,t+1)}(\delta))IV_{(t,t+1)}}{SwV_{(t,t+1)}(\delta)} \Big/ \delta\sqrt{\mu_6 S_{(t,t+1)}}\ ,$$

$$\widehat{J}_{JO} = 3\left[\ln(SwV_{(t,t+1)}(\delta)) - \ln(RV_{(t,t+1)}(\delta))\right] IV_{(t,t+1)} \Big/ \delta\sqrt{\mu_6 S_{(t,t+1)}}\ ,$$

where $S_{(t,t+1)}$ is the sexticity and

$$S_{(t,t+1)} = \int_t^{t+1} \sigma^6(s)ds.$$

All statistics follow standard normal distribution, under the null hypothesis of no jumps in $(t, t+1)$. The difference between the swap variance and realized variance can be positive or negative. Hence, as opposed to the Barndorff-Nielsen and Shephard [4] test, the Jiang and Oomen [5] jump test is a two-sided test. Authors suggest to estimate the unobserved sexticity $S_{(t,t+1)}$ by realized four- or sixpower variation which are defined in the normalized form respectively as

$$SQ(\delta) = \mu_{3/2}^{-6}\delta^{-2} \sum_{j=1}^{[1/\delta]-3} \left|y_{t+j\delta,\delta}\right|^{3/2} \left|y_{t+(j+1)\delta,\delta}\right|^{3/2} \left|y_{t+(j+2)\delta,\delta}\right|^{3/2} \left|y_{t+(j+3)\delta,\delta}\right|^{3/2},$$

and

$$SS(\delta) = \mu_1^{-6}\delta^{-2} \sum_{j=1}^{[1/\delta]-5} \left|y_{t+j\delta,\delta}\right| \left|y_{t+(j+1)\delta,\delta}\right| \cdots \left|y_{t+(j+5)\delta,\delta}\right|.$$

Moreover, similarly as in the case of Barndorff-Nielsen and Shephard [4] tests in the case of rejection hypothesis of no jumps in the period and by the assumption that in this period at the most one jump can occur, the approximate value of jump can be evaluated from the equation

$$SwV_{(t,t+1)}(\delta) - RV_{(t,t+1)}(\delta) \approx 2\exp c(s) - c^2(s) - 2c(s) - 2.$$

3 Testing for Jumps in the Presence of Market Microstructure Noise

It is known that the observed logarithmic price process, and in consequence also the return series, is polluted by microstructure noise. Following Ait-Sahalia et al. [10], Bandi, Russell [19], Zhang et al. [12], and Andersen et al. [2] we suppose that the observed logarithmic price process can be written as

$$X_t = X_t^* + \upsilon_t,$$

where X_t is a real unobserved logarithmic price process and υ_t is a white noise. Therefore, the logarithmic return is given by the formula

$$y_{t,\delta} = X_t^* - X_{t-\delta}^* + \upsilon_t - \upsilon_{t-\delta} = y_{t,\delta}^* + \eta_{t,\delta},$$

where $y_{t,\delta}^*$ is a real unobserved logarithmic return and $\eta_{t,\delta}$ is the MA(1) process. Consequently a negative first-order autocorrelation appears in the observed returns. The dependence between neighboring returns introduced an upward bias in realized bi- and tripower variation. Therefore, Andersen et al. [2] proposed to use a staggered version of realized multipower variations. Their idea based on the skipping of one observation when computing product of adjacent returns. The staggered version of realized bi- and tripower variations are given by the formulas

$$BV^{*}_{(t,t+1)}(\delta) = \frac{1/\delta}{(1/\delta)-2} \sum_{j=1}^{[1/\delta]-2} \left|y_{t+j\delta,\delta}\right| \left|y_{t+(j+2)\delta,\delta}\right|,$$

$$TQ^{*}_{(t,t+1)}(\delta) = \frac{1/\delta}{(1/\delta)-4} \sum_{j=1}^{[1/\delta]-4} \left|y_{t+j\delta,\delta}\right|^{4/3} \left|y_{t+(j+2)\delta,\delta}\right|^{4/3} \left|y_{t+(j+4)\delta,\delta}\right|^{4/3}.$$

In case when the order of the serial autocorrelation was higher, the authors suggest to skip more than one return. The other problem is bias of realized volatility. The bias-reduced measures of Realized Volatility appear in modern literature (e.g. Asai et al.), but empirical studies of Bandi and Russell [19] and Hansen and Lunde [14] show that data for the bias of realized variance determined on the basis of 5-min. is very small. Therefore, we decide to apply standard realized variance measure.

Jiang and Oomen [5] show that, by the assumption of presence of market microstructure noise, statistics $\widehat{G}_{JO}$, $\widehat{H}_{JO}$ and $\widehat{J}_{JO}$ are given by the following formulas

$$\widehat{G}^{*}_{JO} = 3(SwV_{(t,t+1)}(\delta) - RV_{(t,t+1)}(\delta)) \Big/ \delta\sqrt{\Omega^{*}_{SwV}}\,,$$

$$\widehat{H}^{*}_{JO} = \frac{3(SwV_{(t,t+1)}(\delta) - RV_{(t,t+1)}(\delta))}{SwV_{(t,t+1)}(\delta)} \left(IV_{(t,t+1)} + 2\frac{\omega^2}{\delta}\right) \Big/ \delta\sqrt{\Omega^{*}_{SwV}}\,,$$

$$\widehat{J}^{*}_{JO} = 3\left[\ln(SwV_{(t,t+1)}(\delta)) - \ln(RV_{(t,t+1)}(\delta))\right] \left(IV_{(t,t+1)} + 2\frac{\omega^2}{\delta}\right) \Big/ \delta\sqrt{\Omega^{*}_{SwV}},$$

where $\Omega^{*}_{SwV} = 15S_{(t,t+1)} + 72\frac{\omega^2}{\delta}Q_{(t,t+1)} + 72\frac{\omega^4}{\delta^2}IV_{(t,t+1)} + 36\frac{\omega^6}{\delta^3} + 84\frac{\omega^6}{\delta^2}$.

Statistics $\widehat{G}^{*}_{JO}$, $\widehat{H}^{*}_{JO}$ and $\widehat{J}^{*}_{JO}$ have approximately zero mean and unit variance for small but nonzero δ. The crucial issue in the implementation of the statistics $\widehat{G}^{*}_{JO}$, $\widehat{H}^{*}_{JO}$ and $\widehat{J}^{*}_{JO}$ is the estimation of market microstructure noise variance. Bandi and Russell [19] proposed using a consistent estimator $\widehat{\omega}^2 = \delta RV_{(t,t+1)}(\delta)/2$. They also suggested using data sampled at low frequency to obtain estimates of the integrated variance free of noise. Oomen [20, 21] proposed following autocovariance-based noise variance estimator

$$\widehat{\omega} = \frac{1}{(1/\delta)-1} \sum_{j=1}^{[1/\delta]-1} y_{t+j\delta,\delta}\, y_{t+(j+1)\delta,\delta}.$$

Moreover, Jiang and Oomen [5] introduced bias-corrected measure of realized bipower variation. It is given by the formula

$$BV^{**}_{(t,t+1)}(\delta) = (1 + c_V(\gamma))BV_{(t,t+1)}(\delta),$$

where $c_V(\gamma)$ is the bias-correction function and

$$c_V(\gamma) = (1+\gamma)\sqrt{\frac{1+\gamma}{1+3\gamma}} + \gamma\frac{\pi}{2} - 1 + 2\frac{\gamma}{(1+\lambda)\sqrt{1+2\lambda}} + 2\gamma\pi\kappa(\lambda)$$

with $\gamma = \omega^2/\overline{V}$, $\lambda = \gamma/(1+\gamma)$ and

$$\kappa(\lambda) = \int_{-\infty}^{\infty} x^2\Phi(x\sqrt{\lambda})(1-\Phi(x\sqrt{\lambda}))\phi(x\sqrt{\lambda})dx,$$

where $\overline{V}$ is the return variance, and $\Phi(\cdot)$ and $\phi(\cdot)$ are the CDF and PDF of the standard normal distribution respectively. Similarly they introduced bias-correction function for quarlicity and sexticity estimators equaled $c_Q(\gamma) = 5.46648\gamma^2 + 4\gamma$ and $c_S(\gamma) = 13.2968\gamma^3 + 14.4255\gamma^2 + 6\gamma$, respectively.

4 Data

The data applied in this paper consist of 5-min. intraday levels of the indexes CAC40, DAX and WIG20 from the period between January 1, 2004 and December 29, 2006. In the aforementioned period all indexed grew steady and uniform. We avoid considering the period of the global financial crisis, in which the behavior of the indexes levels series were very time-changing. The descriptive statistics of studied logarithmic indexes levels (multiplied by 100) and their returns are presented in the tables 1 and 2 respectively.

Table 1. Descriptive statistics for the multiplied by 100, 5-minute logarithmic levels of CAC40, DAX and WIG20.

series.	obs. num.	mean	std. dev.	skewness	kurtosis	min	max
CAC40 5 min.	78642	4349.2	616.20	0.30621	1.6629	3454.6	5553
DAX 5 min.	78438	4844.6	836.57	0.39903	1.7656	3620.3	6625.4
WIG20 5 min.	57010	2324,6	553.88	0.33721	1.5879	1577.1	3429.8

Table 2. Descriptive statistics for the 5-minute logarithmic returns of CAC40, DAX and WIG20.

series.	obs. num.	mean	std. dev.	skewness	kurtosis	min	max
CAC40 5 min.	78641	0.0003	0.0651	-0.0837	14.717	-1.2192	1.1110
DAX 5 min.	78437	0.0003	0.0778	-0.6293	39.174	-2.1689	1.7562
WIG20 5 min.	57009	0.0007	0.1301	-0.0770	8.9422	-1.6520	1.3182

5 Empirical Research

In the following section we present results of test for jumps. The occurrence of jumps is verified in one-day periods, but multipower variation, realized variance and swap variance are estimated by using 5-min returns. In Tables 3, 4 and 5 there are given numbers of detected days with jumps in logarithmic level process of indexes CAC40, DAX and WIG20 obtained by using Barndorff-Nielsen and Shephard [4] tests with significance levels 0.05, 0.01, 0.005 and 0.001.

Table 3. Number of detected days with jumps in level process of indexes CAC40, DAX and WIG20 in relation with significance level by using Barndorff-Nielsen and Shephard linear statistic.

Time series	Estimators of IV and quarticity	Number of detected days with jumps (Percentage share of days with jumps)			
		0.05	0.01	0.005	0.001
CAC40	BV, TQ	223 (29.00%)	156 (20.29%)	128 (16.64%)	92 (11.96%)
5 min.	BV^s, TQ^s	213 (27.70%)	138 (17.95%)	119 (15.47%)	88 (11.44%)
DAX	BV, TQ	374 (48.51%)	272 (35.28%)	248 (32.17%)	201 (26.07%)
5 min.	BV^s, TQ^s	356 (46.17%)	257 (33.33%)	237 (30.74%)	192 (24.90%)
WIG20	BV, TQ	265 (35.01%)	171 (22.59%)	142 (18.76%)	105 (13.87%)
5 min.	BV^s, TQ^s	232 (30.65%)	149 (19.68%)	130 (17.17%)	92 (12.15%)

Table 4. Number of detected days with jumps in level process of indexes CAC40, DAX and WIG20 in relatio with significance level by using Barndorff-Nielsen and Shephard logarithmic statistic.

Time series	Estimators of IV and quarticity	Number of detected days with jumps (Percentage share of days with jumps)			
		0.05	0.01	0.005	0.001
CAC40	BV, TQ	214 (27,76%)	126 (16,34%)	108 (14,01%)	73 (9,47%)
5 min.	BV^s, TQ^s	192 (24,90%)	119 (15,43%)	97 (12,58%)	64 (8,30%)
DAX	BV, TQ	357 (46,42%)	245 (31,86%)	218 (28,35%)	173 (22,50%)
5 min.	BV^s, TQ^s	329 (42,78%)	236 (30,69%)	207 (26,92%)	159 (20,68%)
WIG20	BV, TQ	242 (31,97%)	140 (18,49%)	120 (15,85%)	66 (8,72%)
5 min.	BV^s, TQ^s	218 (28,80%)	128 (16,91%)	104 (13,74%)	60 (7,93%)

Table 5. Number of detected days with jumps in level process of indexes CAC40, DAX and WIG20 in relation with significance level by using Barndorff-Nielsen and Shephard ratio statistic.

Time series	Estimators of IV and quarticity	Number of detected days with jumps (Percentage share of days with jumps)			
		0.05	0.01	0.005	0.001
CAC40	BV, TQ	186 (24.19%)	99 (12.87%)	81 (10.53%)	52 (6.76%)
5 min.	BV^s, TQ^s	176 (22.89%)	90 (11.70%)	75 (9.75%)	46 (5.98%)
DAX	BV, TQ	321 (41.63%)	210 (27.24%)	179 (23.22%)	117 (15.18%)
5 min.	BV^s, TQ^s	300 (38.91%)	201 (26.07%)	169 (21.92%)	112 (14.53%)
WIG20	BV, TQ	208 (27.48%)	99 (13.08%)	71 (9.38%)	38 (5.02%)
5 min.	BV^s, TQ^s	190 (25.10%)	89 (11.76%)	60 (7.93%)	34 (4.49%)

We can observe that linear test detects slightly higher number of jumps than other statistics. This is most likely result of a big size of the test. We can also observe that ratio statistic detects jumps remarkably less often. This might be result of lower size and power of this test, as shown by empirical studies. Test statistics with unscalled realized bi- and tripower variations applied as integrated variance and quarticity estimators overstate number of detected jumps.

In the table 6, 7 and 8 there are presented number of detected days with jumps in logarithmic level process of indexes CAC40, DAX and WIG20 obtained by using

Jiang and Oomen [5] tests with significance levels 0.05, 0.01, 0.005 and 0.001.

Table 6. Number of detected days with jumps in level process of indexes CAC40, DAX and WIG20 in relation with significance level by using Jiang and Oomen linear statistic.

Time series	Estimator of sexticity	Number of detected days with jumps (Percentage share of days with jumps)			
		0.05	0.01	0.005	0.001
CAC40 5 min.	SS	287 (37.32%)	219 (28.48%)	200 (26.01%)	166 (21.59%)
DAX 5 min.	SS	382 (49.55%)	307 (39.82%)	284 (36.84%)	244 (31.65%)
WIG20 5 min.	SS	268 (35.40%)	208 (27.48%)	182 (24.04%)	151 (19.95%)

Table 7. Number of detected days with jumps in logarithmic level process of indexes CAC40, DAX, and WIG20 in dependence on significance level by using Jiang and Oomen logarithmic statistic.

Time series	Estimators of IV and sexticity	Number of detected days with jumps (Percentage share of days with jumps)			
		0.05	0.01	0.005	0.001
CAC40	BV, SS	258 (33.55%)	189 (24.58%)	173 (22.50%)	121 (15.73%)
5 min.	BV^s, SS^s	261 (33.94%)	193 (25.10%)	175 (22.76%)	125 (16.25%)
DAX	BV, SS	351 (45.53%)	268 (34.76%)	239 (31.00%)	208 (26.98%)
5 min.	BV^s, SS^s	352 (45.65%)	269 (34.89%)	246 (31.91%)	211 (27.37%)
WIG20	BV, SS	242 (31.97%)	176 (23.25%)	159 (21.00%)	120 (15.85%)
5 min.	BV^s, SS^s	245 (32.36%)	180 (23.78%)	162 (21.40%)	122 (16.12%)

Table 8. Number of detected days with jumps in level process of indexes CAC40, DAX and WIG20 in relation with significance level by using Jiang and Oomen ratio statistic.

Time series	Estimators of IV and sexticity	Number of detected days with jumps (Percentage share of days with jumps)			
		0.05	0.01	0.005	0.001
CAC40	BV, SS	258 (33.55%)	189 (24.58%)	173 (22.50%)	121 (15.73%)
5 min.	BV^s, SS^s	260 (33.81%)	194 (25.23%)	175 (22.76%)	125 (16.25%)
DAX	BV, SS	351 (45.53%)	268 (34.76%)	239 (31.00%)	208 (26.98%)
5 min.	BV^s, SS^s	352 (45.65%)	269 (34.89%)	246 (31.91%)	211 (27.37%)
WIG20	BV, SS	242 (31.97%)	176 (23.25%)	159 (21.00%)	120 (15.85%)
5 min.	BV^s, SS^s	245 (32.36%)	180 (23.78%)	162 (21.40%)	122 (16.12%)

Similarly to bipower variation test statistics, swap variance linear statistic detects higher number of days with jumps than other statistics. Logarithmic and ratio statistics detect almost identical number of days with jumps. Although the size and power of the test is not influenced by the type of applied integrated variance estimator, the difference between number of days with jumps detected by using the same statistics with realized bipower and scaled bipower variation as integrated variance estimator is observable.

To determine the existence of market microstructure noise in mentioned intraday series we determine their correlograms.

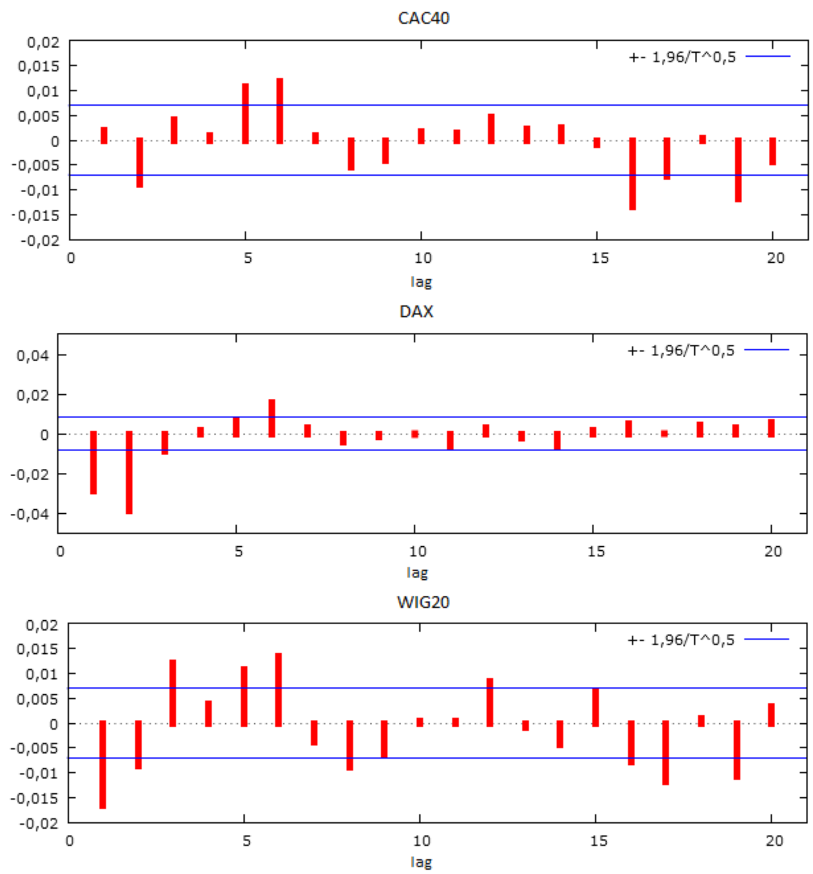

Figure 1. Autocorrelation of 5-min. logarithmic return of CAC40, DAX and WIG20 index.

For every intraday return series there exist significant autocorrelations. The low order negative autocorrelation in high frequency data is the consequence of market microstructure noise. The higher order linear dependences can be a consequence of the market frictions induced by clustering in order flows, delayed information discounting or short-term seasonality. Therefore, to avoid the influence of this spurious autocorrelations on bi- and tripower variations we use staggered version of the realized multipower variation. Based on the figure 1 we decide to skip two returns for CAC40 and WIG20 index and one return for DAX. Number of detected days with jumps by the bipower variation statistics with staggered realized bi- and tripower variations as estimators of integrated variance, and quarticity are given in table 9. To reduce the microstructure noise we decide to use also the $\widehat{G}^*_{JO}$, $\widehat{H}^*_{JO}$ and $\widehat{J}^*_{JO}$ statistics. Number of days with jumps detected by them is given in the table 9 and 10.

Table 9. Numbers of detected days with jumps in level process of indexes CAC40, DAX and WIG20 by using Barndorff-Nielsen and Shephard statistics with staggered realized bi- and tripower variantions as estimators of integrated variance and quarticity.

Time series	Type of the test statistic	Number of detected days with jumps (Percentage share of days with jumps)			
		0.05	0.01	0.005	0.001
CAC40	linear	239 (31.08%)	162 (21.07%)	138 (17.95%)	109 (14.17%)
5 min.	logarithmic	229 (29.78%)	139 (18.08%)	124 (16.13%)	84 (10.92%)
	ratio	195 (29.78%)	139 (18.08%)	124 (16.13%)	84 (10.92%)
DAX	linear	414 (53.70%)	331 (42.93%)	296 (38.39%)	239 (31.00%)
5 min.	logarithmic	396 (51.37%)	299 (38.78%)	267 (34.63%)	197 (25.55%)
	ratio	380 (49.29%)	299 (38.78%)	267 (34.63%)	197 (25.55%)
WIG20	linear	346 (45.71%)	276 (36.46%)	253 (33.42%)	203 (26.82%)
5 min.	logarithmic	331 (43.73%)	246 (32.50%)	217 (28.67%)	163 (21.53%)
	ratio	299 (39.50%)	246 (32.50%)	217 (28.67%)	163 (21.53%)

Table 10. Numbers of detected days with jumps in level process of indexes CAC40, DAX and WIG20 by using modified Jiang and Oomen statistics with bias-corrected measure of realized bipower variation and sexticity.

Time series	Type of the test statistic	Number of detected days with jumps (Percentage share of days with jumps)			
		0.05	0.01	0.005	0.001
CAC40	linear	304 (39.43%)	225 (29.18%)	205 (26.59%)	180 (23.35%)
5 min.	logarithmic	258 (33.46%)	183 (23.74%)	164 (21.27%)	128 (16.60%)
	ratio	258 (33.46%)	183 (23.74%)	164 (21.27%)	128 (16.60%)
DAX	linear	366 (47.66%)	310 (40.36%)	298 (38.80%)	257 (33.46%)
5 min.	logarithmic	344 (44.79%)	260 (33.85%)	244 (31.77%)	211 (27.47%)
	ratio	344 (44.79%)	260 (33.85%)	244 (31.77%)	211 (27.47%)
WIG20	linear	214 (28.27%)	165 (21.80%)	151 (19.95%)	117 (15.46%)
5 min.	logarithmic	196 (25.89%)	137 (18.10%)	110 (14.53%)	86 (11.36%)
	ratio	196 (25.89%)	137 (18.10%)	110 (14.53%)	86 (11.36%)

We can observe that tests which suppose presence of market microstructure noise detect slightly less days with jumps. Only swap variance linear test detects in two cases slightly more days with jumps. This shows that for 5-minute logarithmic levels the scale of market microstructure noise is big enough to have significant influence on the test results. Therefore, applying this type of tests to 5-minute sampled financial data appears to be justified.

6 Summary

Our research shows that occurrence of jumps in processes of stock indexes levels is very common. Both bipower variation tests and swap variance tests detect similar numbers of jumps. Occurrence of jumps in studied processes is confirmed by strong leptokurtosis in logarithmic returns series. It justifies using jump diffusion or jump stochastic volatility models to modeling processes of indexes levels which leads to more precise description of mentioned processes.

Pollution of market microstructure noise can be observed in a 5-minute index logarythmic returns series. We show that using the standard Barndorff-Nielsen and Shephard, and Jiang and Oomen statistics to test for jumps occurence leads to overstating number of detected jumps. So using two approches to test for jumps, both of which assume the presence of market microstructure noise, is justified when a 5-minute sampling was applied.

Using the standard Barndorff-Nielsen and Shephard, and Jiang and Oomen statistics means we need to seek for compromise. Too small frequency leads to not precise measures of realized variance, multipower variation etc., but too big frequency leads to mentioned above bias problem due to market microstructure noise. Applying Barndorff-Nielsen and Shephard statistics with staggered measures of multipower variation and corrected Jiang and Oomen statistics can reduce this problem.

We must take into consideration that the bigger return frequency, the more returns equal zero. Such data will cause understating the multipower variation estimates and in consequence the difference between realized variance and bipower variation will tend to be large and positive, and even more so in the presence of jumps. Fortunately, swap variance does not have this flaw, but understated value of sexticity leads to very big size of the test. Moreover as shown by Bandi and Russell [19], and Hansen and Lunde [14], the bias of realized volatility increases very fast for frequency higher than 2-min. Therefore, applying bias-corrected statistics of Barndorff-Nielsen and Shephard, and Jiang and Oomen for 5-min sampled data is the optimal choice.

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