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B-Statistically A-Summability

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Abstract : Recently, Edely and Mursaleen [1] defined statistically A-summability of a sequence $x = (x_k)$. In this paper we define B-statistically A-summability of x, that is, x is said to be B-statistically A-summable if Ax is B-statistically convergent; where A and B are non-negative regular matrices. We study here other related concepts and provide some interesting examples.

Keywords : statistical convergence; A-statistical convergence; A-summability; statistical A-summability; B-statistical A-summability.
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1 Introduction and Preliminaries

Let l_∞ and c denote the spaces of all bounded and convergent sequences, respectively. Let $A = (a_{nk})_{n,k=1}^{\infty}$ be an infinite matrix and $x = (x_k)_{k=1}^{\infty}$ be a number sequence. By $Ax = (A_n(x))$, we denote the A-transform of the sequence $x = (x_k)$, where $A_n(x) = \sum_{k=1}^{\infty} a_{nk}x_k$. For any two sequence spaces X and Y, we denote by (X, Y) a class of matrices A such that $Ax \in Y$ for $x \in X$, provided that the series $\sum_{k=1}^{\infty} a_{nk}x_k$ converges for each n. If in addition $\lim Ax = \lim x$, then we denote such a class by $(X, Y)_{reg}$. A matrix A is called regular, i.e. $A \in (c, c)_{reg}$ if $A \in (c, c)$ and $\lim_n A_n(x) = \lim_k x_k$ for all $x \in c$. The well-known necessary and sufficient conditions (Silverman-Toeplitz) for A to be regular are

(i) $||A|| = \sup_n \sum_k |a_{nk}| < \infty$;

(ii) $\lim_n a_{nk} = 0$, for each k;

(iii) $\lim_n \sum_k a_{nk} = 1$.

The concept of statistical convergence was first introduced by Fast [2]. In 1953, the concept arises as an example of convergence in density as introduced by Buck [3]. Schoenberg [4] studied statistical convergence as a summability method and Zygmund [5] established a relation between it and strong summability. This idea has grown a little fast after the paper of Salat [6], Fridy [7], Connor [8], Kolk [9], Mursaleen [10], Mursaleen and Edely [11] and many others.

Let $K \subseteq \mathbb{N}$, the set of natural numbers. Then the *natural density* of K is defined by

$$\delta(K) = \lim_n \frac{1}{n} \mid \{k \le n : k \in K\} \mid,$$

if the limit exists, where the vertical bars denote the cardinality of the enclosed set. Notice that

$$\delta(K) = \lim_n \frac{1}{n} \mid K_n \mid = \lim_n (C_1 \chi_K)_n,$$

where $C_1 = (C, 1)$ is the Cesàro matrix of order 1 and χ_K denotes the characteristic sequence of K given by

$$(\chi_K)_i = \begin{cases} 0, & \text{if } \ i \notin K, \\ 1, & \text{if } \ i \in K. \end{cases}$$

A sequence $x = (x_k)$ of real numbers is said to be *statistically convergent* to the number L provided that for every $\epsilon > 0$ the set $K(\epsilon) = \{k \in \mathbb{N} : |x_k - L| \ge \epsilon\}$ has natural density zero; in this case we write $st\text{-}\lim x = L$. By the symbol st we denote the set of all statistically convergent sequences. Notice that every convergent sequence is statistically convergent to the same limit but not conversely, for example let

$$x_k = \begin{cases} k; & \text{if } \ k \ \text{ is a square}, \\ 0; & \text{otherwise}, \end{cases}$$

here x is unbounded even so it is statistically convergent to zero.

Freedmann and Sember [12] generalized the natural density by replacing C_1 with an arbitrary non-negative regular matrix A. A subset K of $\mathbb{N}$ has A-density if

$$\delta_A(K) = \lim_n \sum_{k \in K} a_{nk}$$

exists. Connor [13, 14] and Kolk [9] extended the idea of statistical convergence to A-statistical convergence by using the notion of A-density.

A sequence x is said to be A-*statistically convergent* to L if $\delta_A(K(\epsilon)) = 0$ for every $\epsilon > 0$. In this case we write $st_A\text{-}\lim x_k = L$. By the symbol st_A we denote the set of all A-statistically convergent sequences.

The idea of statistical $(C, 1)$-summability was introduce by Moricz [15]. In [1], Edely and Mursaleen generalized these statistical summability methods by defining the statistical A-summability and studied its relationship with A-statistical convergence. For more general case of A-statistical convergence, we refer to [16].

Let $A = (a_{ij})$ be a non-negative regular matrix. A sequence x is said to be *statistically A-summable* to L if for every $\epsilon > 0$, $\delta(\{i \leq n : |y_i - L| \geq \epsilon\}) = 0$, i.e.

$$\lim_n \frac{1}{n}|\{i \leq n : |y_i - L| \geq \epsilon\}| = 0,$$

where $y_i = A_i(x)$. Thus x is statistically A-summable to L if and only if Ax is statistically convergent to L. In this case we write $L = (A)_{st}\text{-}\lim x = st\text{-}\lim Ax$. By $(A)_{st}$ we denote the set of all statistically A-summable sequences.

2 Some New Definitions and Examples

In this section we define B-statistically A-summable for a non-negative regular matrices A and B.

Definition 2.1. Let $A = (a_{ij})$ and $B = (b_{nk})$ be two non-negative regular matrices. A sequence $x = (x_k)$ of real numbers is said to be *B-statistically A-summable* to L, if for every $\epsilon > 0$, the set $K_\epsilon = \{i : |y_i - L| \geq \epsilon\}$ has B-density zero, thus

$$\delta_B(K_\epsilon) = \lim_n \sum_{k \in K_\epsilon} b_{nk} = \lim_n (B\chi_{K_\epsilon}) = \lim_n \sum_k b_{nk}\chi_{K_\epsilon}(k) = 0,$$

where $y_i = A_i(x) = \sum_j a_{ij}x_j$. In this case we denote by $L = (A)_{st_B}\text{-}\lim x = st_B\text{-}\lim Ax$. The set of all B-statistically A-summable sequences will be denoted by $(A)_{st_B}$.

Remark 2.2.

1. *If $B = (C, 1)$ matrix, then $(A)_{st_B}$ is reduced to the set of statistically A-summable sequences due to Edely and Mursaleen [1].*
2. *If $A = B = (C, 1)$ matrix, then $(A)_{st_B}$ is reduced to the set of statistically $(C, 1)$-summable sequences due to Moricz [15].*
3. *If a sequence is convergent then it is B-statistically A-summable, since Ax converge and has B-density zero, but not conversely.*
4. *The spaces st, st_B, $(A)_{st}$ and $(A)_{st_B}$ are not comparable, even if $A = B\,(\neq (C, 1))$.*
5. *If a sequence is A-summable then it is B-statistically A-summable.*
6. *If a sequence is bounded and A-statistically convergent, then it is A-summable and hence statistically A-summable (see [1, Theorem 2.1]) and B-statistically A-summable but not conversely.*

Example 2.3.

1. *Let us define $A = (a_{ij})$, $B = (b_{nk})$ and $x = (x_k)$ by*

$$a_{ij} = \begin{cases} 1; & \text{if } j = i^2, \\ 0; & \text{otherwise}, \end{cases}$$

$$b_{nk} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & . & . & . & . & . & . & . & . \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & . & . & . & . & . \\ \frac{1}{3} & 0 & \frac{1}{3} & 0 & \frac{1}{3} & 0 & 0 & 0 & . & . & . & . & . \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & . & . \\ \frac{1}{5} & 0 & \frac{1}{5} & 0 & \frac{1}{5} & 0 & \frac{1}{5} & 0 & \frac{1}{5} & 0 & 0 & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . \end{pmatrix},$$

and

$$x_k = \begin{cases} 1; & if \;\; k \;\; \text{is an odd}, \\ 0; & if \;\; k \;\; \text{is an even}. \end{cases}$$

Then

$$\sum_{j=1}^{\infty} a_{ij} x_j = \begin{cases} 1; & \text{if } \; i \;\; \text{is an odd}, \\ 0; & \text{if } i \;\; \text{is an even}. \end{cases}$$

Here $x \notin st$, $x \notin (A)_{st}$, $x \notin st_A$, $x \notin (A)_{st_A}$ *but* x *is* B*-statistically* A*-summable to* 1, *since* $\delta_B \{i : |y_i - 1| \geq \epsilon\} = 0$. *On the other hand we can see that* x *is* B*-summable and hence* x *is* B*-statistically* B*-summable,* A*-statistically* B*-summable,* B*-statistically convergent and statistically* B*-summable.*

2. *Let* $A = (a_{nk})$ *and* $x = (x_k)$ *be defined as*

$$a_{nk} = \begin{cases} 1/2; & \text{if } n \text{ is a nonsquare and } k = n^2, k = n^2 + 1, \\ 1; & \text{if } n \text{ is a square and } \; k = n^2, \\ 0; & \text{otherwise}, \end{cases}$$

and

$$x_k = \begin{cases} 1; & \text{if } k = n^2, \\ k; & \text{otherwise}, \end{cases}$$

then

$$\sum_{k=1}^{\infty} a_{nk} x_k = \begin{cases} 1/2; & \text{if } \; n \text{ is a nonsquare}, \\ n^2; & \text{if } \; n \text{ is a square}. \end{cases}$$

Here x *is unbounded but statistically* A*-summable to* $1/2$, *also* x *is statistically* A*-bounded, since* $\delta\{i : |y_i| > \frac{1}{2}\} = 0$ *but* x *is not* A*-summable. Now if we take* B *as in Example 2.3(1), we see that* x *is not* B*-statistically* A*-summable.*

3. *Let* $A = (a_{nk})$ *and* $x = (x_k)$ *be defined as Example 2.3(2) and define the matrix* B *as*

$$b_{nk} = \begin{cases} 1; & n = k, n \text{ is a nonsquare}, \\ 1; & n = k - 1, n \text{ is a square}, \\ 0; & \text{otherwise}, \end{cases}$$

here x *is* B*-statistically* A*-summable to* $\frac{1}{2}$.

3 B-Statistically A-Cluster and Limit Points

The following definitions are analogue of statistical limit points, cluster points defined by Fridy [17].

Definition 3.1. The number γ is said to be *B-statistically A-cluster point* of a sequence x if for every $\varepsilon > 0$ the set $\{i : \mid y_i - \gamma \mid < \varepsilon\}$ does not have B-density zero.

Definition 3.2. The number λ is said to be *B-statistically A-limit point* of a sequence x if there is a subsequence of (y_i) which converges to λ such that whose indices do not have B-density zero.

We denote by $\Gamma_{Ax}(B)$ the set of B-statistically A-cluster points and by $\Lambda_{Ax}(B)$ the set of B-statistically A-limit points of x.

Like Fridy and Orhan [18], Demirci [19] has defined A-statistical limit superior (inferior) and statistical bounded. Here we define B-statistically A-limit superior (inferior) and B-statistically A-bounded.

Definition 3.3. Let us write

$$G = \{g \in \mathbb{R} : \delta_B(\{i : y_i > g\}) \neq 0\} \text{ and } F = \{f \in \mathbb{R} : \delta_B(\{i : y_i < f\}) \neq 0\},$$

for a number sequence $x = (x_k)$. Then we define the B-statistically A-limit superior and B-statistically A-limit inferior of x as follows:

$$st_B - \limsup Ax = \begin{cases} \sup G, & G \neq \varnothing, \\ -\infty, & G = \varnothing, \end{cases}$$

and

$$st_B - \liminf Ax = \begin{cases} \inf F, & F \neq \varnothing, \\ +\infty, & F = \varnothing. \end{cases}$$

Definition 3.4. The number sequence x is said to be *B-statistically A-bounded* if there is a number M such that $\delta_B(\{i : \mid y_i \mid > M\}) = 0$.

Example 3.5.

1. From Example 2.3(1) we see that $\Lambda_{Ax}(B) = \{1\} = \Gamma_{Ax}(B)$, also st_B-$\limsup Ax = 1 = st_B$-$\liminf Ax$, since $G = (-\infty, 1)$ and $F = (1, \infty)$. Moreover x is bounded and Ax is also bounded and hence x is B-statistically A-bounded.

2. From Example 2.3(2) we see that $\Lambda_{Ax}(B) = \{\frac{1}{2}\} = \Gamma_{Ax}(B)$, also st_B-$\limsup Ax = \infty$, st_B-$\liminf Ax = \frac{1}{2}$, since $G = \mathbb{R}$ and $F = \left(\frac{1}{2}, \infty\right)$. Moreover x is not B-statistically A-bounded, since $\delta_B\left(\{i : |y_i| > M\}\right) \neq 0$ for every number M.

3. From Example 2.3(3) we see that $\Lambda_{Ax}(B) = \{\frac{1}{2}\} = \Gamma_{Ax}(B)$, also st_B-$\limsup Ax = st_B$-$\liminf Ax = \frac{1}{2}$, since $G = \left(-\infty, \frac{1}{2}\right)$ and $F = \left(\frac{1}{2}, \infty\right)$. Moreover x is B-statistically A-bounded, since the set $\left\{i : |y_i| > \frac{1}{2}\right\}$ has B-density zero.

The following result can be proved by straightforward least upper bound argument.

Theorem 3.6.

(a) If $l_1 = st_B\text{-}\limsup Ax$ *is finite, then for every positive number* ε

$$\delta_B(\{i : y_i > l_1 - \varepsilon\}) \neq 0 \text{ and } \delta_B(\{i : y_i > l_1 + \varepsilon\}) = 0. \tag{3.1}$$

Conversely, if (3.1) *holds for every* $\varepsilon > 0$ *then* $l_1 = st_B\text{-}\limsup Ax$.

(b) If $l_2 = st_B\text{-}\liminf Ax$ *is finite, then for every positive number* ε

$$\delta_B(\{i : y_i < l_2 + \varepsilon\}) \neq 0 \text{ and } \delta_B(\{i : y_i < l_2 - \varepsilon\}) = 0. \tag{3.2}$$

Conversely, if (3.2) *holds for every* $\varepsilon > 0$ *then* $l_2 = st_B\text{-}\liminf Ax$.

From the definition we see that the above theorem can be interpreted as saying that st_B-$\limsup Ax$ and st_B-$\liminf Ax$ are the greatest and least B-statistical A-cluster points of x.

Note that B-statistical A-boundedness implies that st_B-$\limsup Ax$ and st_B-$\liminf Ax$ are finite, so that properties (3.1) and (3.2) of Theorem 3.6 hold good.

Now we produce B-analogue of the results of Fridy and Orhan [18]. By $\delta_B(K) \neq 0$ we mean that either $\delta_B(K) > 0$ or K fails to have B-density.

Theorem 3.7. *For any real number sequence* x

$$st_B\text{-}\liminf Ax \leq st_B\text{-}\limsup Ax.$$

Proof. First consider the case in which st_B-$\limsup Ax = -\infty$. This implies that $G = \emptyset$. Therefore for every $g \in \mathbb{R}$, $\delta_B(\{i : y_i > g\}) = 0$, which implies that $\delta_B(\{i : y_i \leq g\}) = 1$. So that for every $f \in \mathbb{R}$, $\delta_B(\{k : x_k < f\}) \neq 0$. Hence st_B-$\liminf Ax = -\infty$.

Now consider st_B-$\limsup x = +\infty$. This implies that for every $g \in \mathbb{R}, \delta_B(\{i : y_i > g\}) \neq 0$. This means that $\delta_B(\{i : y_i \leq g\}) = 0$. Therefore for every $f \in \mathbb{R}, \delta_B(\{i : y_i < f\}) = 0$, which implies that $F = \emptyset$. Hence st_B-$\liminf Ax = +\infty$.

Next we assume that $l_1 = st_B$-$\limsup Ax < \infty$ and let $l_2 = st_B$-$\liminf Ax$. Given $\varepsilon > 0$ we show that $l_1 + \varepsilon \in F$, so that $l_2 \leq l_1 + \varepsilon$. By Theorem 3.6, $\delta_B(\{i : y_i > l_1 + \varepsilon/2\}) = 0$, since $l_1 = lub\, G$. This implies that $\delta_B(\{i : y_i \leq l_1 + \epsilon/2\}) = 1$, which in turn gives $\delta_B(\{i : y_i < l_1 + \varepsilon\}) = 1$. Hence $l_1 + \varepsilon \in F$ and so that $l_2 \leq l_1 + \varepsilon$, i.e. $l_2 \leq l_1$ since ε was arbitrary. □

Remark 3.8. *For any number sequence* x*,*

$$\liminf x \leq \liminf Ax \leq st_B\text{-}\liminf Ax \leq st_B\text{-}\limsup Ax \leq \limsup Ax \leq \limsup x.$$

Theorem 3.9. *The* B*-statistically* A*-bounded sequence* x *is* B*-statistically* A*-summable if and only if*

$$st_B\text{-}\liminf Ax = st_B\text{-}\limsup Ax.$$

Proof. Let $l_1 = st_B\text{-}\limsup Ax$ and $l_2 = st_B\text{-}\liminf Ax$. First assume that $st_B\text{-}\lim Ax = l$ and $\varepsilon > 0$. Then $\delta_B(\{i : \mid y_i - l \mid \geq \varepsilon\}) = 0$, so that $\delta_B(\{i : y_i > l + \varepsilon\}) = 0$, which implies that $l_1 \leq l$. Also $\delta_B(\{i : y_i < l - \varepsilon\}) = 0$, which implies that $l \leq l_2$. By Theorem 3.7, we finally have $l_1 = l_2$.

Conversely, suppose that $l_1 = l_2 = l$ and x are B-statistically A-bounded. Then for $\varepsilon > 0$, by Theorem 3.6, we have $\delta_B(\{i : y_i > l + \varepsilon/2\}) = 0$, and $\delta_B(\{i : y_i < l - \varepsilon/2\}) = 0$. Hence $st_B\text{-}\lim Ax = l$. □

4 Relation Between $(A)_{st}$, $(A)_{st_B}$ and B-Summability

Now we establish some relations between B-summability, B-statistically A-summability and statistically A-summability.

Theorem 4.1. *Let x be bounded sequence and x is statistically A-summable to l. Then Ax is B-summable to l if B is a non-negative regular matrix satisfies*

$$\lim_n \sum_{k\in K} b_{nk} = 0 \text{ for every set } K \subseteq \mathbb{N}, \text{ such that } \delta(K) = 0. \tag{4.1}$$

Proof. Let $x \in l_\infty$. Since A is regular then $Ax \in l_\infty$, and let x be statistically A-summable to l and $K_\epsilon = \{i : |y_i - l| \geq \epsilon\}$. Then

$$\begin{aligned} |(By)_n - l| &\leq \left|\sum_{k\notin K} b_{nk}(y_k - l)\right| + \left|\sum_{k\in K} b_{nk}(y_k - l)\right| \\ &\leq \epsilon \sum_{k\notin K} b_{nk} + \sup_k |y_k - l| \sum_{k\in K} b_{nk}. \end{aligned}$$

Now by using the definition of statistically A-summability and the condition (4.1) we get $\lim_n |(Bx)_n\text{-}l| = 0$, since ϵ was arbitrary. □

Remark 4.2. *We can not replace $x \in l_\infty$ by weaker assumption of a statistical bounded.*

Example 4.3. *Let*

$$a_{nk} = \begin{cases} 1; & \text{if } n \text{ is nonsquare}, k = n^2, \\ \frac{1}{2}; & \text{if } n \text{ is a square, } k = n^2, k = n^2+1, \\ 0; & \text{otherwise}, \end{cases}$$

and

$$x_k = \begin{cases} 0; & \text{if } k = n^2, \\ k; & \text{if } k = n^2+1, \\ 1; & \text{otherwise}, \end{cases}$$

then

$$\sum_{k=1}^{\infty} a_{nk}x_k = \begin{cases} 0; & \text{if } n \text{ is nonsquare}, \\ \frac{1}{2}\left(n^2+1\right); & \text{if } n \text{ is a square.} \end{cases}$$

Here x is statistically bounded since the set $\{k : |x_k| > 1\}$ has density zero, also x is statistically A-summable to zero. Take $B = C_1$ the Cesaro matrix of order 1. It is clear that C_1 satisfies all the conditions of B in Theorem 4.1, but Ax is not B-summable.

Corollary 4.4. *Let x be bounded sequence and x be B-statistically A-summable to l. Then Ax is B-summable to l.*

Theorem 4.5. *If the number sequence Ax is bounded above and B-summable to the number $l = st_B\text{-}\limsup Ax$, then x is B-statistically A-summable to l.*

Proof. Suppose that Ax is not B-statistically A-summable to l. Then $st_B\text{-}\liminf Ax < l$, so there is a number $M < l$ such that $\delta_B(\{i : y_i < M\}) \neq 0$. Let $K_1 = \{i : y_i < M\}$. Then for every $\varepsilon > 0$, $\delta_B(\{i : y_i > l + \varepsilon\}) = 0$. Write $K_2 = \{i : M \leq y_i \leq \ell + \varepsilon\}$, $K_3 = \{i : y_i > \ell + \varepsilon\}$, and let $G = \sup_i y_i < \infty$. Since $\delta_B(K_1) \neq 0$, there are many n such that

$$\limsup_n \sum_{k\in K_1} b_{nk} \geq d > 0,$$

and for each n

$$\sum_{k=1}^{\infty} | \, b_{nk} y_k \, | < \infty.$$

Now

$$\begin{aligned}
\sum_{k=1}^{\infty} b_{nk} y_k &= \left(\sum_{k\in K_1} + \sum_{k\in K_2} + \sum_{k\in K_3} \right) b_{nk} y_k \\
&\leq M \sum_{k\in K_1} b_{nk} + (l+\varepsilon) \sum_{k\in K_2} b_{nk} + G \sum_{k\in K_3} b_{nk} \\
&= M \sum_{k\in K_1} b_{nk} + (l+\varepsilon) \sum_{k=1}^{\infty} b_{nk} - (l+\varepsilon) \sum_{k\in K_1} b_{nk} + O(1) \\
&= - \sum_{k\in K'} b_{nk} \left(-M + (l+\varepsilon) \right) + (l+\varepsilon) \sum_{k=1}^{\infty} b_{nk} + O(1) \\
&\leq l \sum_{k=1}^{\infty} b_{nk} - d\,(l - M) + \varepsilon \left(\sum_{k=1}^{\infty} b_{nk} - d \right) + O(1).
\end{aligned}$$

Since ε is arbitrary, it follows that

$$\liminf By \leq l - d\,(l - M) < l.$$

Hence Ax is not B-summable to l. □

The following is the dual statement of Theorem 4.5.

Theorem 4.6. *If the number sequence Ax is bounded below and B-summable to the number $l = st_B\text{-}\liminf Ax$, then x is B-statistically A-summable to l.*

Remark 4.7. *The above Theorems 4.5 and 4.6, the boundedness of Ax can not be omitted or even replaced by the B-statistical boundedness.*

Example 4.8. *Define the sequence $x = (x_k)$ by*

$$x_k = \begin{cases} 1, \text{ if } k \text{ is an even nonsquare,} \\ 0, \text{ if } k \text{ is an odd nonsquare,} \\ k, \text{ if } k \text{ is an even square,} \\ 1, \text{ if } k \text{ is an odd square,} \end{cases}$$

and

$$a_{nk} = \begin{cases} 1, \text{ if } n \text{ is nonsquare and } k = n^2 + 1, \\ 1, \text{ if } n \text{ is a square and } k = n^2, \\ 0, \text{ otherwise,} \end{cases}$$

then

$$\sum_k a_{nk} x_k = \begin{cases} 1, & \text{if is } n \text{ an odd nonsquare,} \\ 1, & \text{if } n \text{ is an odd square,} \\ 0, & \text{if } n \text{ is an even square,} \\ n^2, & \text{if } n \text{ is an even square.} \end{cases}$$

Now let us define a matrix $B = (b_{nk})$ as

$$b_{nk} = \begin{cases} 1, \text{ if } n = k, \ n \text{ is an even nonsquare,} \\ 1, \text{ if } n = k - 2, \ n \text{ is an even square,} \\ 1, \text{ if } n = k, \ n \text{ is an odd square,} \\ 1, \text{ if } n = k + 1, \ n \text{ is an odd and } (n-1) \text{ nonsquare,} \\ 1, \text{ if } n = k - 1, \ n \text{ is an odd and } (n-1) \text{ square,} \\ 0, \text{ otherwise.} \end{cases}$$

We can see that Ax is B-statistically bounded since the set $\{i : |y_i| > 1\}$ has B-density zero, also Ax is B-summable to 1, but Ax is not B-statistically summable to any number l.

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