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Existence Results for Vector Variational-Like Inequalities[1]

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Abstract : The purpose of this work is to introduce and study a more general form of vector variational-like inequalities in Banach spaces. By using the definitions of h-η-quasimonotone of Stampacchia type and Minty type mappings, some existence results for vector variational-like inequalities are obtained. Some examples supporting the main results are also constructed.

Keywords : Vector variational-like inequality; proper h-η-quasimonotonicity; h-η-pseudomonotonicity; C-convex; Affinity; KKM-mapping.
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1 Introduction

One important generalization of the classical variational inequality is the vector variational inequality, which was introduced by Giannessi [1] in a finite dimentional Euclidean space. Subsequently, vector variational inequalities have been investigated in abstract spaces, see [2–4]. A vector variational-like inequality is a generalization of vector variational inequality related to the class of η-connected sets which is much more general than the class of convex sets, see [5, 6].

Of course, monotonicity of a nonlinear mapping is one of most rapidly used concept in the theory of vector variational inequalities. Some important generalizations of monotonicity, such as quasimonotonicity, proper quasimonotonicity,

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pseudomonotonicity, semi-monotonicity, relaxed η-α-monotonicity, have been introduced and considered in the study of various variational inequalities, see [7–9]. In 2006, Zhao and Xia [10] obtained some existence results for vector variational-like inequalities by using definitions of properly η-quasimonotone of Minty type and properly η-quasimonotone of Stampacchia type mappings. For more details we refer to [11–13].

In this paper, we introduce and study a more general form of vector variational-like inequalities in Banach spaces. Some existence results are established by defining the concept of properly h-η-quasimonotone of Stampacchia type mappings and properly h-η-quasimonotone of Minty type mappings. Some examples are also given.

2 Preliminaries

Throughout this work, unless otherwise specified, let X and Y be two real Banach spaces, $K \subset X$ a nonempty, closed and convex subset, $C \subset Y$ a pointed, closed and convex cone in Y such that $int\, C \neq \emptyset$, where $int\, C$ denotes the interior of C. Then for $y_1, y_2 \in Y$, a partial order $\leq_C$ in Y is defined as

$$y_1 \leq_C y_2 \Longleftrightarrow y_2 - y_1 \in C.$$

Note that $C \neq Y$ iff $0 \notin int\, C$. Denote by $L(X, Y)$ the space of all continuous linear mappings from X to Y. For any $l \in L(X, Y)$, $x \in X$, let $\langle l, x\rangle$ denote the value of l at x. Let $T : K \to L(X, Y)$, $\eta : K \times K \to K$ and $h : K \times K \to Y$ be mappings. Consider the following vector variational-like inequalities:

Find $x \in K$ such that

$$\langle Tx, \eta(y, x)\rangle + h(y, x) \geq_C 0, \quad \forall\, y \in K \tag{2.1}$$

and find $x \in K$ such that

$$\langle Ty, \eta(x, y)\rangle + h(x, y) \leq_C 0, \quad \forall\, y \in K. \tag{2.2}$$

If $h = 0$, then (2.1) and (2.2) reduces to the following vector variational-like inequalities considered and studied by Zhao and Xia [10].

Find $x \in K$ such that

$$\langle Tx, \eta(y, x)\rangle \geq_C 0, \quad \forall\, y \in K \tag{2.3}$$

and find $x \in K$ such that

$$\langle Ty, \eta(x, y)\rangle \leq_C 0, \quad \forall\, y \in K. \tag{2.4}$$

The following concepts and results are needed in the sequel.

Definition 2.1. A mapping $f : K \to Y$ is said to be *hemicontinuous* if, for any fixed $x, y \in K$, the mapping $t \mapsto f(x + t(y - x))$ is continuous at 0^+.

Definition 2.2. Let $C : K \to 2^Y$ be a set-valued mapping, $h : K \times K \to Y$ and $\eta : K \times K \to K$ be single-valued mappings. Then

(i) $h(\cdot, v)$ is said to be *C-convex* in first argument if

$$h(tu_1+(1-t)u_2, v) \in th(u_1, v)+(1-t)h(u_2, v)-C, \ \forall\, u_1, u_2 \in K, \forall\, t \in [0, 1];$$

(ii) If K is an affine set, then $\eta(u, v)$ is said to be *affine* with respect to u if for any given $v \in K$,

$$\eta(tu_1 + (1-t)u_2, v) = t\eta(u_1, v) + (1-t)\eta(u_2, v), \ \forall\, u_1, u_2 \in K, \ \forall\, t \in \mathbb{R};$$

with $u = tu_1 + (1-t)u_2 \in K$.

Definition 2.3. Let $T : K \to L(X, Y)$, $\eta : K \times K \to X$ and $h : K \times K \to Y$ be mappings. Then T is said to be *h-η-pseudomonotone* if for any $x, y \in K$,

$$\langle Tx, \eta(y, x)\rangle + h(y, x) \geq_C 0 \Rightarrow \langle Ty, \eta(x, y)\rangle + h(x, y) \leq_C 0.$$

Remark 2.4.

(i) If $h(\cdot, \cdot) \equiv 0$, then h-η-pseudomonotonicity of T reduces to η-pseudomonotonicity of T.

(ii) If $\eta(y, x) = y - x$ and $h(\cdot, \cdot) \equiv 0$, then h-η-pseudomonotonicity of T reduces to pseudomonotonicity of T.

Example 2.5. *Let $X = \mathbb{R}$, $K = \mathbb{R}_+$, $Y = \mathbb{R}^2$, $C = \mathbb{R}^2_+$ and*

$$T(x) = \begin{pmatrix} 2 + sin2x \\ 2 + cos2x \end{pmatrix}, \ \eta(y, x) = y - 2x, \ h(y, x) = \begin{pmatrix} y^2 - xy - 2x^2 \\ 2y - 4x \end{pmatrix},$$

$\forall\, x, y \in K$. Then $\forall\, x, y \in K$,

$$\begin{aligned} \langle T(x), \eta(y, x)\rangle + h(y, x) &= \begin{pmatrix} 2 + sin2x \\ 2 + cos2x \end{pmatrix} (y - 2x) + \begin{pmatrix} y^2 - xy - 2x^2 \\ 2y - 4x \end{pmatrix} \\ &= \begin{pmatrix} (2 + sin2x)(y - 2x) \\ (2 + cos2x)(y - 2x) \end{pmatrix} + \begin{pmatrix} y^2 - xy - 2x^2 \\ 2y - 4x \end{pmatrix} \\ &= (y - 2x) \begin{pmatrix} (2 + sin2x) + (x + y) \\ (2 + cos2x) + 2 \end{pmatrix} \geq_C 0 \end{aligned}$$

implies $y > 2x$, so it follows that

$$\begin{aligned} \langle T(y), \eta(x, y)\rangle + h(x, y) &= \begin{pmatrix} 2 + sin2y \\ 2 + cos2y \end{pmatrix} (x - 2y) + \begin{pmatrix} x^2 - xy - 2y^2 \\ 2x - 4y \end{pmatrix} \\ &= (x - 2y) \begin{pmatrix} (2 + sin2y) + (x + y) \\ (2 + cos2y) + 2 \end{pmatrix} \leq_C 0 \end{aligned}$$

$\Longrightarrow$ *T is h-η-pseudomonotone.*

Definition 2.6 ([14])**.** A multivalued operator $T : X \to 2^{X^*}$ is called *quasimonotone* if, for all $x, y \in X$, the following implication holds:

$$\exists\, x^* \in T(x) : \langle x^*, y - x\rangle > 0 \Rightarrow \forall y^* \in T(y) : \langle y^*, y - x\rangle \geq 0.$$

Definition 2.7 ([14])**.** An operator $T : X \to 2^{X^*}$ is called *properly quasimonotone* if, for every $x_1, x_2, ..., x_n \in X$ and every $y \in Conv\{x_1, x_2, ..., x_n\}$, there exists i such that

$$\forall x_i^* \in T(x_i) : \langle x_i^*, y - x_i\rangle \leq 0.$$

Choosing $y = \frac{(x_1+x_2)}{2}$, we see that a properly quasimonotone operator is quasimonotone.

Remark 2.8. *The adjective "quasimonotone" suggests a relationship to quasiconvex function which indeed exists.*

Definition 2.9. Let $T : K \to L(X, Y)$ be mapping. Then

(i) T is said to be *properly quasimonotone of Stampacchia type* if for all $n \in \mathbb{N}$, for all vectors $v_1, ..., v_n \in K$ and scalars $\lambda_i \geq 0$, $i = 1, 2, ..., n$ with $\sum_{i=1}^n \lambda_i = 1$ and $u := \sum_{i=1}^n \lambda_i v_i$,

$$\langle Tu, v_i - u\rangle \geq_C 0, \qquad \text{holds for some } i.$$

(ii) T is said to be *properly quasimonotone of Minty type* if for all vectors $v_1, ..., v_n \in K$ and scalars $\lambda_i \geq 0$, $i = 1, 2, ..., n$ with $\sum_{i=1}^n \lambda_i = 1$ and $u := \sum_{i=1}^n \lambda_i v_i$,

$$\langle Tv_i, v_i - u\rangle \geq_C 0, \qquad \text{holds for some } i.$$

Definition 2.10. Let $T : K \to L(X, Y)$ and $\eta : K \times K \to X$ be mappings. Then

(i) T is said to be *properly η-quasimonotone of Stampacchia type* if for all $n \in \mathbb{N}$, for all vectors $v_1, ..., v_n \in K$ and scalars $\lambda_i \geq 0$, $i = 1, 2, ..., n$ with $\sum_{i=1}^n \lambda_i = 1$ and $u := \sum_{i=1}^n \lambda_i v_i$,

$$\langle Tu, \eta(v_i, u)\rangle \geq_C 0, \qquad \text{holds for some } i.$$

(ii) T is said to be *properly η-quasimonotone of Minty type* if for all vectors $v_1, ..., v_n \in K$ and scalars $\lambda_i \geq 0$, $i = 1, 2, ..., n$ with $\sum_{i=1}^n \lambda_i = 1$ and $u := \sum_{i=1}^n \lambda_i v_i$,

$$\langle Tv_i, \eta(v_i, u)\rangle \geq_C 0, \quad \text{holds for some } i.$$

Definition 2.11. Let $T : K \to L(X, Y)$, $\eta : K \times K \to X$ and $h : K \times K \to Y$ be mappings. Then

(i) T is said to be *properly h-η-quasimonotone of Stampacchia type* if for all $n \in \mathbb{N}$, for all vectors $v_1, ..., v_n \in K$ and scalars $\lambda_i \geq 0$, $i = 1, 2, ..., n$ with $\sum_{i=1}^{n} \lambda_i = 1$ and $u := \sum_{i=1}^{n} \lambda_i v_i$,

$$\langle Tu, \eta(v_i, u)\rangle + h(v_i, u) \geq_C 0, \qquad \text{holds for some } i.$$

(ii) T is said to be *properly h-η-quasimonotone of Minty type* if for all vectors $v_1, ..., v_n \in K$ and scalars $\lambda_i \geq 0$, $i = 1, 2, ..., n$ with $\sum_{i=1}^{n} \lambda_i = 1$ and $u := \sum_{i=1}^{n} \lambda_i v_i$,

$$\langle Tv_i, \eta(u, v_i) + h(u, v_i)\rangle \leq_C 0, \qquad \text{holds for some } i.$$

Example 2.12. *Let X, K, Y, C be same as in Example 2.5 and*

$$T(x) = \begin{pmatrix} 2x^2 \\ 8x^3 \end{pmatrix},\ \eta(y, x) = y - (x - x^2),\ h(y, x) = \begin{pmatrix} y + 2x^2 \\ y + x \end{pmatrix}.$$

We claim that T is properly h-η-quasimonotone of Stampacchia type. Suppose to the contrary that there exists $x_i \in K$, $t_i \geq 0$, $i = 1, 2, ..., n$ with $\sum_{i=1}^{n} t_i = 1$ such that

$$\langle Tx, \eta(x_i, x)\rangle + h(x_i, x) \not\geq_C 0, \quad i = 1, 2, ..., n,$$

where $x = \sum_{i=1}^{n} t_i x_i$. It follows that

$$\langle Tx, \eta(x_i, x)\rangle + h(x_i, x) = \begin{pmatrix} 2x^2(x_i - x + x^2) + (x_i + 2x^2) \\ 8x^3(x_i - x + x^2) + (x_i + x) \end{pmatrix} \not\geq_C 0, \quad i = 1, 2, ..., n,$$

which is a contradiction, since

$$2x^2(x_i - x + x^2) + (x_i + 2x^2) \geq_C 0$$

and

$$8x^3(x_i - x + x^2) + (x_i + x) \geq_C 0, \text{ for atleast one } i.$$

Thus, T is properly h-η-quasimonotone of Stampacchia type.

Lemma 2.13. *Let $T : K \to L(X, Y)$, $\eta : K \times K \to X$ and $h : K \times K \to Y$ be mappings. If T is h-η-pseudomonotone and properly h-η-quasimonotone of Stampacchia type, then T is properly h-η-quasimonotone of Minty type.*

Proof. The fact directly follows from the Definition 2.3 and Definition 2.11. □

Definition 2.14. Let D be a nonempty subset of a topological Häusdorff space E. A mapping $G : D \to 2^E$ (where 2^E is the family of all nonempty subsets of E) is said to be a *KKM mapping* if, for any finite set $\{x_1, ..., x_n\} \subset D$, $conv\{x_1, ..., x_n\} \subset \bigcup_{i=1}^{n} G(x_i)$, where *conv* denotes the convex hull operator.

Lemma 2.15 ([15])**.** *Let D be a nonempty subset of a topological Häusdorff vector space E and $G : D \to 2^E$ a KKM mapping. If $G(x)$ is closed for any $x \in D$ and compact for some $x \in D$, then $\bigcap_{x\in D} G(x) \neq \emptyset$.*

Lemma 2.16. *Let Y be topological vector space with a pointed, closed and convex cone such that int $C \neq \emptyset$. Then, for all x, y, $z \in Y$,*

(i) $x - y \in -C$ *and* $x \notin -int\ C \implies y \notin -int\ C$;

(ii) $x \in -int\ C$ *and* $y \notin int\ C \implies x + y \notin C$.

3 Existence Results

In this section, we establish some existence results for (2.1) and (2.2) by using Lemma 2.15.

Lemma 3.1. *Let $T : K \to L(X,Y)$, $\eta : K \times K \to X$ and $h : K \times K \to Y$ be mappings satisfying the following conditions:*

(i) T *is* h-η-*pseudomonotone;*

(ii) for any fixed $y \in X$, *the mapping* $y \to \langle Ty, \eta(x,y)\rangle$ *is hemicontinuous and* $h(x,y)$ *is continuous with* $\{z_t\} \to x_0 \in K$, $z_t \in K$;

(iii) $h(\cdot, y)$ *is* C-*convex in the first variable and* $h(x,x) = 0$, $\forall x \in K$;

(iv) $\eta(\cdot, y)$ *is affine in the first variable and* $\eta(x,x) = 0$, $\forall x \in K$.

Then for any $x_0 \in K$, *the following statements are equivalent:*

(I) $\langle Tx_0, \eta(x, x_0)\rangle + h(x, x_0) \geq_C 0, \quad \forall x \in K$;

(II) $\langle Tx, \eta(x_0, x)\rangle + h(x_0, x) \leq_C 0, \quad \forall x \in K$.

Proof. As T is h-η-pseudomonotone, it follows that (I)⇒(II).
Conversely, suppose that (II) holds i.e. for any $x_0 \in K$,

$$\langle Tx, \eta(x_0, x)\rangle + h(x_0, x) \leq_C 0, \quad \forall x \in K. \tag{3.1}$$

For any arbitrary $z \in K$, letting $z_t = (1-t)x_0 + tz$, $0 < t < 1$, we have $z_t \in K$ by convexity of K. Hence, we have

$$\langle Tz_t, \eta(x_0, z_t)\rangle + h(x_0, z_t) \leq_C 0. \tag{3.2}$$

Now we prove that

$$\langle Tz_t, \eta(z, z_t)\rangle + h(z, z_t) \geq_C 0. \tag{3.3}$$

Suppose that (3.3) is not true, then

$$\langle Tz_t, \eta(z, z_t)\rangle + h(z, z_t) \not\geq_C 0. \tag{3.4}$$

As C is a convex cone and in view of (iii), (iv), we get

$$\begin{aligned}
0 &= \langle Tz_t, \eta(z_t, z_t)\rangle + h(z_t, z_t) \\
&= \langle Tz_t, \eta((1-t)x_0 + tz, z_t)\rangle + h((1-t)x_0 + tz, z_t) \\
&= t\{\langle Tz_t, \eta(z, z_t)\rangle + h(z, z_t)\} + (1-t)\{\langle Tz_t, \eta(x_0, z_t)\rangle + h(x_0, z_t)\} \\
&\in t\{\langle Tz_t, \eta(z, z_t)\rangle + h(z, z_t)\} + (1-t)\{\langle Tz_t, \eta(x_0, z_t)\rangle + h(x_0, z_t)\} - C
\end{aligned}$$

which implies that

$$t\{\langle Tz_t, \eta(z, z_t)\rangle + h(z, z_t)\} + (1-t)\{\langle Tz_t, \eta(x_0, z_t)\rangle + h(x_0, z_t)\} \in C. \qquad (3.5)$$

In view of Lemma 2.16, (3.2) and (3.4), we have

$$t\{\langle Tz_t, \eta(z, z_t)\rangle + h(z, z_t)\} + (1-t)\{\langle Tz_t, \eta(x_0, z_t)\rangle + h(x_0, z_t)\} \notin C$$

which is a contradiction to (3.5) and hence (3.3) is true. Condition (ii) implies that

$$\langle Tx_0, \eta(x, x_0)\rangle + h(x, x_0) \geq_C 0, \quad \forall x \in K.$$

□

Theorem 3.2. *Let X and Y be real Banach spaces and $K \subset X$ a nonempty, compact and convex set. Let $T : K \to L(X, Y)$, $\eta : K \times K \to X$ and $h : K \times K \to Y$ be mappings satisfying the following conditions:*

(i) for any fixed $y \in K$, the mappings $x \to \langle Tx, \eta(y, x)\rangle$ and $h(\cdot, x)$ are continuous;

(ii) T is properly h-η-quasimonotone of Stampacchia type;

(iii) for all $x \in K$, $\eta(x, x) = 0 = h(x, x)$.

Then there exists $x \in K$ such that

$$\langle Tx, \eta(y, x)\rangle + h(y, x) \geq_C 0, \quad \forall\, y \in K.$$

Proof. Define a multivalued mapping $H_1 : K \to 2^K$ by

$$H_1(z) = \{x \in K : \langle Tx, \eta(z, x)\rangle + h(z, x) \geq_C 0\}, \quad \forall\, z \in K.$$

Then $H_1(z)$ is nonempty for each $z \in K$. Note that H_1 is a KKM mapping on K. Infact, if it is not the case, then there exists $\{x_1, ..., x_n\} \subset K$, $x = \sum_{i=1}^n t_i x_i$ with $t_i > 0$, $i = 1, 2, ..., n$ and $\sum_{i=1}^n t_i = 1$ such that $x \notin \bigcup_{i=1}^n H_1(x_i)$. This implies that

$$\langle Tx, \eta(x_i, x)\rangle + h(x_i, x) \ngeq_C 0, \quad i = 1, ..., n.$$

This contradicts condition (ii). Therefore, H_1 is a KKM mapping. Now, we prove that for any $z \in K$, $H_1(z)$ is closed. In view of (i), let there exists a net $\{x_n\} \subset H_1(z)$ such that $x_n \longrightarrow x \in K$.Because

$$\langle Tx, \eta(z, x_n)\rangle + h(z, x_n) \geq_C 0, \quad \text{for all } n,$$

we have

$$\langle Tx, \eta(z, x)\rangle + h(z, x) \geq_C 0.$$

Hence $x \in H_1(z)$ and so $H_1(z)$ is closed. It follows from the compactness of K and closedness of $H_1(z) \subset K$, that $H_1(z)$ is compact. Thus by Lemma 2.15, we have

$$\bigcap_{z \in K} H_1(z) \neq \emptyset.$$

Hence there exists $x \in K$ such that

$$\langle Tx, \eta(y,x)\rangle + h(y,x) \geq_C 0, \quad \forall\ y \in K.$$

This completes the proof. □

Theorem 3.3. *Let K be a nonempty, bounded, closed and convex subset of a real reflexive Banach space X and Y a real Banach space. Let $T : K \to L(X,Y)$, $\eta : K \times K \to X$ and $h : K \times K \to Y$ be mappings satisfying the following conditions:*

(i) T is properly h-η-quasimonotone of Minty type;

(ii) for all $x \in K$, $\eta(x,x) = 0$ and $h(x,x) = 0$.

Then there exists $x \in K$ such that

$$\langle Ty, \eta(x,y)\rangle + h(x,y) \leq_C 0, \quad \forall\ y \in K.$$

Proof. Define multivalued mapping $H_2 : K \to 2^K$ by

$$H_2(z) = \{x \in K : \langle Tz, \eta(x,z)\rangle + h(x,z) \leq_C 0\}, \quad \forall\ z \in K.$$

Then for each $z \in K$, $H_2(z)$ is nonempty. Suppose that H_2 is not a KKM mapping, then there exists $\{x_1, ..., x_n\} \subset K$, $x = \sum_{i=1}^n t_i x_i$ with $t_i > 0$, $i = 1, 2, ..., n$ and $\sum_{i=1}^n t_i = 1$ such that $x \notin \bigcup_{i=1}^n H_2(x_i)$. This implies that

$$\langle Tx_i, \eta(x,x_i)\rangle + h(x,x_i) \not\leq_C 0, \quad i = 1, ..., n$$

which contradicts condition (i). Therefore, H_2 is a KKM mapping. In addition, it is easy to verify that $H_2(z)$ is bounded, closed and convex for all $z \in K$. Since X is reflexive, $H_2(z)$ is weakly compact for all $z \in K$. It follows from Lemma 2.15 that

$$\bigcap_{z \in K} H_2(z) \neq \emptyset.$$

Hence there exists $x \in K$ such that

$$\langle Ty, \eta(x,y)\rangle + h(x,y) \leq_C 0, \quad \forall\ y \in K.$$

This completes the proof. □

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