Thai Journal of Mathematics
Volume 10 (2012) Number 1 : 123–135

www.math.science.cmu.ac.th/thaijournal
Online ISSN 1686-0209

Fixed Point Theorems for a Generalized Intuitionistic Fuzzy Contraction in Intuitionistic Fuzzy Metric Spaces

Wutiphol Sintunavarat and Poom Kumam[1]

Department of Mathematics, Faculty of Science
King Mongkut's University of Technology Thonburi
Bangkok 10140, Thailand
e-mail : poom_teun@hotmail.com (W. Sintunavarat)
poom.kum@kmutt.ac.th (P. Kumam)

Abstract : In 2006, Rafi and Noorani [M. Rafi, M.S.M. Noorani, Fixed point theorem on intuitionistic fuzzy metric spaces, Iranian Journal of Fuzzy Systems 3 (1) (2006) 23–29] has introduced the notion of intuitionistic fuzzy contraction mappings. The aim of this paper is to develop and generalize the notion of intuitionistic fuzzy contraction mappings and establish a fixed point theorem for new mappings in intuitionistic fuzzy metric spaces.

Keywords : Intuitionistic fuzzy metric spaces; Intuitionistic fuzzy contraction mappings; Fixed point; Weakly compatible.
2010 Mathematics Subject Classification : 47H09; 47H10.

1 Introduction

The theory of fuzzy sets was introduced simultaneously by Zadeh's [1] in 1965. It gives the foundation of fuzzy mathematics. Later, several researchers have applied this theory to the well-known results in the classical set theory. The concept of an intuitionistic fuzzy set was first introduced by Atanassov in [2] and many works by the same author in [3–5]. It has been used and applied extensively in many areas of mathematics and sciences. Further, it is especially useful in the

[1]Corresponding author email: poom.kum@kmutt.ac.th (P. Kumam)

study of phenomena having two probabilistic aspects, for instance, in the context of two-slit experiment as the foundation of E-infinity theory of high energy physics, introduced and studied by El Naschie [6, 7]. In another example in this way, Liu and Wang [8] used the intuitionistic fuzzy point operators to reduce the degree of uncertainty of the elements in the universe corresponding to an intuitionistic fuzzy set.

One of the most important problems in fuzzy topology is to obtain an appropriate concept of an intuitionistic fuzzy metric space and an intuitionistic fuzzy normed space. This problems have been studies by Park [9] and Saadati and Park [10]. Park [9] introduced a notion of an intuitionistic fuzzy metric spaces which is an extension of fuzzy metric spaces of George and Veeramani [11] and proved some known results of metric spaces including Baires theorem and the Uniform limit theorem for intuitionistic fuzzy metric spaces. Saadati and Park [10] have been investigated and studied the theory of intutionistic fuzzy topology (both in metric and normed) spaces. These bring the most valuable applications which applied from an intuitionistic fuzzy sets. In fact, an intuitionistic fuzzy metric is very useful tool in modeling some phenomena which is necessary to study the relationship between two probability functions as will be observed in [12].

On the other hand, the fixed point theorem, generally known as the Banach contraction mapping principle, appeared in explicit form in Banach's thesis in 1922 which was used to establish the existence of a solution for an integral equation. Since its simplicity and usefulness, it has become the most famous mathematical theories with application in several branches of sciences, especially in chaos theory, game theory, nonlinear programming, economics, theory of differential equations, etc. A number of articles in this field have been dedicated to the improvement and generalization of the Banach's contraction mapping principle in many ways (see [13–21]).

Fixed point theory in fuzzy metric spaces has been developed starting with the work of Heilpern [22]. He introduced the concept of fuzzy contraction mappings and proved some fixed point theorems for fuzzy contraction mappings in metric linear spaces, which is a fuzzy extension of the Banach's contraction principle by setting the distance between two points to be a non-negative fuzzy number, and investigated some connections between fuzzy metric spaces and probabilistic metric spaces. Afterward, a number of fixed point theorems in fuzzy metric, intuitionnistic fuzzy metric spaces have been considered by many authors [23–29].

In 2006, Rafi and Noorani [30] introduced an intuitionistic fuzzy contraction mapping as follows:

Definition 1.1 (Rafi and Noorani [30])**.** Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. A mapping $T : X \to X$ is called that *an intuitionistic fuzzy contraction* if there exists $k \in (0, 1)$ such that

$$\frac{1}{M(Tx, Ty, t)} - 1 \leq k\left(\frac{1}{M(x, y, t)} - 1\right) \tag{1.1}$$

and

$$N(Tx, Ty, t) \leq kN(x, y, t) \tag{1.2}$$

for all $x, y \in X$ and $t > 0$.

They proved the existence fixed point in intuitionistic fuzzy metric spaces for an intuitionistic fuzzy contraction mapping in next Theorem.

Theorem 1.2 ([30, Theorem 3.6]). *Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space and $T : X \to X$. If T is an intuitionistic fuzzy contraction mapping, then T has a unique fixed point.*

The above theorem of Rafi and Noorani is very important and necessary theorem to claim the existence of fixed point in the intuitionistic fuzzy metric spaces. Occasionally, the constant number which satisfies Rafi and Noorani's Theorem is difficult and complicated to find. Therefore, it is the most interesting to search another auxiliary tool to claim the existence of a fixed point in the intuitionistic fuzzy metric spaces.

In our paper we will introduce a new intuitionistic fuzzy contraction mapping which is more general than a Rafi and Noorani's intuitionistic fuzzy contraction mapping and establish the new fixed point and common fixed point theorems in intuitionistic metric spaces.

The structure of this paper is as follows. Section 2 states some definitions and property of intuitionistic fuzzy metric spaces. This is followed by concept of a new intuitionistic fuzzy contraction mappings in Section 3. We also give the fixed point and common fixed point theorems in intuitionistic fuzzy metric spaces in this section.

2 Preliminaries

For the reader's convenience we recall some terminologies from the theory of fuzzy metric spaces, which will be used in what follows.

Definition 2.1 (Schweizer and Sklar [31]). A continuous t-norm is a binary operation $*$ on [0, 1] satisfying the following conditions:

(i) $*$ is commutative and associative;

(ii) $*$ is continuous;

(iii) $a * 1 = a$ for all $a \in [0, 1]$;

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0, 1])$.

Example 2.2. *The following examples are classical examples of a continuous t-norms:*

(TL) *(The Lukasiewicz t-norm) A mapping $*_L : [0, 1] \times [0, 1] \to [0, 1]$ which defined through*

$$a *_L b = \max\{a + b - 1, 0\}.$$

(TP) *(The product t-norm) A mapping* $*_P : [0,1] \times [0,1] \to [0,1]$ *which defined through*

$$a *_P b = ab.$$

(TM) *(The minimum t-norm) A mapping* $*_M : [0,1] \times [0,1] \to [0,1]$ *which defined through*

$$a *_M b = \min\{a, b\}.$$

(TD) *(The weakest t-norm) A mapping* $*_D : [0,1] \times [0,1] \to [0,1]$ *which defined through*

$$a *_D b = \begin{cases} \min\{a, b\}, & \max\{a, b\} = 1; \\ 0, & \textit{otherwise.} \end{cases}$$

Definition 2.3 (Schweizer and Sklar [31])**.** A continuous t-conorm is a binary operation $\diamond$ on [0, 1] satisfying the following conditions:

(i) $\diamond$ is commutative and associative;

(ii) $\diamond$ is continuous;

(iii) $a \diamond 0 = a$ for all $a \in [0,1]$;

(iv) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ $(a, b, c, d \in [0,1])$.

Remark 2.4. *The concepts of triangular t-norms and t-conorms are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions, respectively. These concepts were originally introduced by Menger [32] in his study of statistical metric spaces.*

Definition 2.5 (George and Veeramani [11])**.** A fuzzy metric space is a triple $(X, M, *)$ where X is a nonempty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty)$ and the following conditions are satisfied for all $x, y \in X$ and $t, s > 0$:

(GV-1) $M(x, y, t) > 0$;

(GV-2) $M(x, y, t) = 1 \iff x = y$;

(GV-3) $M(x, y, t) = M(y, x, t)$;

(GV-4) $M(x, y, \cdot) : (0, \infty) \to [0, 1]$ is continuous;

(GV-5) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$.

Definition 2.6 (Park [9])**.** A intutionistic fuzzy metric space is a 5-tuple $(X, M, N, *, \diamond)$ where X is a nonempty set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M, N is a fuzzy sets on $X^2 \times (0, \infty)$ such that the following axioms hold for $x, y, z \in X$ and $s, t > 0$:

(P-1) $M(x, y, t) + N(x, y, t) \leq 1$;

(P-2) $M(x, y, t) > 0$;

(P-3) $M(x, y, t) = 1 \Longleftrightarrow x = y$;

(P-4) $M(x, y, t) = M(y, x, t)$;

(P-5) $M(x, y, \cdot) : (0, \infty) \to [0, 1]$ is continuous;

(P-6) $M(x, z, s + t) \geq M(x, y, s) * M(y, z, t)$;

(P-7) $N(x, y, t) > 0$;

(P-8) $N(x, y, t) = 0 \Longleftrightarrow x = y$;

(P-9) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(P-10) $N(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous;

(P-11) $N(x, z, s + t) \leq N(x, y, s) \diamond N(y, z, t)$.

The pair (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2.7. *Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1 - M, *, \diamond)$ such that t-norm $*$ and t-conorm $\diamond$ are associated (see [33]), i.e. $x \diamond y = (1 - (1 - x) * (1 - y))$ for any $x, y \in X$.*

Remark 2.8. *In intuitionistic fuzzy metric space X, $M(x, y, \cdot)$ is non-decreasing and $N(x, y, \cdot)$ is non-increasing for all $x, y \in X$.*

Definition 2.9 (Park [9])**.** Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space, and let $r \in (0, 1), t > 0$ and $x \in X$. The set $B(x, r, t) = \{y \in X | M(x, y, t) > 1 - r$ and $N(x, y, t) < 1\}$ is called the open ball with center x and radius r with respect to t.

Definition 2.10 (Park [9])**.** Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. A sequence $\{x_n\}$ in X converges to $x \in X$ if for $r \in (0, 1)$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that $x_n \in B(x, r, t)$ for all $n \geq n_0$.

Lemma 2.11 (Park [9])**.** *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. A sequence $\{x_n\}$ in X converges to $x \in X$ if and only if*

$$M(x_n, x, t) \to 1 \text{ and } N(x_n, x, t) \to 0 \text{ as } n \to \infty \tag{2.1}$$

for all $t > 0$.

Definition 2.12 (Park [9])**.** Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. A sequence $\{x_n\}$ in X is called a Cauchy if for $0 < \epsilon < 1$ and $t > 0$ there exists $n_0 \in \mathbb{N}$ such that

$$M(x_n, x_m, t) > 1 - \epsilon \text{ and } N(x_n, x_m, t) < \epsilon \tag{2.2}$$

for $n, m \geq n_0$.

Definition 2.13 (Park [9])**.** The intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be complete if every Cauchy sequence is convergent.

Definition 2.14. Let S and T be self mappings of a nonempty set X.

(i) A point $x \in X$ is said to be a fixed point of T if $Tx = x$.

(ii) A point $x \in X$ is said to be a coincidence point of S and T if $Sx = Tx$ and we shall called $w = Sx = Tx$ that a point of coincidence of S and T.

(iii) A point $x \in X$ is said to be a common fixed point of S and T if $x = Sx = Tx$.

Definition 2.15 (Jungck [34])**.** Let S and T be self mappings of a nonempty set X. The mapping S and T are weakly compatible if $STx = TSx$ whenever $Sx = Tx$.

Lemma 2.16 (Haghi et al. [35])**.** *Let X be a nonempty set and $T : X \to X$ a function. Then there exists a subset $E \subseteq X$ such that $T(E) = T(X)$ and $T : E \to X$ is one-to-one.*

3 Main Results

We first introduce the notion of an intuitionistic fuzzy contraction depend on Δ mappings.

Definition 3.1. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. A mapping $T : X \to X$ is called that an *intuitionistic fuzzy contraction depend on* Δ (IFC$_\triangle$) if there exists a mapping $\Delta : X \to [0, 1)$ which $\Delta(Tx) \leq \Delta(x)$ such that

$$\frac{1}{M(Tx, Ty, t)} - 1 \leq \Delta(x)\left(\frac{1}{M(x, y, t)} - 1\right) \tag{3.1}$$

and

$$N(Tx, Ty, t) \leq \Delta(x)N(x, y, t) \tag{3.2}$$

for all $x, y \in X$ and $t > 0$.

Lemma 3.2. *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $T : X \to X$. If T is an (IFC$_\triangle$) mapping, then T is a continuous mapping.*

Proof. Let $\{x_n\}$ be a sequence in X such that $x_n \to z$ as $n \to \infty$ for some $z \in X$. From the notion of (IFC$_\triangle$), we get

$$\frac{1}{M(Tz, Tx_n, t)} - 1 \leq \Delta(z)\left(\frac{1}{M(z, x_n, t)} - 1\right) \tag{3.3}$$

for all $n \in \mathbb{N}$. Taking $n \to \infty$, we have

$$\frac{1}{M(Tz, Tx_n, t)} - 1 \to 0 \tag{3.4}$$

that is

$$M(Tz, Tx_n, t) \to 1 \tag{3.5}$$

as $n \to \infty$. Again, using the notion of (IFC$_\triangle$), we get

$$N(Tz, Tx_n, t) \leq \Delta(z) N(z, x_n, t) \tag{3.6}$$

for all $n \in \mathbb{N}$. We take $n \to \infty$, we get

$$N(Tz, Tx_n, t) \to 0. \tag{3.7}$$

From (3.5) and (3.7), we have $Tx_n \to Tz$ as $n \to \infty$. Therefore, we can conclude that T is continuous. □

Next, we proved the existence of fixed point theorem for (IFC$_\Delta$) mapping.

Theorem 3.3. *Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space and $T : X \to X$. If T is an (IFC$_\triangle$) mapping, then T has a unique fixed point.*

Proof. Let x_0 be an arbitrary point in X. We can construct the sequence $\{x_n\}$ in X by

$$x_n = T^n x_0 = Tx_{n-1} \tag{3.8}$$

for $n = 1, 2, ...$ If there exists $n_0 \in \{1, 2, ...\}$ such that $Tx_{n_0-1} = Tx_{n_0}$, then we get $x_{n_0} = Tx_{n_0}$. Thus x_{n_0} is a fixed point of T and then the proof is complete. Now we suppose that $Tx_{n-1} \neq Tx_n$ for all $n \geq 1$. Now, for positive integer n and m such that $m > n$ and $t > 0$, we have

$$\begin{aligned}
\frac{1}{M(x_n, x_m, t)} - 1 &= \left(\frac{1}{M(Tx_{n-1}, Tx_{m-1}, t)} - 1\right) \\
&\leq \Delta(x_{n-1})\left(\frac{1}{M(x_{n-1}, x_{m-1}, t)} - 1\right) \\
&= \Delta(Tx_{n-2})\left(\frac{1}{M(Tx_{n-2}, Tx_{m-2}, t)} - 1\right) \\
&\leq \Delta(x_{n-2})\Delta(x_{n-2})\left(\frac{1}{M(x_{n-2}, x_{m-2}, t)} - 1\right) \\
&= \Delta(Tx_{n-3})\Delta(Tx_{n-3})\left(\frac{1}{M(Tx_{n-3}, Tx_{m-3}, t)} - 1\right) \\
&\leq \Delta(x_{n-3})\Delta(x_{n-3})\Delta(x_{n-3})\left(\frac{1}{M(x_{n-3}, x_{m-3}, t)} - 1\right) \\
&\vdots \\
&\leq (\Delta(x_0))^n\left(\frac{1}{M(x_0, x_{m-n}, t)} - 1\right).
\end{aligned} \tag{3.9}$$

Since $\Delta(x_0) \in [0,1)$, if we taking limit as $m,n \to \infty$, then $\frac{1}{M(x_n,x_m,t)} - 1 \to 0$, that is

$$M(x_n,x_m,t) \to 1 \tag{3.10}$$

as $m,n \to \infty$. Also for positive integer n and m such $m > n$ and $t > 0$, we get

$$\begin{aligned} N(x_n,x_m,t) &= N(Tx_{n-1},Tx_{m-1},t) \\ &\leq \Delta(x_{n-1})N(x_{n-1},x_{m-1},t) \\ &= \Delta(Tx_{n-2})N(Tx_{n-2},Tx_{m-2},t) \\ &\leq \Delta(x_{n-2})\Delta(x_{n-2})N(x_{n-2},x_{m-2},t) \\ &= \Delta(Tx_{n-3})\Delta(Tx_{n-3})N(Tx_{n-3},Tx_{m-3},t) \\ &\leq \Delta(x_{n-3})\Delta(x_{n-3})\Delta(x_{n-3})N(x_{n-3},x_{m-3},t) \\ &\vdots \\ &\leq (\Delta(x_0))^n N(x_0,x_{m-n},t). \end{aligned} \tag{3.11}$$

Since $\Delta(x_0) \in [0,1)$, we have

$$N(x_n,x_m,t) \to 0 \tag{3.12}$$

as $m,n \to \infty$. From (3.10) and (3.12), we get the sequence $\{x_n\}$ is a Cauchy sequence in intuitionistic fuzzy metric X. As X is a complete, there exists a point $z \in X$ such that $x_n \to z$ as $n \to \infty$, which is implies that

$$M(x_n,z,t) \to 1 \text{ and } N(x_n,z,t) \to 0 \text{ as } n \to \infty \tag{3.13}$$

for all $t > 0$. Next, we show that z is a fixed point of T. Since T is (IFC$_\Delta$), for all $n \in \mathbb{N}$ we get

$$\frac{1}{M(Tz,Tx_n,t)} - 1 \leq \Delta(z)\left(\frac{1}{M(z,x_n,t)} - 1\right) \tag{3.14}$$

for all $t > 0$. Taking $n \to \infty$, we have for every $t > 0$, $\frac{1}{M(Tz,Tx_n,t)} - 1 \to 0$ and then $M(Tz,Tx_n,t) \to 1$ as $n \to \infty$ for all $t > 0$. Again, using the fact of T is (IFC$_\Delta$), we have

$$N(Tz,Tx_n,t) \leq \Delta(z)N(z,x_n,t) \tag{3.15}$$

for all $t > 0$ and for all $n \in \mathbb{N}$. Taking $n \to \infty$, we get $N(Tz,Tx_n,t) \to 0$ for all $t > 0$. In both case, it can be concluded that $Tx_n \to Tz$. Therefore, $x_{n+1} \to Tz$ and then $z = Tz$.

Finally, we show that z is a unique fixed point of T. We suppose that $z_1 \in X$ is an another fixed point of T. We use the notion of (IFC$_\Delta$) for $t > 0$, we have

$$\begin{aligned} \frac{1}{M(z,z_1,t)} - 1 &= \frac{1}{M(Tz,Tz_1,t)} - 1 \\ &\leq \Delta(z)\left(\frac{1}{M(z,z_1,t)} - 1\right). \end{aligned} \tag{3.16}$$

Since $\Delta(z) \in [0,1)$, we obtain that $\frac{1}{M(z,z_1,t)} - 1 = 0$ that is $M(z, z_1, t) = 1$. The previous statement implies that $z = z_1$. Therefore, z is a unique fixed point of T. □

Corollary 3.4 ([30, Theorem 3.6])**.** *Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space and $T : X \to X$. If T is an intuitionistic fuzzy contraction mapping, then T has a unique fixed point.*

Proof. We can prove this results by apply Theorem 3.3 with $\Delta(x) = k$ where $k \in (0, 1)$. □

Theorem 3.5. *Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space and $T : X \to X$. If T^n is an (IFC$_\Delta$) mapping for some $n \in \mathbb{N}$, then T has a unique fixed point.*

Proof. From Theorem 3.3, we get T^n has a unique fixed point z. But $T^n(Tz) = T(T^n z) = Tz$, so Tz is a fixed point of T^n and then $Tz = z$ by the uniqueness of a fixed point of T^n. Therefore, z is also a fixed point of T. Since the fixed point of T is also fixed point of T^n, the fixed point of T is unique. □

Next, we shall prove the common fixed point theorem in intuitionistic fuzzy metric space. We introduce the notion of mapping which is more general than an (IFC$_\Delta$) in the below definition.

Definition 3.6. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $S, T : X \to X$. A mapping T is called that an *intuitionistic fuzzy contraction depend on Δ with respect to S* if there exists a mapping $\Delta : X \to [0, 1)$ which $\Delta(Tx) \le \Delta(Sx)$ for all $x \in X$ such that

$$\frac{1}{M(Tx,Ty,t)} - 1 \le \Delta(Sx)\left(\frac{1}{M(Sx,Sy,t)} - 1\right) \tag{3.17}$$

and

$$N(Tx, Ty, t) \le \Delta(Sx)N(Sx, Sy, t) \tag{3.18}$$

for all $x, y \in X$ and $t > 0$.

Remark 3.7. *If we take S is identity mapping, then T is reduce to an (IFC$_\Delta$) mapping.*

Theorem 3.8. *Let $(X, M, N, *, \diamond)$ be a intuitionistic fuzzy metric space and $S, T : X \to X$ such that $S(X)$ is a complete subspace of X. If T is an (IFC$_\Delta$) w.r.t to S and the mappings S and T are weakly compatible, then S and T have a unique common fixed point.*

Proof. By Lemma 2.16, there exists $E \subseteq X$ such that $S(E) = S(X)$ and $S : E \to X$ is one-to-one. Now, define a mapping $U : S(E) \to S(E)$ by $U(Sx) = Tx$. Since

S is one-to-one on E, U is well-defined. From the fact of T is ($\mathrm{IFC}_\triangle$) w.r.t to S and $U \circ S = T$, we get

$$\frac{1}{M(U(Sx), U(Sy), t)} - 1 \leq \Delta(Sx)\left(\frac{1}{M(Sx, Sy, t)} - 1\right) \tag{3.19}$$

and

$$N(U(Sx), U(Sy), t) \leq \Delta(Sx) N(Sx, Sy, t) \tag{3.20}$$

for all $Sx, Sy \in S(E)$ and $t > 0$. Therefore, U is (IFC_Δ). Since $S(E) = S(X)$ is complete, by using Theorem 3.3, there exists a unique fixed point $z \in S(X)$ such that $Uz = z$. Since $z \in S(X)$, we get $z = Sw$ for some $w \in X$ and then $U(Sw) = Sw$ that is $Tw = Sw$. Hence, T and S have a point of coincidence, which is also unique.

Next, we show that S and T have a common fixed point. Since $z = Tw = Sw$ and S and T are weakly compatible, we get $Sz = STw = TSw = Tz$ and so $Sz = Tz$ is a point of coincidence of S and T. However, z is the only point of coincidence of S and T, so $z = Sz = Tz$ that is z is a common fixed point of S and T. Moreover, if z_1 is another common fixed point of S and T that is $z_1 = Sz_1 = Tz_1$, then z_1 is a point of coincidence of S and T, and then $z_1 = z$ by uniqueness of point of coincidence. Therefore, z is a unique common fixed point of S and T. □

4 Conclusion

In this paper, we modify an intuitionistic fuzzy contraction mapping of Rafi and Noorani in [30] and proved some fixed point and common fixed point theorem for new contraction mappings in intuitionistic fuzzy metric spaces. Although, Theorem 3.6 of Rafi and Noorani in [30] is an essential tool in the intuitionistic fuzzy metric space to claim the existence of common fixed points of two mappings. Sometimes the constant numbers which satisfy Rafi and Noorani's Theorem are very difficult to find. Therefore, it is the most interesting to define such mappings Δ as another auxiliary tool to claim the existence of a common fixed point. However, all the main results in this paper are some of the choices for solving problems in an intuitionistic fuzzy metric spaces. Our results may be the motivation to other authors for extending and improving these results to be suitable tools for their applications.

Acknowledgements : The first author would like to thank the Research Professional Development Project Under the Science Achievement Scholarship of Thailand (SAST) and the second author would like to thank the National Research University Project of Thailand's Office of the Higher Education Commission for financial support under the project NRU-CSEC no. 54000267 for financial support during the preparation of this manuscript.

References

[1] L.A. Zadeh, Fuzzy Sets, Information and Control 8 (1965) 338–353.

[2] K. Atanassov, lntuitionistic fuzzy sets, in: V. Sgurev, Ed., VII ITKR's Session, Sofia (June 1983 Central Sci. and Techn. Library, Bulg. Academy of Sciences, 1984).

[3] K. Atanassov, lntuitionistic fuzzy sets, Fuzzy, Sets and Systems 20 (1986) 87–96.

[4] K. Atanassov, Review and new results on intuitionistic fuzzy sets, Preprint IM-MFAIS-1-88, Sofia, 1988.

[5] K. Atanassov, S. Stoeva, Intuitionistic fuzzy sets, in: Polish Syrup. on Interval & Fuzzy Mathematics, Poznan (August 1983) 23–26.

[6] MS. El Naschie, On the uncertainty of Cantorian geometry and two-slit experiment, Chaos, Soliton and Fractals 9 (3) (1998) 17–29.

[7] MS. El Naschie, On the verifications of heterotic strings theory and $\epsilon^{(\infty)}$ theory, Chaos, Soliton and Fractals 11 (2) (2000) 397–407.

[8] H.-W. Liu, G.-J. Wang, Multi-criteria decision-making methods based on intuitionistic fuzzy sets, European Journal of Operational Research 179 (1) (2007) 220–233.

[9] J.H. Park, Intutionistic fuzzy metric spaces, Chaos, Solitons and Fractals 22 (2004) 1039–1046.

[10] R. Saadati, J.H. Park, On the intuitionistic fuzzy topological spaces, Chaos, Soliton & Fractals 27 (2006) 331–344.

[11] A. George, P. Veeramani, On some results in fuzzy metric spaces, Fuzzy Sets and Systems 64 (1994) 395–399.

[12] V. Gregori, S. Romaguera, P. Veereamani, A note on intuitionistic fuzzy metric spaces, Chaos, Solitons and Fractals 28 (4) (2006) 902–905.

[13] C. Mongkolkeha, P. Kumam, Fixed point and common fixed point theorems for generalized weak contraction mappings of integral type in Modular spaces, International Journal of Mathematics and Mathematical Sciences, Vol. 2011, Article ID 705943, 12 pages.

[14] C. Mongkolkeha, W. Sintunavarat, P. Kumam, Fixed point theorems for contraction mappings in modular metric spaces, Fixed Point Theory and Applications 2011, 2011:93.

[15] W. Sintunavarat, P. Kumam, Coincidence and common fixed points for hybrid strict contractions without the weakly commuting condition, Appl. Math. Lett. 22 (2009) 1877–1881.

[16] W. Sintunavarat, P. Kumam, Weak condition for generalized multi-valued (f, α, β)-weak contraction mappings, Appl. Math. Lett. 24 (2011) 460–465.

[17] W. Sintunavarat, P. Kumam, Coincidence and common fixed points for generalized contraction multi-valued mappings, J. Comput. Anal. Appl. 13 No 2 (2011) 362–367.

[18] W. Sintunavarat, P. Kumam, Gregus-type Common fixed point theorems for tangential multivalued mappings of integral type in metric spaces, Internat. J. Math. Math. Sci., Vol. 2011, Article ID 923458, 12 pages.

[19] W. Sintunavarat, P. Kumam, Gregus type fixed points for a tangential multi-valued mappings satisfying contractive conditions of integral type, Journal of Inequalities and Applications 2011, 2011:3.

[20] W. Sintunavarat, Y.J. Cho, P. Kumam, Common fixed point theorems for c-distance in ordered cone metric spaces, Comp. Math. Appl. 62 (2011) 1969–1978.

[21] W. Sintunavarat, P. Kumam, Common fixed point theorems for generalized $\mathcal{JH}$-operator classes and invariant approximations, Journal of Inequalities and Applications 2011, 2011:67.

[22] S. Heilpern, Fuzzy mappings and fixed point theorems, J. Math. Anal. Appl. 83 (1981) 566–569.

[23] J.X. Fang, A note on fixed point theorems of Hadzic, Fuzzy Sets and Systems 48 (1991) 391–395.

[24] O. Hadzic, Fixed point theorems for multi-valued mappings in some classes of fuzzy metric spaces, Fuzzy Sets and Systems 29 (1989) 115–125.

[25] P.J. He, The variational principle in fuzzy metric spaces and its applications, Fuzzy Sets and Systems 45 (1992) 389–394.

[26] J.S. Jung, Y.J. Cho, J.K. Kim, Minimization theorems for fixed point theorems in fuzzy metric spaces and applications, Fuzzy Sets and Systems 61 (1994) 199–207.

[27] O. Kaleva, S. Seikkala, On fuzzy metric spaces, Fuzzy Sets and Systems 12 (1984) 215–229.

[28] W. Sintunavarat, P. Kumam, Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric spaces, Journal of Applied Mathematics, Vol. 2011, Article ID 637958, 14 pages, 2011.

[29] W. Sintunavarat, Y.J. Cho, P. Kumam, Coupled coincidence point theorems for contractions without commutative condition in intuitionistic fuzzy normed spaces, Fixed Point Theory and Applications 2011, 2011:81.

[30] M. Rafi, M.S.M. Noorani, Fixed point theorem on intuitionistic fuzzy metric spaces, Iranian Journal of Fuzzy Systems 3 (1) (2006) 23–29.

[31] B. Schweizer, A. Sklar, Probabilistic Metric Spaces, North Holland, New York, 1983.

[32] K. Menger, Statistical metrics, Proc. Nat. Acad. Sci. U.S.A. 28 (1942) 535–537.

[33] R. Lowen, Fuzzy set theory, Dordrecht: Kluwer Academic Publishers, 1996.

[34] G. Jungck, Common fixed points for non-continuous non-self mappings on a non-numeric spaces, Far East J. Math. Sci. 4 (2) (1996) 199–212.

[35] R.H. Haghi, Sh. Rezapour, N. Shahzadb, Some fixed point generalizations are not real generalizations, Nonlinear Anal. 74 (2011) 1799–1803.

(Received 16 November 2011)
(Accepted 30 November 2011)