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Submersion of CR-Submanifolds of Nearly Trans-Sasakian Manifold[1]

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Abstract : In this paper, we discuss the submersion of CR-submanifolds of a nearly trans-Sasakian manifold. We show that if $\pi : M \longrightarrow M'$ is a submersion of CR-submanifold M of a nearly trans-Sasakian manifold $\bar{M}$ then M' is a nearly trans-Sasakian manifold. Also we derive some curvature relations by means of which we deduce the compactness of M' under slight condition.

Keywords : CR-submanifolds; Nearly trans-Sasakian manifold; Submersion.
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1 Introduction

In 1981, Kobayashi [1] initiated the study of submersion of a CR-submanifolds of Sasakian manifold whereas Bejancu [2, 3] introduced the notion of CR-submanifolds of a Kaehler manifold. Moreover, Papaghuic [4] studied the submersion of semi-invariant submanifolds of a Sasakian manifold. Later in 1985, Oubina [5] gave a new class of almost contact Riemannian manifold known as trans-Sasakian manifold. A trans-Sasakian manifold is a generalization of both α-Sasakian and β-Kenmotsu manifolds. Based on the fundamental equations of submersion given by O'Neill [6]. In [7], Matsumoto et al. studied the submersion of semi-invariant submanifolds of trans-Sasakian manifold. In 2002, Al-Solamy [8] obtained some

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results regarding CR-submanifolds of nearly trans-Sasakian manifold. As nearly trans-Sasakian structure is a natural generalization of trans-Sasakian structure, in this article we study the submersion of CR-submanifolds of nearly trans-Sasakian manifold.

2 Preliminaries

Let $\bar{M}$ be an n-dimensional almost contact metric manifold with almost contact metric structure (ϕ, ξ, η, g) [9].

$$\phi^2 = -1 + \eta \otimes \xi, \phi \circ \xi = 0, \eta \circ \phi = 0, \eta(\xi) = 1,$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y),$$

for any vector fields X, Y on $\bar{M}$. An almost contact metric structure (ϕ, ξ, η, g) on $\bar{M}$ is called nearly trans-Sasakian if

$$(\bar{\nabla}_X \phi)Y + (\bar{\nabla}_Y \phi)X = \alpha\,[2g(X, Y)\xi - \eta(Y)X - \eta(X)Y] - \beta\,[\eta(Y)\phi X + \eta(X)\phi Y]$$

for functions α and β on $\bar{M}$, and we say that the trans-Sasakian structure is of type (α, β).

Definition 2.1. An m-dimensional Riemannian submanifold M of a nearly trans-Sasakian manifold $\bar{M}$ is called a contact CR-submanifold if ξ is tangent to M and there exist on M a differential distribution $D : x \longrightarrow D_x \subset T_xM$ such that (i) D_x is invariant under ϕ i.e. $\phi D_x \subset D_x$ for each $x \in M$, (ii) the orthogonal complementary distribution $D^\perp : x \longrightarrow D^\perp \subset T_xM$ of the distribution D on M is totally real, i.e. $\phi D^\perp \subset T_x^\perp M$ (iii) $TM = D \oplus D^\perp \oplus \{\xi\}$, where T_xM, $T_x^\perp M$ are the tangent space and the normal space of M at x respectively and $\oplus$ denotes the orthogonal direct sum.

We call D (resp. $D^\perp$) the horizontal (resp. vertical) distribution. We denote by g the metric tensor field of $\bar{M}$ as well as that induced on M. Let $\bar{\nabla}$ (resp. ∇) be the covariant differentiation with respect to the Levi-Civita connection on $\bar{M}$ (resp. M). The Gauss and Weingarten formulas for M are respectively given by

$$\bar{\nabla}_X Y = \nabla_X Y + h(X, Y) \tag{2.1}$$

and

$$\bar{\nabla}_X N = -A_N X + \nabla_X^\perp N \tag{2.2}$$

for $X, Y \in TM$, $N \in T^\perp M$, where h(resp.A) is the second fundamental form (resp. tensor) of M in $\bar{M}$, and $\nabla^\perp$ denotes the operator of the normal connection. Moreover, we have

$$g(h(X, Y), N) = g(A_N X, Y).$$

The projection of TM to D and $D^\perp$ are denoted by h and v respectively i.e., for any $X \in TM$ we have

$$X = hX + vX + \eta(X).$$

The normal bundle to M has the decomposition

$$T^{\perp}M = \phi D^{\perp} \oplus n_1 \text{ and } g(\phi D^{\perp}, n_1) = \{0\}.$$

For any $U \in T^{\perp}M$, we put

$$U = nU + mU \tag{2.3}$$

where $nU \in \phi D^{\perp}$, $mU \in n_1$. From the above equation we have

$$\phi U = \phi nU + \phi mU, \;\; U \in T^{\perp}M, \;\; \phi nU \in D^{\perp}, \;\; \phi mU \in n_1. \tag{2.4}$$

Definition 2.2. Let M be a contact CR-submanifold of a nearly trans-Sasakian manifold $\bar{M}$ and M' be an almost contact metric manifold with the almost contact metric structure (ϕ', ξ', η', g'). Assume that there is a submersion $\pi : M \longrightarrow M'$ such that:

(i) $D^{\perp} = \ker \pi_*$, where $\pi_* \colon TM \longrightarrow TM'$ is the tangent mapping to π.

(ii) $\pi_* \colon D_p \oplus \{\xi\} \longrightarrow T_{\pi(p)}M'$ is an isometry for each $p \in M$ which satisfies: $\pi_* \circ \phi = \phi' \circ \pi_*$; $\eta = \eta' \circ \pi_*$; $\pi_*(\xi_p) = \xi'_{\pi(p)}$, where $T_{\pi(p)}M'$ denotes the tangent space of M' at $\pi(p)$.

A vector field X on M is said to be basic if $X \in D_p \oplus \{\xi\}$ and X is $\pi-$related to a vector field on M' i.e., there exists a vector field $X_* \in TM'$ such that $\pi_*(X_p) = X_{*\pi(p)}$ for each $p \in M$. Note that, by condition (ii) of the above definition we have that the structural vector field ξ is a basic vector field.

Lemma 2.3 ([4]). *Let X, Y be basic vector fields on M. Then*

(i) $g(X, Y) = g'(X_*, Y_*) \circ \pi$;

(ii) the component $h([X, Y]) + \eta([X, Y])\xi$ of $[X, Y]$ is a basic vector field and corresponds to $[X_*, Y_*]$, i.e., $\pi_*(h([X, Y]) + \eta([X, Y])\xi) = [X_*, Y_*]$;

(iii) $[U, X] \in D^{\perp}$ *for any* $U \in D^{\perp}$;

(iv) $h(\nabla_X Y) + \eta(\nabla_X Y)\xi$ is a basic vector field corresponding to $\nabla^*_{X_*} Y_*$, where ∇^* denotes the Levi-Civita connection on M'.

For basic vector fields on M, we define the operator $\tilde{\nabla}^*$ corresponding to ∇^* by setting $\tilde{\nabla}^*_X Y = h(\nabla_X Y) + \eta(\nabla_X Y)\xi$ for $X, Y \in (D \oplus \{\xi\})$.

By (iv) of Lemma 2.3, $\tilde{\nabla}^*_X Y$ is a basic vector field, and we have

$$\pi_*(\tilde{\nabla}^*_X Y) = \nabla^*_{X_*} Y_*. \tag{2.5}$$

Define the tensor field C by

$$\nabla_X Y = \tilde{\nabla}^*_X Y + C(X, Y), \quad X, Y \in (D \oplus \{\xi\}) \tag{2.6}$$

where $C(X,Y)$ is the vertical part of $\nabla_X Y$. It is known that C is skew-symmetric and satisfies

$$C(X,Y) = \frac{1}{2}v[X,Y], \quad X,Y \in (D \oplus \{\xi\}). \tag{2.7}$$

The curvature tensors R, R^* of the connection ∇, ∇^* on M and M' respectively are related by [4]

$$R(X,Y,Z,W) = R^*(X_*,Y_*,Z_*,W_*) - g(C(Y,Z),C(X,W)) + g(C(X,Z),C(Y,W)) + 2g(C(X,Y),C(Z,W)) \tag{2.8}$$

$X,Y,Z,W \in (D \oplus \{\xi\})$, where $\pi_* X = X_*$, $\pi_* Y = Y_*$, $\pi_* Z = Z_*$ and $\pi_* W = W_* \in \chi(M')$. First we have

Proposition 2.4. *Let $\pi : M \longrightarrow M'$ be a submersion of contact CR-submanifold of a nearly trans-Sasakian manifold $\bar{M}$ onto an almost contact metric manifold M'. Then we have*

(i) $$(\tilde{\nabla}^*_X \phi)Y + (\tilde{\nabla}^*_Y \phi)X = \alpha\left[2g(X,Y)\xi - \eta(Y)X - \eta(X)Y\right] - \beta\left[\eta(Y)\phi X + \eta(X)\phi Y\right], \tag{2.9}$$

(ii) $$C(X,\phi Y) + C(Y,\phi X) = 2\phi nh(X,Y), \tag{2.10}$$

(iii) $$nh(X,\phi Y) = -nh(Y,\phi X), \tag{2.11}$$

(iv) $$\phi mh(X,Y) = h(X,\phi Y), \quad \textit{if} \quad mh(X,\phi Y) = mh(Y,\phi X). \tag{2.12}$$

for any $X,Y \in (D \oplus \{\xi\})$.

Proof. For any $X,Y \in (D \oplus \{\xi\})$ and by using Gauss formula (2.1), decomposition (2.3) and equation (2.6) we have

$$\begin{aligned} \bar{\nabla}_X Y &= \nabla_X Y + h(X,Y) \\ &= \nabla_X Y + nh(X,Y) + mh(X,Y) \\ &= \tilde{\nabla}^*_X Y + C(X,Y) + nh(X,Y) + mh(X,Y). \end{aligned} \tag{2.13}$$

Hence

$$\phi\bar{\nabla}_X Y = \phi\tilde{\nabla}^*_X Y + \phi C(X,Y) + \phi nh(X,Y) + \phi mh(X,Y). \tag{2.14}$$

Putting $Y = \phi Y$ in equation (2.13), we obtain

$$\bar{\nabla}_X \phi Y = \tilde{\nabla}^*_X \phi Y + C(X,\phi Y) + nh(X,\phi Y) + mh(X,\phi Y). \tag{2.15}$$

Similarly, we find

$$\phi\bar{\nabla}_Y X = \phi\tilde{\nabla}^*_Y X + \phi C(Y,X) + \phi nh(Y,X) + \phi mh(Y,X) \tag{2.16}$$

and

$$\bar{\nabla}_Y \phi X = \tilde{\nabla}^*_Y \phi X + C(Y,\phi X) + nh(Y,\phi X) + mh(Y,\phi X). \tag{2.17}$$

On the other hand, using the definition of nearly trans-Sasakian manifold we have

$$\begin{aligned}(\bar{\nabla}_X\phi)Y + (\bar{\nabla}_Y\phi)X &= \bar{\nabla}_X\phi Y - \phi\bar{\nabla}_X Y + \bar{\nabla}_Y\phi X - \phi\bar{\nabla}_Y X \\ &= \alpha\,[2g(X,Y)\xi - \eta(Y)X - \eta(X)Y] - \beta\,[\eta(Y)\phi X + \eta(X)\phi Y]\,.\end{aligned} \tag{2.18}$$

Substituting equations (2.14), (2.15), (2.16) and (2.17) in equation (2.18) we get

$$\begin{aligned}&\tilde{\nabla}^*_X\phi Y + C(X,\phi Y) + nh(X,\phi Y) + mh(X,\phi Y) - \phi\tilde{\nabla}^*_X Y - \phi C(X,Y) \\ &-\phi nh(X,Y) - \phi mh(X,Y) + \tilde{\nabla}^*_Y\phi X + C(Y,\phi X) + nh(Y,\phi X) + mh(Y,\phi X) \\ &-\phi\tilde{\nabla}^*_Y X - \phi C(Y,X) - \phi nh(Y,X) - \phi mh(Y,X) \\ &\qquad = \alpha\,[2g(X,Y)\xi - \eta(Y)X - \eta(X)Y] - \beta\,[\eta(Y)\phi X + \eta(X)\phi Y]\,.\end{aligned}$$

Comparing components of $(D \oplus \{\xi\})$, $D^{\perp}$, $\phi D^{\perp}$ and n_1 respectively on both sides we find

$$(\tilde{\nabla}^*_X\phi)Y + (\tilde{\nabla}^*_Y\phi)X = \alpha\,[2g(X,Y)\xi - \eta(Y)X - \eta(X)Y] - \beta\,[\eta(Y)\phi X + \eta(X)\phi Y]\,,$$

$$C(X,\phi Y) + C(Y,\phi X) = 2\phi nh(X,Y),$$

$$nh(X,\phi Y) = -nh(Y,\phi X),$$

$$\phi mh(X,Y) = h(X,\phi Y) \ \text{ if } \ mh(X,\phi Y) = mh(Y,\phi X).$$

□

Proposition 2.5. *Let $\pi : M \longrightarrow M'$ be a submersion of contact CR-submanifold of a nearly trans-Sasakian manifold $\bar{M}$ onto an almost contact metric manifold M'. Then M' is also a nearly trans-Sasakian manifold.*

Proof. Using (i) of the last proposition, we have

$$\begin{aligned}(\tilde{\nabla}^*_X\phi)Y + (\tilde{\nabla}^*_Y\phi)X = &\ \alpha\,[2g(X,Y)\xi - \eta(Y)X - \eta(X)Y] \\ &-\beta\,[\eta(Y)\phi X + \eta(X)\phi Y]\,.\end{aligned}$$

Applying π_* to the above equation and using Lemma 2.3, equation (2.5) and definition of submersion, we derive

$$\begin{aligned}(\tilde{\nabla}^*_{X_*}\phi')Y_* + (\tilde{\nabla}^*_{Y_*}\phi')X_* = &\ \alpha\left[2g'(X_*,Y_*)\xi' - \eta'(Y_*)X_* - \eta'(X_*)Y_*\right] \\ &-\beta\left[\eta'(Y_*)\phi' X_* + \eta'(X_*)\phi' Y_*\right].\end{aligned}$$

The above equation shows that M' is a nearly trans-Sasakian manifold. □

Proposition 2.6. *Let $\pi : M \longrightarrow M'$ be a submersion of contact CR-submanifold of a nearly trans-Sasakian manifold $\bar{M}$ onto an almost contact metric manifold M'. Then* *(i)* $nh(\phi X, \phi Y) = nh(X, Y)$,

(ii) $mh(\phi X, \phi Y) = -mh(X, Y)$,

(iii) $C(\phi X, \phi Y) = \frac{1}{2}C(X, Y)$. (2.19)

Proof. From part (iii) of Proposition 2.4 we may write $nh(X, \phi Y) = -nh(Y, \phi X)$. Putting $X = \phi X$, we obtain part (i) of the proposition.

Now let $mh(X, \phi Y) = mh(Y, \phi X)$. Then using part (iv) of the Proposition 2.4 we write $mh(X, \phi Y) = \phi mh(X, Y)$. Putting $X = \phi X$ in this relation gives

$$mh(\phi X, \phi Y) = -mh(X, Y).$$

Making similar computations as in [10] we find easily (iii). □

3 Curvature Relations

Proposition 3.1. *Let $\pi : M \longrightarrow M'$ be a submersion of contact CR-submanifold of a nearly trans-Sasakian manifold $\bar{M}$ onto an almost contact metric manifold M' such that $mh(X, \phi Y) = mh(Y, \phi X)$. Then the ϕ-bisectional curvature of $\bar{M}$ and M' are related by*

$$\begin{aligned} \bar{B}(X,Y) = {} & B'(X_*, Y_*) - \frac{25}{9}\|nh(X,Y)\|^2 - \frac{17}{9}\|nh(X,\phi Y)\|^2 \\ & - \frac{32}{9} g(nh(X,X), nh(Y,Y)) + 2\|mh(X,Y)\|^2 \end{aligned}$$

for all $X, Y \in (D \oplus \{\xi\})$.

Proof. We know $\bar{B}(X, Y) = \bar{R}(X, \phi X, \phi Y, Y)$. Put $Y = \phi X,\ Z = \phi Y,\ W = Y$ in Gauss equation

$$\bar{R}(X, Y, Z, W) = R(X, Y, Z, W) - g(h(X, W), h(Y, Z)) + g(h(X, Z), h(Y, W))$$

we get

$$\bar{R}(X, \phi X, \phi Y, Y) = R(X, \phi X, \phi Y, Y) - g(h(X, Y), h(\phi X, \phi Y)) + g(h(X, \phi Y), h(\phi X, Y)).$$

Substituting $h = nh + mh$, in the above equation, we obtain

$$\begin{aligned} \bar{R}(X, \phi X, \phi Y, Y) = {} & R(X, \phi X, \phi Y, Y) \\ & - g(nh(X, Y) + mh(X, Y), nh(\phi X, \phi Y) + mh(\phi X, \phi Y)) \\ & + g(nh(X, \phi Y) + mh(X, \phi Y), nh(\phi X, Y) + mh(\phi X, Y)) \end{aligned}$$

$$
\begin{aligned}
&= R(X,\phi X,\phi Y,Y) - g(nh(X,Y),nh(\phi X,\phi Y)) \\
&\quad - g(nh(X,Y),mh(\phi X,\phi Y)) - g(mh(X,Y),nh(\phi X,\phi Y)) \\
&\quad - g(mh(X,Y),mh(\phi X,\phi Y)) + g(nh(X,\phi Y),nh(\phi X,Y)) \\
&\quad + g(nh(X,\phi Y),mh(\phi X,Y)) + g(mh(X,\phi Y),nh(\phi X,Y)) \\
&\quad + g(mh(X,\phi Y),mh(\phi X,Y)) \\
&= R(X,\phi X,\phi Y,Y) - g(nh(X,Y),nh(\phi X,\phi Y)) \\
&\quad - g(mh(X,Y),mh(\phi X,\phi Y)) + g(nh(X,\phi Y),nh(\phi X,Y)) \\
&\quad + g(mh(X,\phi Y),mh(\phi X,Y)) \\
&= R(X,\phi X,\phi Y,Y) - g(nh(X,Y),nh(X,Y)) \\
&\quad + g(mh(X,Y),mh(X,Y)) - g(nh(X,\phi Y),nh(X,\phi Y)) \\
&\quad + g(\phi mh(X,Y),\phi mh(X,Y)) \\
&= R(X,\phi X,\phi Y,Y) - \|nh(X,Y)\|^2 + 2\|mh(X,Y)\|^2 - \|nh(X,\phi Y)\|^2 .
\end{aligned} \tag{3.1}
$$

Now put $Y=\phi X$, $Z=\phi Y$, $W=Y$ in equation (2.8) we find

$$
\begin{aligned}
R(X,\phi X,\phi Y,Y) = {} & R^*(X_*,\phi' X_*,\phi' Y_*,Y_*) - g(C(\phi X,\phi Y),C(X,Y)) \\
& + g(C(X,\phi Y),C(\phi X,Y)) + 2g(C(X,\phi X),C(\phi Y,Y))
\end{aligned}
$$

or

$$
\begin{aligned}
R(X,\phi X,\phi Y,Y) = {} & R^*(X_*,\phi' X_*,\phi' Y_*,Y_*) - g(C(\phi X,\phi Y),C(X,Y)) \\
& - g(C(X,\phi Y),C(Y,\phi X)) - 2g(C(X,\phi X),C(Y,\phi Y)).
\end{aligned} \tag{3.2}
$$

From (ii) of Proposition 2.4 and (iii) of Proposition 2.6 we derive

$$
C(Y,\phi X) = \frac{4}{3}\phi nh(X,Y). \tag{3.3}
$$

Therefore equation (3.2) implies, jointly with (2.19) and (3.3), that

$$
\begin{aligned}
R(X,\phi X,\phi Y,Y) = {} & R^*(X_*,\phi' X_*,\phi' Y_*,Y_*) - \frac{16}{9}\|nh(X,Y)\|^2 \\
& - \frac{1}{2}\|C(X,Y)\|^2 - \frac{32}{9} g(nh(X,X),nh(Y,Y)).
\end{aligned} \tag{3.4}
$$

Further equation (3.3) yields

$$
C(X,Y) = -\frac{4}{3}\phi nh(X,\phi Y). \tag{3.5}
$$

Making use of the last equation in equation (3.4) we see

$$
\begin{aligned}
R(X,\phi X,\phi Y,Y) = {} & R^*(X_*,\phi' X_*,\phi' Y_*,Y_*) - \frac{16}{9}\|nh(X,Y)\|^2 \\
& - \frac{8}{9}\|nh(X,\phi Y)\|^2 - \frac{32}{9} g(nh(X,X),nh(Y,Y)).
\end{aligned}
$$

Put this value of $R(X, \phi X, \phi Y, Y)$ in equation (3.1) we get

$$\begin{aligned}\bar{R}(X, \phi X, \phi Y, Y) =& R^*(X_*, \phi' X_*, \phi' Y_*, Y_*) - \frac{16}{9}\|nh(X,Y)\|^2 - \frac{8}{9}\|nh(X,\phi Y)\|^2 \\ & - \frac{32}{9} g(nh(X,X), nh(Y,Y)) - \|nh(X,Y)\|^2 \\ & + 2\|mh(X,Y)\|^2 - \|nh(X,\phi Y)\|^2 \\ =& R^*(X_*, \phi' X_*, \phi' Y_*, Y_*) - \frac{25}{9}\|nh(X,Y)\|^2 - \frac{17}{9}\|nh(X,\phi Y)\|^2 \\ & - \frac{32}{9} g(nh(X,X), nh(Y,Y)) + 2\|mh(X,Y)\|^2 .\end{aligned}$$

which implies that

$$\begin{aligned}\bar{B}(X,Y) =& B'(X_*, Y_*) - \frac{25}{9}\|nh(X,Y)\|^2 - \frac{17}{9}\|nh(X,\phi Y)\|^2 \\ & - \frac{32}{9} g(nh(X,X), nh(Y,Y)) + 2\|mh(X,Y)\|^2 .\end{aligned}$$

□

Corollary 3.2. *Let $\pi : M \longrightarrow M'$ be a submersion of contact CR-submanifold of a positively curved nearly trans-Sasakian manifold $\bar{M}$ onto an almost contact metric manifold M' such that the second fundamental form $h(X,Y)$ lies entirely in $\phi D^{\perp}$. Then M' has positive ϕ-holomorphic sectional curvature.*

Proof. Putting $X = Y$ in the above expression of $\bar{B}(X,Y)$ we obtain

$$\begin{aligned}\bar{B}(X,X) = \bar{H}(X) =& H'(X_*) - \frac{25}{9}\|nh(X,X)\|^2 - \frac{17}{9}\|nh(X,\phi X)\|^2 \\ & - \frac{32}{9} g(nh(X,X), nh(X,X)) + 2\|mh(X,X)\|^2 \\ =& H'(X_*) - \frac{57}{9}\|nh(X,X)\|^2 - \frac{17}{9}\|nh(X,\phi X)\|^2 + 2\|mh(X,X)\|^2 .\end{aligned}$$

From equation (3.5) we deduce that $nh(X, \phi X) = 0$. Also since $h(X,Y)$ lies entirely in $\phi D^{\perp}$, $mh(X,X) = 0$. Thus the last expression of $\bar{H}(X)$ turns out to be

$$\bar{H}(X) = H'(X_*) - \frac{57}{9}\|nh(X,X)\|^2 .$$

Now from hypothesis $\bar{M}$ is positively curved which implies that $\bar{H}(X) > 0$. Using this inequality in the above equation gives $H'(X_*) > 0$ which is the required result.

□

We now come to the main theorem of the article giving the compactness of M'.

Theorem 3.3. *Let $\pi : M \longrightarrow M'$ be a submersion of contact CR-submanifold of a positively curved nearly trans-Sasakian manifold $\bar{M}$ onto a complete almost contact metric manifold M' such that the second fundamental form $h(X, Y)$ lies entirely in $\phi D^{\perp}$. Then M' is compact.*

Proof. The theorem follows from a theorem in [11] and the last corollary. □

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