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On ϕ-Concircularly Symmetric Kenmotsu Manifolds

Uday Chand De[†] **and Krishnendu De**[‡]

[†]Department of Pure Mathematics, Calcutta University
35 Ballygunge Circular Road, Kol 700019, West Bengal, India
e-mail : uc_de@yahoo.com

[‡]Konnagar High School (H.S.)
68 G.T. Road (West), Konnagar, Hooghly
Pin.712235, West Bengal, India
e-mail : krishnendu_de@yahoo.com

Abstract : We study locally and globally ϕ-concircularly symmetric Kenmotsu manifolds. At first we show that in a Kenmotsu manifold globally ϕ-symmetry and globally ϕ-concircularly symmetry are equivalent. Next we study 3-dimensional locally ϕ-concircularly symmetric Kenmotsu manifolds. Finally, we give some examples of ϕ-concircularly symmetric Kenmotsu manifolds.

Keywords : Kenmotsu manifold; Globally ϕ-concircularly symmetric manifold; Locally ϕ-concircularly symmetric manifold; Einstein manifold.
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1 Introduction

The product of an almost contact manifold M and the real line $\mathbb{R}$ carries a natural almost complex structure. However if one takes M to be an almost contact metric manifold and suppose that the product metric G on $M \times \mathbb{R}$ is Kaehlerian, then the structure on M is cosymplectic [1] and not Sasakian. On the other hand, Oubina [2] pointed out that if the conformally related metric $e^{2t}G$, t being the coordinates on $\mathbb{R}$, is Kaehlerian, then M is Sasakian and conversely.

In [3], Tanno classified almost contact metric manifolds whose automorphism groups possesses the maximum dimension. For such a manifold M, the sectional curvature of plane section containing ξ is a constant, say c. If $c > 0$, M is a homogeneous Sasakian manifold of constant sectional curvature. If $c = 0$, M is the product of a line or a circle with a Kaehler manifold of constant holomorphic sectional curvature. If $c < 0$, M is a warped product space $\mathbb{R} \times f^{C^n}$. In 1972, Kenmotsu [4] abstracted the differential geometric properties of the third case. We call it Kenmotsu manifold.

In general, a geodesic circle (a curve whose first curvature is constant and second curvature is identically zero) does not transform into a geodesic circle by the conformal transformation

$$\tilde{g}_{ij} = \psi^2 g_{ij}, \tag{1.1}$$

of the fundamental tensor g_{ij}. The transformation which preserves geodesic circles was first introduced by Yano [5]. The conformal transformation (1.1) satisfying the partial differential equation

$$\psi_{;i;j} = \phi g_{ij}, \tag{1.2}$$

changes a geodesic circle into a geodesic circle. Such a transformation is known as the concircular transformation and the geometry which deals with such transformation is called the concircular geometry [5].

A $(1,3)$ type tensor $\widetilde{C}(X,Y)Z$ which remains invariant under concircular transformation, for an n-dimensional Riemannian manifold M^n, is given by Yano and Kon [6, 7].

$$\tilde{C}(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y], \tag{1.3}$$

where $R(X,Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z$ (∇ being the Riemannian connection) is the Riemannian curvature tensor and r, the scalar curvature. From (1.3) we obtain

$$(\nabla_W \widetilde{C})(X,Y)Z = (\nabla_W R)(X,Y)Z - \frac{dr(W)}{n(n-1)}[g(Y,Z)X - g(X,Z)Y]. \tag{1.4}$$

The importance of concircular transformation and concircular curvature tensor is very well known in the differential geometry of certain F-structure such as complex, almost complex, Kahler, almost Kahler, contact and almost contact structure etc. ([7–9]). In a recent paper, Ahsan and Siddiqui [10] studied the application of concircular curvature tensor in fluid space time.

In this paper, we study locally ϕ-concircularly symmetric and globally ϕ-concircularly symmetric contact metric manifolds. A contact metric manifold (M,g) is called *locally ϕ-concircularly symmetric* if the condition

$$\phi^2 \left(\left(\nabla_X \widetilde{C} \right) (Y,Z,W) \right) = 0 \tag{1.5}$$

holds on M, where X, Y, Z and W are horizontal vectors. If X, Y, Z and W are arbitrary vectors then the manifold is called *globally ϕ-concircularly symmetric*. Kenmotsu manifold were studied by many authors such as Pitis [11], De and Pathak [12], Binh et al. [13], Begewadi et al. [14–16], Ozgur [17, 18] and many others. The paper is organized as follows: In section 2, some preliminary results are recalled.After preliminaries,we study globally ϕ-concircularly symmetric Kenmotsu manifolds. We prove that if a Kenmotsu manifold is globally ϕ-concircularly symmetric, then the manifold is an Einstein manifold. We also show that a globally ϕ-concircularly symmetric Kenmotsu manifold is globally ϕ-symmetric. In the next section, we study 3-dimensional locally ϕ-concircularly symmetric Kenmotsu manifolds. We prove that a 3-dimensional Kenmotsu manifold is locally ϕ-concircularly symmetric if and only if the scalar curvature is constant.Finally, we cited some examples of ϕ-concircularly symmetric Kenmotsu manifolds.

2 Preliminaries

Let M be a connected almost contact metric manifold with an almost contact metric structure (ϕ,ξ,η,g), that is, ϕ is an (1,1) tensor field, ξ is a vector field, η is a 1-form and g is a compatible Riemannian metric such that

$$\phi^2(X) = -X + \eta(X)\xi, \eta(\xi) = 1, \phi\xi = 0, \eta\phi = 0 \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2.2}$$

$$g(X, \xi) = \eta(X) \tag{2.3}$$

for all $X, Y \in T(M)$ [19, 20]. If an almost contact metric manifold satisfies

$$(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{2.4}$$

then M is called a Kenmotsu manifold [4], where ∇ is the Levi-Civita connection of g. From the above equation it follows that

$$\nabla_X \xi = X - \eta(X)\xi, \tag{2.5}$$

and

$$(\nabla_X \eta)Y = g(X, Y) - \eta(X)\eta(Y). \tag{2.6}$$

Moreover, the curvature tensor R and the Ricci tensor S satisfy

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X \tag{2.7}$$

and

$$S(X, \xi) = -(n - 1)\eta(X). \tag{2.8}$$

From [12], we know that for a 3-dimensional Kenmotsu manifold

$$
\begin{aligned}
R(X,Y)Z = & \left(\frac{r+4}{2}\right)[g(Y,Z)X - g(X,Z)Y] \\
& - \left(\frac{r+6}{2}\right)[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi \\
& + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y],
\end{aligned} \tag{2.9}
$$

$$
S(X,Y) = \frac{1}{2}[(r+2)g(X,Y) - (r+6)\eta(X)\eta(Y)], \tag{2.10}
$$

where S is the Ricci tensor of type $(0,2)$, R is the curvature tensor of type $(1,3)$ and r is the scalar curvature of the manifold M.

3 Globally ϕ-Concircularly Symmetric Kenmotsu Manifolds

Definition 3.1. A Kenmotsu manifold M is said to be *globally ϕ-concircularly symmetric* if the concircular curvature tensor $\widetilde{C}$ satisfies

$$
\phi^2\left(\left(\nabla_X \widetilde{C}\right)(Y,Z,W)\right) = 0, \tag{3.1}
$$

for all vector fields $X, Y, Z \in \chi(M)$.

It is well-known that if the Ricci tensor S of the manifold is of the form $S(X,Y) = \lambda g(X,Y)$, where λ is a constant and $X, Y \in \chi(M)$, then the manifold is called an Einstein manifold.

Let us suppose that M is a globally ϕ-concircularly symmetric Kenmotsu manifold. Then by definition

$$
\phi^2\left(\left(\nabla_W \widetilde{C}\right)(X,Y,Z)\right) = 0.
$$

Using (2.1) we have

$$
-\left(\nabla_W \widetilde{C}\right)(X,Y)Z + \eta\left(\left(\nabla_W \widetilde{C}\right)(X,Y)Z\right)\xi = 0.
$$

From (1.4) it follows that

$$
\begin{aligned}
0 = & -g((\nabla_W R)(X,Y)Z,U) + \frac{dr(W)}{n(n-1)}[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)] \\
& + \eta((\nabla_W R)(X,Y)Z)\eta(U) - \frac{dr(W)}{n(n-1)}[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]\eta(U).
\end{aligned}
$$

Putting $X = U = e_i$, where $\{e_i\}$, $(i = 1, 2, ..., n)$ is an orthonormal basis of the tangent space at each point of the manifold, and taking summation over i, we get

$$\begin{aligned} 0 = &- (\nabla_W S)(Y, Z) + \frac{dr(W)}{n} g(Y, Z) + \eta((\nabla_W R)(e_i, Y)Z)\eta(e_i) \\ &- \frac{dr(W)}{n(n-1)}[g(Y, Z) - \eta(Y)\eta(Z)]. \end{aligned}$$

Putting $Z = \xi$, we obtain

$$- (\nabla_W S)(Y, \xi) + \frac{dr(W)}{n}\eta(Y) + \eta((\nabla_W R)(e_i, Y)\xi)\eta(e_i) = 0. \tag{3.2}$$

Now

$$\eta((\nabla_W R)(e_i, Y)\xi)\eta(e_i) = g((\nabla_W R)(e_i, Y)\xi, \xi) g(e_i, \xi). \tag{3.3}$$

$$\begin{aligned} g((\nabla_W R)(e_i, Y)\xi, \xi) = &\, g(\nabla_W R(e_i, Y)\xi, \xi) - g(R(\nabla_W e_i, Y)\xi, \xi) \\ &- g(R(e_i, \nabla_W Y)\xi, \xi) - g(R(e_i, Y)\nabla_W \xi, \xi). \end{aligned}$$

Since $\{e_i\}$ is an orthonormal basis $\nabla_X e_i = 0$ and using (2.7) we find

$$\begin{aligned} g(R(e_i, \nabla_W Y)\xi, \xi) &= g(\eta(e_i)\nabla_W Y - \eta(\nabla_W Y)e_i, \xi) \\ &= \eta(e_i)\eta(\nabla_W Y) - \eta(\nabla_W Y)\eta(e_i) \\ &= 0. \end{aligned}$$

As

$$g(R(e_i, Y)\xi, \xi) + g(R(\xi, \xi)Y, e_i) = 0$$

we have

$$g(\nabla_W R(e_i, Y)\xi, \xi) + g(R(e_i, Y)\xi, \nabla_W \xi) = 0.$$

Using this we get

$$g((\nabla_W R)(e_i, Y)\xi, \xi) = 0. \tag{3.4}$$

By the use of (3.3) and (3.4) , from (3.2) we obtain

$$(\nabla_W S)(Y, \xi) = \frac{1}{n} dr(W)\eta(Y), \tag{3.5}$$

Putting $Y = \xi$ in (3.5), we get $dr(W) = 0$. This implies r is constant. So from (3.5), we have $\nabla_W S(Y, \xi) = 0$. This implies that

$$S(Y, W) = (1 - n)g(Y, W). \tag{3.6}$$

Hence we can state the following:

Theorem 3.2. *If a Kenmotsu manifold is globally ϕ-concircularly symmetric, then the manifold is an Einstein manifold.*

Next suppose $S(X,Y) = \lambda g(X,Y)$, that is, the manifold is an Einstein manifold. Then from (1.3) we have

$$\left(\nabla_W \widetilde{C}\right)(X,Y)Z = (\nabla_W R)(X,Y)Z.$$

Applying ϕ^2 on both sides of the above equation we have

$$\phi^2 \left(\nabla_W \widetilde{C}\right)(X,Y)Z = \phi^2 (\nabla_W R)(X,Y)Z.$$

Hence we can state:

Theorem 3.3. *A globally ϕ-concircularly symmetric Kenmotsu manifold is globally ϕ-symmetric.*

Remark 3.4. *Since a globally ϕ-symmetric Kenmotsu manifold is always a globally ϕ-concircularly symmetric manifold, from Theorem 3.3, we conclude that on a Kenmotsu manifold, globally ϕ-symmetry and globally ϕ-concircularly symmetry are equivalent.*

4 3-Dimensional Locally ϕ-Conccircularly Symmetric Kenmotsu Manifolds

Using (2.9) in (1.3), in a 3-dimensional Kenmotsu manifold the concircular curvature tensor is given by

$$\begin{aligned}
\widetilde{C}(X,Y)Z = {} & \left(\frac{r+4}{2}\right)[g(Y,Z)X - g(X,Z)Y] \\
& - \left(\frac{r+6}{2}\right)[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi \\
& + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y] - \left(\frac{r}{6}\right)[g(Y,Z)X - g(X,Z)Y].
\end{aligned} \tag{4.1}$$

Taking the covariant differentiation to the both sides of the equation (4.1), we have

$$\begin{aligned}
(\nabla_W \widetilde{C})(X,Y)Z = {} & \frac{dr(W)}{2}[g(Y,Z)X - g(X,Z)Y] \\
& - \frac{dr(W)}{2}[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi \\
& + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y] - \left(\frac{r+6}{2}\right)[g(Y,Z)(\nabla_W\eta)(X)\xi \\
& - g(X,Z)(\nabla_W\eta)(Y)\xi + g(Y,Z)\eta(X)\nabla_W\xi - g(X,Z)\eta(Y)\nabla_W\xi \\
& + (\nabla_W\eta)(Y)\eta(Z)X + \eta(Y)(\nabla_W\eta)(Z)X - (\nabla_W\eta)(X)\eta(Z)Y \\
& - \eta(X)(\nabla_W\eta)(Z)Y] - \left(\frac{dr(W)}{6}\right)[g(Y,Z)X - g(X,Z)Y].
\end{aligned} \tag{4.2}$$

Now assume that X, Y and Z are horizontal vector fields. So the equation (4.2) becomes

$$(\nabla_W \widetilde{C})(X,Y)Z = \frac{dr(W)}{3}[g(Y,Z)X - g(X,Z)Y] - \left(\frac{r+6}{2}\right)[g(Y,Z)(\nabla_W \eta)(X)\xi - g(X,Z)(\nabla_W \eta)(Y)\xi]. \tag{4.3}$$

From (4.3) it follows that

$$\phi^2((\nabla_W \widetilde{C})(X,Y)Z) = -\frac{dr(W)}{3}[g(Y,Z)X - g(X,Z)Y]. \tag{4.4}$$

Hence we can state the following:

Theorem 4.1. *A 3-dimensional Kenmotsu manifold is locally ϕ-concircularly symmetric if and only if the scalar curvature r is constant.*

In [12], De and Pathak prove that

Corollary 4.2. *A 3-dimensional Kenmotsu manifold is locally ϕ-symmetric if and only if the scalar curvature r is constant.*

Using Corollary 4.2, we can state the following theorem:

Theorem 4.3. *A 3-dimensional Kenmotsu manifold is locally ϕ-concircularly symmetric if and only if it is locally ϕ-symmetric.*

5 Examples

Example 5.1. *In [21], the authors prove that if $R(\xi, X)\widetilde{C} = 0$ for any $X \in \chi(M)$, then M has constant sectional curvature -1. Hence the manifold is an Einstein manifold. Therefore from the definition of concircular curvature tensor we find that globally ϕ-symmetry and globally ϕ-concircularly symmetry are equivalent. Hence in a concircular semi-symmetric [$R\widetilde{C} = 0$] Kenmotsu manifold globally ϕ-symmetry and globally ϕ-concircularly symmetry are equivalent. Thus Theorem 3.3 is verified.*

Example 5.2. *In [4], Kenmotsu prove that a conformally flat Kenmotsu manifold of dimension ≥ 5 has constant sectional curvature equal to -1. Hence the manifold is an Einstein manifold.Therefore by the same argument as in Example 5.1, in a conformally flat Kenmotsu manifold of dimension ≥ 5 globally ϕ-symmetry and globally ϕ-concircularly symmetry are equivalent. Thus Theorem 3.3 is verified.*

Example 5.3. *In [22], Jun et al. prove that any η-Einstein [$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y)$] Kenmotsu manifold of dimension $n \geq 5$ with $b =$ constant is Einstein.Hence by the similar argument as in Example 5.1, in an η-Einstein Kenmotsu manifold of dimension ≥ 5 globally ϕ-symmetry and globally ϕ-concircularly symmetry are equivalent.Thus Theorem 3.3 is verified.*

Example 5.4. *In [22], the authors prove that a Ricci recurrent [$\nabla S = \alpha \otimes S$] manifold is an Einstein manifold. Hence by the similar argument as in Example 5.1,in a Ricci-recurrent Kenmotsu manifold globally ϕ-symmetry and globally ϕ-concircularly symmetry are equivalent.Thus Theorem 3.3 is verified.*

Example 5.5. *We consider the 3-dimensional manifold $M = \{(x, y, z)\varepsilon\mathbb{R}^3, z \neq 0\}$, where (x, y, z) are standard coordinate of $\mathbb{R}^3$.*

The vector fields

$$e_1 = z\frac{\partial}{\partial x}, \quad e_2 = z\frac{\partial}{\partial y}, \quad e_3 = -z\frac{\partial}{\partial z}$$

are linearly independent at each point of M.

Let g be the Riemannian metric defined by

$$g(e_1, e_3) = g(e_1, e_2) = g(e_2, e_3) = 0,$$

$$g(e_1, e_1) = g(e_2, e_2) = g(e_3, e_3) = 1.$$

Let η be the 1-form defined by $\eta(Z) = g(Z, e_3)$ for any $Z\varepsilon\chi(M)$.

Let ϕ be the $(1, 1)$ tensor field defined by

$$\phi(e_1) = -e_2, \quad \phi(e_2) = e_1, \quad \phi(e_3) = 0.$$

Then using the linearity of ϕ and g, we have

$$\eta(e_3) = 1,$$

$$\phi^2 Z = -Z + \eta(Z)e_3,$$

$$g(\phi Z, \phi W) = g(Z, W) - \eta(Z)\eta(W),$$

for any $Z, W\varepsilon\chi(M)$. Then for $e_3 = \xi$, the structure (ϕ, ξ, η, g) defines an almost contact metric structure on M.

Let ∇ be the Levi-Civita connection with respect to metric g. Then we have

$$\begin{aligned}[e_1, e_3] &= e_1e_3 - e_3e_1 \\ &= z\frac{\partial}{\partial x}\left(-z\frac{\partial}{\partial z}\right) - \left(-z\frac{\partial}{\partial z}\right)\left(z\frac{\partial}{\partial x}\right) \\ &= -z^2\frac{\partial^2}{\partial x\partial z} + z^2\frac{\partial^2}{\partial z\partial x} + z\frac{\partial}{\partial x} \\ &= e_1. \end{aligned} \tag{5.1}$$

Similarly, $[e_1, e_2] = 0$ and $[e_2, e_3] = e_2$.

The Riemannian connection ∇ of the metric g is given by

$$\begin{aligned}2g(\nabla_X Y, Z) &= Xg(Y, Z) + Yg(Z, X) - Zg(X, Y) \\ &\quad - g(X, [Y, Z]) - g(Y, [X, Z]) + g(Z, [X, Y]), \end{aligned} \tag{5.2}$$

which known as Koszul's formula. Using (5.2) we have

$$\begin{aligned} 2g(\nabla_{e_1} e_3, e_1) &= -2g(e_1, -e_1) \\ &= 2g(e_1, e_1). \end{aligned} \tag{5.3}$$

Again by (5.2), we have

$$2g(\nabla_{e_1} e_3, e_2) = 0 = 2g(e_1, e_2) \tag{5.4}$$

and

$$2g(\nabla_{e_1} e_3, e_3) = 0 = 2g(e_1, e_3). \tag{5.5}$$

From (5.3), (5.4) and (5.5), we obtain

$$2g(\nabla_{e_1} e_3, X) = 2g(e_1, X), \tag{5.6}$$

for all $X \varepsilon \chi(M)$. *Thus* $\nabla_{e_1} e_3 = e_1$. *Therefore, (5.2) further yields*

$$\begin{aligned} &\nabla_{e_1} e_3 = e_1, \quad \nabla_{e_1} e_2 = 0, \quad \nabla_{e_1} e_1 = -e_3, \\ &\nabla_{e_2} e_3 = e_2, \quad \nabla_{e_2} e_2 = e_3, \quad \nabla_{e_2} e_1 = 0, \\ &\nabla_{e_3} e_3 = 0, \quad \nabla_{e_3} e_2 = 0, \quad \nabla_{e_3} e_1 = 0. \end{aligned} \tag{5.7}$$

From the above it follows that the manifold satisfies $\nabla_X \xi = X - \eta(X)\xi$, *for* $\xi = e_3$. *Hence the manifold is a Kenmotsu manifold. It is known that*

$$R(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z. \tag{5.8}$$

With the help of the above results and using (5.8), it can be easily verified that

$$\begin{aligned} &R(e_1, e_2)e_3 = 0, \quad R(e_2, e_3)e_3 = -e_2, \quad R(e_1, e_3)e_3 = -e_1, \\ &R(e_1, e_2)e_2 = -e_1, \quad R(e_2, e_3)e_2 = e_3, \quad R(e_1, e_3)e_2 = 0, \\ &R(e_1, e_2)e_1 = e_2, \quad R(e_2, e_3)e_1 = 0, \quad R(e_1, e_3)e_1 = e_3. \end{aligned}$$

From the above expressions of the curvature tensor R *we obtain*

$$\begin{aligned} S(e_1, e_1) &= g(R(e_1, e_2)e_2, e_1) + g(R(e_1, e_3)e_3, e_1) \\ &= -2. \end{aligned} \tag{5.9}$$

Similarly, we have $S(e_2, e_2) = S(e_3, e_3) = -2$. *Therefore,* $r = S(e_1, e_1) + S(e_2, e_2) + S(e_3, e_3) = -6$. *We note that here* r *is constant. Thus Theorem 4.1 is verified.*

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