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Meromorphic Functions That Share One Finite Value DM with Their First Derivative

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Abstract : This paper has studied the uniqueness of meromorphic functions that share one finite value DM (different multiplicities) with first derivatives and obtains some results which improve a result given by Zhang [1].

Keywords : Nevanlinna theory; uniqueness theorem; share DM; meromorphic function.
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1 Introduction

We say that two nonconstant meromorphic functions f and g share the finite value a IM (ignoring multiplicities), if $f-a$ and $g-a$ have the same zeros. If $f-a$ and $g-a$ have the same zeros with the same multiplicities, we say that f and g share the value a CM (counting multiplicities). If $f-a$ and $g-a$ have the same zeros with the different multiplicities, we say that f and g share the value a DM (different multiplicities). In this paper the term "meromorphic" will always mean meromorphic in the complex plane. We use the standard notations and results of the Nevanlinna theory (see [2], for example). In particular, $S(r,f)$ denotes any quantity satisfying $S(r,f)=o(T(r,f))$ as $r\to\infty$, except possibly for a set E of r of finite linear measure. Let k be a positive integer, we denote by $N_{k)}(r,\frac{1}{f-a})$ the counting function of zeros of $f-a$ with multiplicity $\leq k$ and by $N_{(k+1}(r,\frac{1}{f-a})$ the counting function of zeros of $f-a$ with multiplicity $> k$. Definitions of the terms $N_{k)}(r,f)$ and $N_{(k+1}(r,f)$ can be similarly formulated. Finally $N_2(r,\frac{1}{f})$ denotes the counting function of zeros of f where a zero of multiplicity k is counted with multiplicity $\min\{k,2\}$.

Rubel and Yang [3] proved the following result:

Theorem 1.1. *If a nonconstant entire function f and its derivative f' share two finite values CM, then $f \equiv f'$.*

Mues and Steinmetz [4] have shown that "CM" can be replaced by "IM" in Theorem 1.1 and Gundersen [5] have shown that "entire" can be replaced by "meromorphic" in Theorem 1.1.

On the other hand, the meromorphic function [4]

$$f(z) = \left[\frac{1}{2} - \frac{\sqrt{5}}{2} i tan\left(\frac{\sqrt{5}}{4} iz\right)\right]^2 \tag{1.1}$$

shares 0 by DM and 1 by IM (neither CM nor DM) with f', while the meromorphic function [6]

$$f(z) = \frac{2a}{1-ce^{-2z}} \tag{1.2}$$

shares 0 CM and a DM with f', where c and a are nonzero constants. It immediately yields from (1.1) and (1.2) that $f \not\equiv f'$.

Zhang [1] proved the following theorem:

Theorem 1.2. *Let f be a nonconstant meromorphic function, a be a nonzero finite complex constant. If f and f' share 0 CM, and share a IM, then $f \equiv f'$ or f is given as (1.2).*

From example (1.2) we also see that $N(r,\frac{1}{f}) = N(r,\frac{1}{f'}) = 0$.

2 Main Results

The purpose of this paper is to prove:

Theorem 2.1. *Let f be a nonconstant meromorphic function. Suppose that f and f' share the value a $(\neq 0,\infty)$ DM. Then either*

$$f(z) = \frac{a[1+b+(b-1)ce^{2b\ell z}]}{1-ce^{2b\ell z}}, \tag{2.1}$$

where b, c, ℓ are nonzero constants and $b^2\ell = -1$, or

$$T(r,f') \le 12\bar{N}\left(r,\frac{1}{f'}\right) + S(r,f) \tag{2.2}$$

and

$$T(r,f) \le \frac{11}{2} N_2\left(r,\frac{1}{f}\right) + S(r,f). \tag{2.3}$$

Proof. Suppose that $a = 1$ (the general case follows by considering $\frac{1}{a}f$ instead of f). We consider the following function

$$\psi = \frac{2f'}{f-1} - \frac{3f''}{2(f'-1)} + \frac{f'''}{f''} - \frac{f''}{f'}. \tag{2.4}$$

From the fundamental estimate of logarithmic derivative it follows that

$$m(r, \psi) = S(r, f). \tag{2.5}$$

Since f and f' share 1 DM, all zeros of $f - 1$ are simple and all zeros of $f' - 1$ with multiplicities not less than two. And so

$$N\left(r, \frac{1}{f-1}\right) = N_{1)}\left(r, \frac{1}{f-1}\right) \tag{2.6}$$

and

$$N\left(r, \frac{1}{f-1}\right) = \bar{N}\left(r, \frac{1}{f'-1}\right) = \bar{N}_{(2}\left(r, \frac{1}{f'-1}\right). \tag{2.7}$$

Suppose that z_2 is a zero of $f' - 1$ with multiplicity 2. Since f and f' share 1 DM, we see from (2.6) and (2.4) that

$$\psi(z_2) = 0. \tag{2.8}$$

If z_∞ is a simple pole of f, then an elementary calculation gives that

$$\psi(z_\infty) = O(1). \tag{2.9}$$

It follows from (2.6) - (2.9) that the poles of ψ can only occur at zeros of f', or zeros of f'' which are not zeros of $f'(f'-1)$, zeros of $f' - 1$ with multiplicities not less than three and multiple poles of f. Thus

$$N(r, \psi) \le \bar{N}\left(r, \frac{1}{f'}\right) + \bar{N}_{(3}\left(r, \frac{1}{f'-1}\right) + \bar{N}_{(2}(r, f) + \bar{N}_0\left(r, \frac{1}{f''}\right), \tag{2.10}$$

where $\bar{N}_0(r, \frac{1}{f''})$ denotes the counting function corresponding to the zeros of f'' that are not zeros of $f'(f'-1)$, each zero in this counting function is counted only once.

We distinguish the following two cases

Case 1. $\psi \equiv 0$. Then, by integrating two sides of (2.4) we obtain

$$\frac{(f-1)^4}{(f'-1)^3} = c\left(\frac{f'}{f''}\right)^2, \tag{2.11}$$

where c is a nonzero constant. If z_q is a zero of $f' - 1$ with multiplicity q (≥ 3), then from (2.6) and (2.11) we see that

$$O((z - z_q)^{2-q}) = c.$$

This implies that $q = 2$, a contradiction. Therefore

$$N_{(3}\left(r, \frac{1}{f'-1}\right) = 0. \tag{2.12}$$

Also if z_p is a pole of f with multiplicity p (≥ 2), then from (2.11) we find that

$$O((z - z_p)^{1-p}) = c.$$

Hence $p = 1$, a contradiction. Therefore

$$N_{(2}(r, f) = 0. \tag{2.13}$$

It follows from f and f' share 1 DM, (2.6), (2.7), (2.12) and (2.13) that

$$\frac{f'-1}{(f-1)^2} = e^{\alpha}, \tag{2.14}$$

where α is some entire function. Combining (2.11) and (2.14) we get

$$\left(\frac{f''}{f'}\right)\left(\frac{f''}{f'-1} - \frac{f''}{f'}\right) = ce^{2\alpha}. \tag{2.15}$$

Consequently,

$$T(r, e^{\alpha}) = S(r, f). \tag{2.16}$$

Also we know from (2.15) that

$$\bar{N}\left(r, \frac{1}{f'}\right) = S(r, f). \tag{2.17}$$

Suppose that z_1 is a simple zero of $f - 1$. Then by (2.7) and (2.12) we have

$$f(z) - 1 = (z - z_1) + a_3(z - z_1)^3 + \cdots, a_3 \neq 0 \tag{2.18}$$

Substituting (2.18) into (2.11) and (2.14) we find that

$$3a_3c = 4 \quad \textit{and} \quad 3a_3 = e^{\alpha(z_1)},$$

which implies

$$e^{\alpha(z_1)} = \frac{4}{c}. \tag{2.19}$$

If $e^{\alpha} \not\equiv \frac{4}{c}$, then we have from (2.6) and (2.16) that

$$N\left(r, \frac{1}{f-1}\right) \leq N\left(r, \frac{1}{e^{\alpha} - \frac{4}{c}}\right) \leq T(r, e^{\alpha}) + O(1) = S(r, f). \tag{2.20}$$

By (2.7), (2.17), (2.20) and the second fundamental theorem we have

$$\begin{aligned} T(r, f') &\leq \bar{N}\left(r, \frac{1}{f'}\right) + \bar{N}\left(r, \frac{1}{f'-1}\right) + \bar{N}(r, f) + S(r, f) \\ &\leq \bar{N}(r, f) + S(r, f). \end{aligned}$$

Since

$$\begin{aligned} T(r,f') &= m(r,f') + N(r,f') \\ &= m(r,f') + N(r,f) + \bar{N}(r,f), \end{aligned}$$

it follows from the last inequality that

$$m(r,f') + N(r,f) = S(r,f),$$

and so $T(r,f') = S(r,f)$. From this, (2.17) and (2.14) we get $T(r,f) = S(r,f)$ which is impossible. Therefore $e^{\alpha} \equiv \frac{4}{c}$. Together with (2.14) we arrive at the conclusion (2.1).

Case 2. $\psi \not\equiv 0$. Then from (2.8), (2.5) and (2.10) we conclude that

$$\begin{aligned} \bar{N}_{(2}\left(r,\frac{1}{f'-1}\right) - \bar{N}_{(3}\left(r,\frac{1}{f'-1}\right) &\le N\left(r,\frac{1}{\psi}\right) \le T(r,\psi) + O(1) \\ &\le N(r,\psi) + m(r,\psi) + O(1) \\ &\le \bar{N}\left(r,\frac{1}{f'}\right) + \bar{N}_{(3}\left(r,\frac{1}{f'-1}\right) + \bar{N}_{(2}(r,f) \\ &\quad + \bar{N}_0\left(r,\frac{1}{f''}\right) + S(r,f). \end{aligned} \tag{2.21}$$

Since $N(r,f') = N(r,f) + \bar{N}(r,f)$, from the second fundamental theorem for f'

$$T(r,f') \le \bar{N}\left(r,\frac{1}{f'}\right) + \bar{N}\left(r,\frac{1}{f'-1}\right) + \bar{N}(r,f) - \bar{N}_0\left(r,\frac{1}{f''}\right) + S(r,f), \tag{2.22}$$

we have

$$N(r,f) \le \bar{N}\left(r,\frac{1}{f'}\right) + \bar{N}\left(r,\frac{1}{f'-1}\right) - \bar{N}_0\left(r,\frac{1}{f''}\right) + S(r,f). \tag{2.23}$$

Also, we know from (2.22) that

$$N\left(r,\frac{1}{f'-1}\right) \le \bar{N}\left(r,\frac{1}{f'}\right) + \bar{N}\left(r,\frac{1}{f'-1}\right) + \bar{N}(r,f) - \bar{N}_0\left(r,\frac{1}{f''}\right) + S(r,f).$$

Combining this with (2.23) we obtain

$$\begin{aligned} N(r,f) - \bar{N}(r,f) + N\left(r,\frac{1}{f'-1}\right) - 2\bar{N}\left(r,\frac{1}{f'-1}\right) + 2\bar{N}_0\left(r,\frac{1}{f''}\right) \\ \le 2\bar{N}\left(r,\frac{1}{f'}\right) + S(r,f). \end{aligned} \tag{2.24}$$

Obviously,

$$N(r,f) - \bar{N}(r,f) \ge \bar{N}_{(2}(r,f), \tag{2.25}$$

and

$$N\left(r,\frac{1}{f'-1}\right)-2\bar{N}\left(r,\frac{1}{f'-1}\right)\geq \bar{N}_{(3}\left(r,\frac{1}{f'-1}\right), \tag{2.26}$$

by (2.7). Thus from (2.24) - (2.26) we obtain

$$\bar{N}_{(2}(r,f)+\bar{N}_{(3}\left(r,\frac{1}{f'-1}\right)+2\bar{N}_0\left(r,\frac{1}{f''}\right)\leq 2\bar{N}\left(r,\frac{1}{f'}\right)+S(r,f).$$

From this and (2.21) we deduce that

$$\bar{N}_{(2}\left(r,\frac{1}{f'-1}\right)\leq 5\bar{N}\left(r,\frac{1}{f'}\right)+S(r,f).$$

Together with (2.7) we have

$$\bar{N}\left(r,\frac{1}{f'-1}\right)\leq 5\bar{N}\left(r,\frac{1}{f'}\right)+S(r,f). \tag{2.27}$$

From (2.27) and (2.23), it follows that

$$\bar{N}(r,f)\leq 6\bar{N}\left(r,\frac{1}{f'}\right)+S(r,f). \tag{2.28}$$

Finally, Combining (2.22), (2.27) and (2.28) we find that

$$T(r,f')\leq 12\bar{N}\left(r,\frac{1}{f'}\right)+S(r,f).$$

This is the conclusion (2.2).

We set

$$G=\frac{1}{f}\left(\frac{f''}{f'-1}-2\frac{f'}{f-1}\right). \tag{2.29}$$

Then

$$\begin{aligned} m(r,G) &\leq m\left(r,\frac{f'}{f}\left(\frac{f''}{f'(f'-1)}\right)\right)+m\left(r,\frac{f'}{f(f-1)}\right)+O(1)\\ &\leq 2m\left(r,\frac{f'}{f}\right)+m\left(r,\frac{f''}{f'}\right)+m\left(r,\frac{f''}{f'-1}\right)+m\left(r,\frac{f'}{f-1}\right)+O(1)\\ &= S(r,f). \end{aligned} \tag{2.30}$$

Suppose z_2 be a zero of $f'-1$ with multiplicity 2. Since f and f' share 1 DM, we see from (2.29), (2.6) and (2.7) that

$$G(z_2)=O(1). \tag{2.31}$$

If z_∞ is a pole of f with multiplicity p (≥ 1), then an elementary calculation gives that

$$G(z)=O((z-z_\infty)),\quad if\quad p=1 \tag{2.32}$$

$$G(z) = O((z - z_\infty)^{p-1}), \quad if \quad p \geq 2. \tag{2.33}$$

It follows from (2.6), (2.7), (2.31), (2.32) and (2.33) that the pole of G can only occur at zeros of $f' - 1$ with multiplicities not less than three and zeros of f. Thus

$$N(r, G) \leq N_2\left(r, \frac{1}{f}\right) + \bar{N}_{(3}\left(\frac{1}{f'-1}\right).$$

Together with (2.30) we have

$$T(r, G) \leq N_2\left(r, \frac{1}{f}\right) + \bar{N}_{(3}\left(\frac{1}{f'-1}\right) + S(r, f). \tag{2.34}$$

We consider two cases:

Case I. $G \equiv 0$. Then (2.29) becomes

$$\frac{f''}{f'-1} - 2\frac{f'}{f-1} = 0.$$

By integration, we get $f' - 1 = \ell(f-1)^2$. We rewrite this in the form

$$\frac{f'}{f-1-b} - \frac{f'}{f-1+b} = 2b\ell, \tag{2.35}$$

where $b^2\ell = -1$. Integrating this once we arrive at the conclusion (2.1).

Case II. $G \not\equiv 0$. From (2.32), (2.33) and (2.34) we see that

$$\begin{aligned} N(r, f) - \bar{N}_{(2}(r, f) &\leq N\left(r, \frac{1}{G}\right) \leq -m\left(r, \frac{1}{G}\right) + T(r, G) + O(1) \\ &\leq -m\left(r, \frac{1}{G}\right) + N_2\left(r, \frac{1}{f}\right) + \bar{N}_{(3}\left(r, \frac{1}{f'-1}\right) + S(r, f). \end{aligned} \tag{2.36}$$

By rewriting (2.29) we have

$$f = \frac{1}{G}\left(\frac{f''}{f'-1} - 2\frac{f'}{f-1}\right).$$

Therefore

$$\begin{aligned} m(r, f) &\leq m\left(r, \frac{1}{G}\right) + m\left(r, \frac{f''}{f'-1}\right) + m\left(r, \frac{f'}{f-1}\right) + O(1) \\ &\leq m\left(r, \frac{1}{G}\right) + S(r, f). \end{aligned}$$

Combining this with (2.36) we have

$$T(r, f) \leq N_2\left(r, \frac{1}{f}\right) + \bar{N}_{(3}\left(\frac{1}{f'-1}\right) + \bar{N}_{(2}(r, f) + S(r, f). \tag{2.37}$$

From (2.37) and (2.36), we obtain

$$\begin{aligned} N\left(r,\frac{1}{f'-1}\right) &\le T(r,f') + O(1) = m(r,f') + N(r,f') + O(1) \\ &\le m\left(r,\frac{f'}{f}\right) + m(r,f) + N(r,f) + \bar{N}(r,f) + O(1) \\ &\le T(r,f) + \bar{N}(r,f) + S(r,f) \\ &\le 2N_2\left(r,\frac{1}{f}\right) + 2\bar{N}_{(3}\left(\frac{1}{f'-1}\right) + \bar{N}_{(2}(r,f) + S(r,f). \qquad (2.38) \end{aligned}$$

Set

$$W = \frac{1}{f}\left(\frac{f''}{f'-1} - 3\frac{f'}{f-1}\right). \qquad (2.39)$$

Proceeding as above, we have

$$m(r,W) = S(r,f), \qquad (2.40)$$

$$W(z_3) = O(1), \qquad (2.41)$$

$$W(z) = O((z-z_\infty)^{p-1}), \qquad (2.42)$$

where z_3 is a zero of $f'-1$ with multiplicity 3 and z_∞ is a pole of f with multiplicity p (≥ 1). Thus

$$N(r,W) \le N_2\left(r,\frac{1}{f}\right) + \bar{N}_{2)}\left(\frac{1}{f'-1}\right) + \bar{N}_{(4}\left(\frac{1}{f'-1}\right).$$

Together with (2.40) we find

$$T(r,W) \le N_2\left(r,\frac{1}{f}\right) + \bar{N}_{2)}\left(\frac{1}{f'-1}\right) + \bar{N}_{(4}\left(\frac{1}{f'-1}\right) + S(r,f). \qquad (2.43)$$

If $W \equiv 0$, then

$$\frac{f''}{f'-1} - 3\frac{f'}{f-1} = 0.$$

Therefore, we get $f'-1 = c(f-1)^3$. This imply that

$$N(r,f) = 0, \qquad (2.44)$$

and $m(r,f') = 3m(r,f) + O(1)$. Hence $m(r,f) = S(r,f)$. This together with (2.44) gives the contradiction $T(r,f) = S(r,f)$. Therefore $W \not\equiv 0$. From this, (2.42) and (2.43) we see that

$$\begin{aligned} \bar{N}_{(2}(r,f) &\le N\left(r,\frac{1}{W}\right) \le T(r,W) + O(1) \\ &\le N_2\left(r,\frac{1}{f}\right) + \bar{N}_{2)}\left(r,\frac{1}{f'-1}\right) + \bar{N}_{(4}\left(r,\frac{1}{f'-1}\right) + S(r,f). \qquad (2.45) \end{aligned}$$

It follows from (2.7), (2.38) and (2.45) that

$$N\left(r, \frac{1}{f-1}\right) = \bar{N}\left(r, \frac{1}{f'-1}\right) \leq 3N_2\left(r, \frac{1}{f}\right) + S(r,f). \tag{2.46}$$

Also, from (2.37), (2.45) and (2.7) we find that

$$m\left(r, \frac{1}{f-1}\right) \leq 2N_2\left(r, \frac{1}{f}\right) + \bar{N}_{(4}\left(r, \frac{1}{f'-1}\right) + S(r,f). \tag{2.47}$$

Set

$$L = \frac{f''}{f(f-1)}. \tag{2.48}$$

It is clear that

$$m(r,L) \leq m\left(r, \frac{f''}{f'}\left(\frac{f'}{f(f-1)}\right)\right) = S(r,f). \tag{2.49}$$

If z_∞ is a pole of f with multiplicity p (≥ 1), then from (2.48) we see that

$$L(z) = O((z - z_\infty)^{p-2}). \tag{2.50}$$

Also, if z_q is a zero of $f' - 1$ with multiplicity q (≥ 2), then from (2.48) we get

$$L(z) = O((z - z_q)^{q-2}). \tag{2.51}$$

Therefore from (2.48), (2.50) and (2.51) we conclude that

$$N(r,L) \leq N_2\left(r, \frac{1}{f}\right) + N_{1)}(r,f).$$

Together with (2.49) we have

$$T(r,L) \leq N_2\left(r, \frac{1}{f}\right) + N_{1)}(r,f) + S(r,f). \tag{2.52}$$

If $L \equiv 0$, then f is a linear function. So f and f' can not share 1 DM which contradicts the condition of Theorem 2.1. Next we assume that $L \not\equiv 0$. From this, (2.51) and (2.52) we see that

$$\begin{aligned} N_{(3}\left(r, \frac{1}{f'-1}\right) - 2\bar{N}_{(3}\left(r, \frac{1}{f'-1}\right) &\leq N\left(r, \frac{1}{L}\right) \leq T(r,L) + O(1) \\ &\leq N_2\left(r, \frac{1}{f}\right) + N_{1)}(r,f) + S(r,f). \end{aligned}$$

That is

$$\begin{aligned} N_{(3}\left(r, \frac{1}{f'-1}\right) + \bar{N}_{(2}(r,f) \leq N_2\left(r, \frac{1}{f}\right) &+ 2\bar{N}_{(3}\left(r, \frac{1}{f'-1}\right) \\ &+ \bar{N}(r,f) + S(r,f). \end{aligned} \tag{2.53}$$

Hence from this and (2.36) we obtain

$$\bar{N}_{(4}\left(r, \frac{1}{f'-1}\right) + \bar{N}_{(2}(r, f) \le 2N_2\left(r, \frac{1}{f}\right) + S(r, f),$$

and eliminating $\bar{N}_{(2}(r, f)$ between this and (2.37) gives

$$m\left(r, \frac{1}{f-1}\right) + \bar{N}_{(4}\left(r, \frac{1}{f'-1}\right) \le 3N_2\left(r, \frac{1}{f}\right) + S(r, f), \qquad (2.54)$$

and eliminating $\bar{N}_{(4}(r, \frac{1}{f'-1})$ between (2.54) and (2.47) leads to

$$m\left(r, \frac{1}{f-1}\right) \le \frac{5}{2}N_2\left(r, \frac{1}{f}\right) + S(r, f).$$

Combining this with (2.46) we will arrive at the conclusion (2.3). This completes the proof of Theorem 2.1. □

Remark 2.2. *From (2.1) we find that*

(1) If $\ell = -1$, then $b = \pm 1$. Hence (2.1) becomes $f(z) = \frac{2a}{1-ce^{-2z}}$. This is (1.2).

(2) If $c = 1$, then $f(z) = a[1 - b\ coth(b\ell z)]$.

(3) If $c = -1$, then $f(z) = a[1 - b\ tanh(b\ell z)]$.

(4) If $b \ne \pm 1$, then $T(r, f) = N(r, \frac{1}{f}) + S(r, f)$.

(5) $N(r, \frac{1}{f'}) = 0$.

From Theorem 2.1 and Remarks 2.2 (3), we deduce readily the following corollaries:

Corollary 2.3. *Let f be a nonconstant meromorphic function. If f and f' share the value a $(\ne 0, \infty)$ DM and if $\bar{N}(r, \frac{1}{f'}) = S(r, f)$, then f is given as (2.1).*

Corollary 2.4. *Let f be a nonconstant meromorphic function. If f and f' share the value a $(\ne 0, \infty)$ DM and if $\bar{N}(r, \frac{1}{f}) = S(r, f)$, then f is given as (1.2).*

It is obvious that Corollary 2.3 is extension and improvement for Theorem 1.2 and Corollary 2.4 is improvement for Theorem 1.2.

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