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Convergence of Modified Ishikawa Iterative Process for Nonself I-Asymptotically Quasi-Nonexpansive Mappings in Banach spaces[1]

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Abstract : Let E be a uniformly convex Banach space and K be a nonempty closed convex subset of E. Let $T : K \to E$ be a nonself mapping. In this paper,we establish the weak and strong convergence of a sequence of a modified Ishikawa iterative process of a nonself I-asymptotically quasi-nonexpansive mapping in a Banach space. The results obtained in this paper extend and improve the results in the existing lituratures.

Keywords : Nonself I-asymptotically quasi-nonexpansive mapping; Nonself asymptotically quasi-nonexpansive mapping; Nonself nonexpansive mapping.
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1 Introduction

Let E be a normed linear space, K be a nonempty, convex subset of E, and T be a self map of K. Three most popular iteration procedures for obtaining fixed points of T, if they exist, are Mann iteration [1], defined by

$$u_1 \in K, u_{n+1} = (1 - \alpha_n)u_n + \alpha_n T u_n, n \geq 1 \tag{1.1}$$

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Ishikawa iteration [2], defined by

$$\begin{aligned} z_1 \in K, z_{n+1} &= (1-\alpha_n)z_n + \alpha_n T y_n, \\ y_n &= (1-\beta_n)z_n + \beta_n T z_n, n \geq 1 \end{aligned} \tag{1.2}$$

Noor iteration [3], defined by

$$\begin{aligned} v_1 \in K, v_{n+1} &= (1-\alpha_n)v_n + \alpha_n T w_n, \\ w_n &= (1-\beta_n)v_n + \beta_n T t_n, \\ t_n &= (1-\gamma_n)v_n + \gamma_n T v_n, n \geq 1 \end{aligned} \tag{1.3}$$

for certain choices of $\{\alpha_n\}, \{\beta_n\}$ and $\{\gamma_n\} \subset [0,1]$.

The multi-step iteration [4], arbitrary fixed order $p \geq 2$, defined by

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T y_n^1, \\ y_n^i &= (1-\beta_n^i)x_n + \beta_n^i T y_n^{i+1}, i = 1, 2, \ldots, p-2 \\ y_n^{p-1} &= (1-\beta_n^{p-1} x_n + \beta_n^{p-1} T x_n. \end{aligned} \tag{1.4}$$

where, for all $n \in N$,

$$\{\alpha_n\} \subset (0,1), \lim_{n\to\infty} \alpha_n = 0, \sum_{n=1}^{\infty} \alpha_n = \infty.$$

and for all $n \in N$,

$$\{\beta_n^i\} \subset [0,1), 1 \leq i \leq p-1, \lim_{n\to\infty} \beta_n^i = 0.$$

Taking $p = 3$ in (1.4) we obtain iteration (1.3). Taking $p = 2$ in (1.4) we obtain iteration (1.2).

Let K be a subset of normed linear space E and T be a self-mapping of K. Then T is called nonexpansive on K if

$$\|Tx - Ty\| \leq \|x - y\| \tag{1.5}$$

for all $x, y \in K$. Let $F(T) := \{x \in K : Tx = x\}$ denotes the set of fixed points of a mapping T.

The concept of a quasi-nonexpansive mapping was initiated by Tricomi in 1941 for real functions. Diaz and Metcalf [5] and Dotson [6] studied quasi-nonexpansive mappings in Banach spaces. Recently, this concept was given by Kirk [7] in metric spaces which we adapt to a normed space as follows:

T is called a quasi-nonexpansive mapping provided that

$$\|Tx - p\| \leq \|x - p\| \tag{1.6}$$

for all $x \in K$ and $p \in F(T)$.

T is called asymptotically quasi-nonexpansive if $\{\lambda_n\} \subset [0, \infty)$ with $\lim_{n\to\infty} \lambda_n = 0$ such that

$$\|T^n x - p\| \leq (1 + \lambda_n)\|x - p\| \tag{1.7}$$

for all $x \in K$ and $p \in F(T)$ and $n \geq 1$.

Let K be a subset of normed linear space E, T and I be self-mappings on K. Then T is called I-nonexpansive on K if

$$\|Tx - Ty\| \leq \|Ix - Iy\| \tag{1.8}$$

for all $x, y \in K[8]$. T is called I-quasi-nonexpansive on K if

$$\|Tx - p\| \leq \|Ix - p\| \tag{1.9}$$

for all $x \in K$ and $p \in F(T) \cap F(I)$.

T is called I-asymptotically nonexpansive on K if there exists a sequence $\{\lambda'_n\} \subset [0, \infty)$ with $\lim_{n\to\infty} \lambda'_n = 0$ such that

$$\|T^n x - T^n y\| \leq (1 + \lambda'_n)\|I^n x - I^n y\| \tag{1.10}$$

for all $x, y \in K$ and $n = 1, 2, ...$

T is called I-asymptotically quasi-nonexpansive on K if there exists a sequence $\{u_n\} \subset [0, \infty)$ with $\lim_{n\to\infty} u_n = 0$ such that

$$\|T^n x - p\| \leq (1 + u_n)\|I^n x - p\| \tag{1.11}$$

for all $x \in K, p \in F = F(T) \cap F(I)$ and $n = 1, 2, ...$

Remark 1.1. *There are many results of fixed points on nonexpansive and quasi-nonexpansive mappings in Banach spaces and metric spaces. For example, the strong and weak convergence of the sequence of certain iterates to a fixed point of quasi- nonexpansive maps was studied by Petryshyn and Williamson [9]. Their analysis was related to the convergence of Mann iterates studied by Dotson [6]. Subsequently, the convergence of Ishikawa iterates of quasi-nonexpansive mappings in Banach spaces was discussed by Ghosh and Debnath [10]. In [11], the weakly convergence theorem for I-asymptotically quasi-nonexpansive mapping defined in Hilbert space was proved. In [12], convergence theorems of iterative schemes for nonexpansive mappings have been presented and generalized.*

In [13], Rhoades and Temir considered T and I self-mappings of K, where T is an I-nonexpansive mapping and K a nonempty closed convex subset of a uniformly convex Banach space. They established the weak convergence of the sequence of Mann iterates to a common fixed point of T and I. However, if the domain K of T is a proper subset of E and T maps K into E then, the iteration formula (1.1) may fail to be well defined. One method that has been used to overcome this in the case of single operator T is to introduce a retraction $P : E \to K$ in the recursion formula (1.1) as follows: $u_1 \in K$,

$$u_{n+1} = (1 - \alpha_n)u_n + \alpha_n PTu_n, n \geq 1.$$

In [14], Kiziltunc and Ozdemir considered T and I nonself-mappings of K, where T is an I-nonexpansive mapping. They established the weak convergence of the sequence of modified Ishikawa iterates to a common fixed point of T and I. In [15], Kiziltunc and Yildirim considered T and I nonself-mappings of K, where T is an I-nonexpansive mapping. They established the weak convergence of the sequence of modified multi-step iterative scheme $\{x_n\}$ defined by, arbitrary fixed order $p \geq 2$

$$\begin{aligned} x_{n+1} &= P((1-\alpha_n)x_n + \alpha_n T y_n^1), \\ y_n^i &= P((1-\beta_n^i)x_n + \beta_n^i T y_n^{i+1}), i = 1, 2, \ldots, p-2 \\ y_n^{p-1} &= P((1-\beta_n^{p-1})x_n + \beta_n^{p-1} T x_n). \end{aligned} \tag{1.12}$$

where, for all $n \in N$,

$$\{\alpha_n\} \subset (0,1), \lim_{n\to\infty} \alpha_n = 0, \sum_{n=1}^{\infty} \alpha_n = \infty,$$

and for all $n \in N$,

$$\{\beta_n^i\} \subset [0,1), 1 \leq i \leq p-1, \lim_{n\to\infty} \beta_n^i = 0.$$

Remark 1.2. *Clearly, if T is a self-map, then* (1.12) *reduces to an iterative scheme* (1.4).

In [16], Nantadilok considered T and I nonself-mappings of K, where T is an I-quasi-nonexpansive mapping, and established the weak convergence of the sequence of modified multi-step iterative scheme $\{x_n\}$ defined by (1.12).

2 Preliminaries

Let E be a real Banach space. A subset K of E is said to be a retract of E if there exists a continuous map $P : E \to K$ such that $Px = x$ fo all $x \in K$. A map $P : E \to E$ is said to be retraction if $P^2 = P$. It follows that if a map P is a retraction, then $Py = y$ for all y in the range of P. Recall that a Banach space E is said to satisfy Opial's condition [17] if, for each sequence $\{x_n\}$ in E, the condition $x_n \rightharpoonup x$ implies that

$$\liminf_{n\to\infty} \|x_n - x\| < \liminf_{n\to\infty} \|x_n - y\| \tag{2.1}$$

for all $y \in E$ with $y \neq x$.

Define the Ishikawa iterative process for nonself I-asymptotically quasi-nonexpansive mappings in uniformly convex Banach space E as follows

$$\begin{aligned} x_{n+1} &= P((1-\alpha_n)x_n + \alpha_n I^n y_n), \\ y_n &= P((1-\beta_n)x_n + \beta_n T^n x_n), \end{aligned} \tag{2.2}$$

for all $n \in N$,where $0 \leq \alpha_n, \beta_n \leq 1$.

Remark 2.1. *Clearly, if T is a self-map, then* (2.2) *reduces to a modified Ishikawa iterative scheme.*

Lemma 2.2 ([18])**.** *Let $\{a_n\}, \{b_n\}$ and $\{\sigma_n\}$ be sequences of nonnegative real numbers satisfying the following inequality*

$$a_{n+1} \leq (1+\sigma_n)a_n + b_n, \ \forall n \geq 1.$$

If $\sum_{n=1}^{\infty} b_n < \infty$ and $\sum_{n=1}^{\infty} \sigma_n < \infty$, then

(i) $\lim_{n\to\infty} a_n$ *exists.*

(ii) $\lim_{n\to\infty} a_n = 0$, *if $\{a_n\}$ has a subsequence converging to zero.*

Lemma 2.3. *Let E be a uniformly convex Banach space, K be a nonempty closed convex subset of E. Let T, I be nonself mappings of K with P a nonexpansive retraction, where T is a nonself I-asymptotically quasi-nonexpansive mapping with $\{u_n\} \subset [0,\infty)$ such that $\sum_{n=1}^{\infty} u_n < \infty$ and I is a nonself asymptotically quasi-nonexpansive mapping of K with $\{v_n\} \subset [0,\infty)$ such that $\sum_{n=1}^{\infty} v_n < \infty$. Let $\{x_n\}$ be the sequence defined by* (2.2) *with $F = F(T)\cap F(I) \neq \phi$. Then $\lim_{n\to\infty} \|x_n - p\|$ exists for any fixed point p of T and I.*

Proof. For any $p \in F(T) \cap T(I)$.

$$\begin{aligned} \|x_{n+1} - p\| &= \|P((1-\alpha_n)x_n + \alpha_n I^n y_n) - P(p)\| \\ &\leq \|(1-\alpha_n)x_n + \alpha_n I^n y_n - p\| \\ &\leq (1-\alpha_n)\|x_n - p\| + \alpha_n \|I^n y_n - p\| \\ &\leq (1-\alpha_n)\|x_n - p\| + \alpha_n(1+v_n)\|y_n - p\| \qquad (2.3) \end{aligned}$$

and

$$\begin{aligned} \|y_n - p\| &= \|P((1-\beta_n)x_n + \beta_n T^n x_n) - P(p)\| \\ &\leq \|(1-\beta_n)x_n + \beta_n T^n x_n - p\| \\ &\leq (1-\beta_n)\|x_n - p\| + \beta_n\|T^n x_n - p\| \\ &\leq (1-\beta_n)\|x_n - p\| + \beta_n\Big[(1+u_n)\|I^n x_n - p\|\Big] \\ &\leq (1-\beta_n)\|x_n - p\| + \beta_n\Big[(1+u_n)(1+v_n)\|x_n - p\|\Big] \\ &= \big(1+\beta_n u_n + \beta_n v_n + \beta_n u_n v_n\big)\|x_n - p\| \qquad (2.4) \end{aligned}$$

From (2.3) and (2.4), we get

$$\begin{aligned} \|x_{n+1} - p\| \leq & \Big[1 + \alpha_n\beta_n(u_n + v_n) + 2\alpha_n\beta_n u_n v_n + \alpha_n v_n \\ & + \alpha_n\beta_n v_n^2 + \alpha_n\beta_n u_n v_n^2\Big]\|x_n - p\| \qquad (2.5) \end{aligned}$$

We can rewrite (2.5) as follows

$$\|x_{n+1} - p\| \leq (1+\gamma_n)\|x_n - p\| + \psi_n,$$

where

$$\gamma_n = \alpha_n\beta_n(u_n + v_n) + 2\alpha_n\beta_n u_n v_n + \alpha_n v_n + \alpha_n\beta_n v_n^2 + \alpha_n\beta_n u_n v_n^2$$

with $\sum_{n=1}^{\infty} \gamma_n < \infty$, and $\psi_n = 0$ with $\sum_{n=1}^{\infty} \psi_n < \infty$.

By Lemma(2.2), $\lim_{n\to\infty} \|x_n - p\|$ exists for each $p \in F(T) \cap F(I)$. □

Lemma 2.4. *Let E be a uniformly convex Banach space, K be a nonempty closed convex subset of E. Let T, I be nonself mappings of K where T is a nonself I-asymptotically quasi-nonexpansive mapping, I is a nonself asymptotically quasi-nonexpansive mapping of K, with $F = F(T) \cap F(I) \neq \phi$. Then $F = F(T) \cap F(I)$ is closed.*

Proof. Let $\{p_k\}$ be a sequence in F such that $p_k \to p$ as $k \to \infty$. Since K is closed and $\{p_k\}$ is a sequence in K, we must have $p \in K$. Since $T : K \to E$ is a nonself I-asymptotically quasi-nonexpansive mapping, we obtain

$$\begin{aligned} \|T^n p_k - p\| &\leq (1 + u_n)\|I^n p_k - p\|, \\ &\leq (1 + u_n)(1 + v_n)\|p_k - p\|, \end{aligned} \tag{2.6}$$

for all $p_k \in K, p \in F = F(T) \cap F(I)$ and $n \geq 1$.
Taking lim both sides of (2.6), we get

$$\lim_{k\to\infty} \|T^n p_k - p\| \leq 0, n \geq 1$$

which implies that

$$\lim_{k\to\infty} \|T^n p_k - p\| = 0, n \geq 1.$$

Therefore, we get $\|T^n p - p\| = 0$. In particular, we have $\|Tp - p\| = 0$. Thus $p \in F = F(T) \cap F(I)$. This completes our proof. □

In this paper, we consider T and I nonself mappings of K, where T a nonself I-asymptotically quasi-nonexpansive mapping, more general class of mappings than those mentioned in the existing literatures. We established weak and strong convergence theorem of sequence of modified Ishikawa iterative scheme $\{x_n\}$ defined by (2.2) for nonself I-asymptotically quasi-nonexpansive mapping T, where I a nonself asymptotically quasi-nonexpansive mapping.

3 Main Results

Theorem 3.1. *Let K be a closed convex bounded subset of uniformly convex Banach space E, which satisfies Opial's condition, and let T,I be nonself mappings of K with T an I-asymptotically quasi-nonexpansive mapping, I a nonself asymptotically quasi-nonexpansive mapping on K. Then, for $x_0 \in K$, the sequence $\{x_n\}$ of modified Ishikawa iterates converges weakly to a common fixed point p of $F(T) \cap F(I)$.*

Proof. Let $p \in F(T) \cap F(I)$. Then, as in Lemma 2.3, it follows that $\lim_{n\to\infty} \|x_n - p\|$ exists and so for $n \geq 1$, the sequence $\{x_n\}$ is bounded on K. Then by the reflexibility of E and the boundedness of $\{x_n\}$, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $x_{n_k} \rightharpoonup p$ weakly. If $p \in F(T) \cap F(I)$ is nonempty and a singleton, then the proof is complete.We will assume that $F(T) \cap F(I)$ is not a singleton. We show that $\{x_n\}$ converges weakly to a common fixed point of T and I. Let $\{x_{n_k}\}$ and $\{x_{n_j}\}$ be two subsequences of $\{x_n\}$ which coverge weakly to p and q, respectively. We show that $p = q$. Suppose that E satisfies Opial's condition and that $p \neq q$ belong to weak limit set of the sequence $\{x_n\}$. Then $x_{n_k} \rightharpoonup p$ and $x_{n_j} \rightharpoonup q$ respectively. Since $\lim_{n\to\infty} \|x_n - p\|$ exists for any $p \in F(T) \cap F(I)$, by Opial's condition, we conclude that

$$\begin{aligned}
\lim_{n\to\infty} \|x_n - p\| &= \lim_{k\to\infty} \|x_{n_k} - p\| \\
&< \lim_{k\to\infty} \|x_{n_k} - q\| \\
&= \lim_{j\to\infty} \|x_{n_j} - q\| \\
&< \lim_{j\to\infty} \|x_{n_j} - p\| \\
&= \lim_{n\to\infty} \|x_n - p\|.
\end{aligned}$$

This is a contradiction. Thus $\{x_n\}$ converges weakly to a common fixed point of $F(T) \cap F(I)$. □

Theorem 3.2. *Let E be a real Banach space, and let K be a nonempty closed convex nonexpansive retract of E with P a nonexpansive retraction. Let $T : K \to E$ be a nonself I-asymptotically quasi-nonexpansive mapping with $\{u_n\} \subset [0, \infty)$ such that $\sum_{n=1}^{\infty} u_n < \infty$ and I a nonself asymptotically quasi-nonexpansive mapping of K with $\{v_n\} \subset [0, \infty)$ such that $\sum_{n=1}^{\infty} v_n < \infty$. Let $\{x_n\}$ be the sequence defined by (2.2) with $F = F(T) \cap F(I) \neq \phi$. Then $\{x_n\}$ converges strongly to a common fixed point of T and I if and only if*

$$\liminf_{n\to\infty} d(x_n, F) = 0,$$

where $d(x_n, F) = \inf\{\|x - p\| : p \in F = F(T) \cap F(I)\}$.

Proof. The neccesity is obvious, so it is omitted. We now prove the sufficiency. Let $p \in F = F(T) \cap F(I)$. Furthermore, from Lemma 2.3, we obtain

$$\|x_{n+1} - p\| \leq (1 + \gamma_n)\|x_n - p\| \tag{3.1}$$

where

$$\gamma_n = \alpha_n\beta_n(u_n + v_n) + 2\alpha_n\beta_n u_n v_n + \alpha_n v_n + \alpha_n\beta_n v_n^2 + \alpha_n\beta_n u_n v_n^2$$

with $\sum_{n=1}^{\infty} \gamma_n < \infty$, and $\psi_n = 0$ with $\sum_{n=1}^{\infty} \psi_n < \infty$.
By Lemma 2.2, $\lim_{n\to\infty} \|x_n - p\|$ exists for each $p \in F(T) \cap F(I)$.
By (3.1), we get

$$d(x_{n+1}, F)| \leq (1 + \gamma_n)d(x_n, F).$$

Then, by Lemma2.2, $\lim_{n\to\infty} d(x_n, F)$ exists.

Next, we show that $\{x_n\}$ is a Cauchy sequence in E. In fact, $\sum_{n=1}^{\infty} \gamma_n < \infty$, and $1 + x \leq e^x$, for all $x > 0$. From (3.1), for $m, n \geq 1$ and $p \in F(T) \cap F(I)$, we have

$$\begin{aligned} \|x_{n+m} - p\| &\leq (1 + \gamma_{n+m-1})\|x_{n+m-1} - p\| \\ &\leq (1 + \gamma_{n+m-1})(1 + \gamma_{n+m-2})\|x_{n+m-2} - p\| \\ &\leq (1 + \gamma_{n+m-1})(1 + \gamma_{n+m-2})\|x_{n+m-2} - p\| \\ &\leq \exp(\gamma_{n+m-1} + \gamma_{n+m-2})\|x_{n+m-2} - p\| \\ &\vdots \\ &\leq \exp\left(\sum_{i=n}^{n+m-1} \gamma_i\right)\|x_n - p\| \\ &\leq M\|x_n - p\| \end{aligned} \tag{3.2}$$

where $M = \exp(\sum_{i=n}^{\infty} \gamma_i) < \infty$.

The assumption $\liminf_{n\to\infty} d(x_n, F) = 0$ implies that there exists a subsequence $\{d(x_{n_k}, F)\}$ converging to zero. Therefore, by Lemma 2.1, we get

$$\lim_{n\to\infty} d(x_n, F) = 0$$

Let $\epsilon > 0$, there exists a postive number n_o such that for all $n \geq n_o$, $d(x_n, F) < \frac{\epsilon}{2M}$ and from this inequality, there exists $p_o \in F = F(T) \cap F(I)$ such that $\|x_{n_o} - p_o\| < \frac{\epsilon}{2M}$. Hence, for all $n \geq n_0$ and $m \geq 1$, and from (3.2) we have

$$\begin{aligned} \|x_{n+m} - x_n\| &\leq \|x_{n+m} - p_o\| + \|x_n - p_o\| \\ &\leq M\|x_{n_o} - p_o\| + M\|x_{n_o} - p_o\| \\ &\leq 2M\|x_{n_o} - p_o\| \\ &\leq 2M\frac{\epsilon}{2M} = \epsilon. \end{aligned}$$

This means that $\{x_n\}$ is a Cauchy sequence in E. Since E is a Banach space which is complete, implies that $\{x_n\}$ is convergent. Assume that $x_n \to p \in E$. Since K is closed and $\{x_n\}$ is a sequence in K converging to p, we have that $p \in K$. Therefore, by Lemma 2.3, $F = F(T) \cap F(I)$ is closed. Now $\lim_{n\to\infty} d(x_n, F) = 0$ and $x_n \to p$ as $n \to \infty$, the continuity of $d(x, F)$ implies that $d(p, F) = 0$. Then $p \in F$. This completes our proof. □

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References

[1] W.R. Mann, Mean value in iteration, Proc. Amer. Math. Soc. 4 (1953) 506–510.

[2] S. Ishikawa, Fixed points by a new iteration method, Proc. Amer. Math. Soc. 44 (1974) 147–150.

[3] M.A. Noor, New approximation schemes for general variational inequalities, J. Math. Anal. Appl. 251 (2000) 217–229.

[4] B.E. Rhoades, S.M. Soltuz, The equivalence between Mann-Ishikawa iterations and multistep iteration, Nonlinear Analysis 58 (2004) 219–228.

[5] J.B. Diaz, F.T. Metcalf, On the set of subsequential limit points of successive approximations, Trans. Amer. Math. Soc. 135 (1969) 459–485.

[6] W.G. Dotson Jr., On Mann iterative process, Trans. Amer. Math. Soc. 149 (1) (1970) 65–73.

[7] W.A. Kirk, Remarks on approximation and approximate fixed points in metric fixed point theory, Annales Universitatis Mariae Curie-Sklodowska. Section A 51 (2) (1997) 167–178.

[8] N. Shahzad, Generalized I-nonexpansive maps and best approximations in Banach spaces, Demon-stratio Mathematica 37 (3) (2004) 597–600.

[9] W.V. Petryshin, T.E. Williamson Jr., Strong and weak convergence of the sequence of successive approximations for quasi-nonexpansive mappings, J. Math. Anal. Appl. 43 (1973) 459–497.

[10] M.K. Ghosh, L.Debnath, Convergence of Ishikawa iterates of quasi-nonexpansive mappings, J. Math. Anal. Appl. 207 (1) (1997) 96–103.

[11] S. Temir, O. Gul, Convergence theorem for I-asymptotically quasi- nonexpansive mapping in Hilbert space, J. Math. Anal. Appl. 329 (2007) 759–765.

[12] H. Zhou, R.P. Agarwal, Y.J. Cho, Y.S. Kim, Nonexpansive mappings and iterative methods in uniformly convex Banach spaces, Georgian Mathematical Journal 9 (3) (2002) 591–600.

[13] B.E. Rhoades, S. Temir, Convergence theorem for I-nonexpansive mapping, Int. J. Math. Math. Sci. 2006 (2006), Article ID 63435, 4 pages.

[14] H. Kiziltunc and M. Ozdemir, On convergence theorem for nonsef I-nonexpansive mapping in Banach spaces, Applied Mathematical Sciences 1 (48) (2007) 2379–2383.

[15] H. Kiziltunc, I. Yildirim, On common fixed point of nonself-nonexpansive mappings for multistep iteration in Banach spaces, Thai J. Math. 6 (2) (2008) 343–349.

[16] J. Nantadilok, Common fixed points of nonself I-quasi- nonexpansive mappings for multi-step iteration in Banach spaces, J. Pure and Applied Math. : Advances and Applications 2 (1) (2009) 87–95.

[17] Z. Opial, Weak convergence of the sequence of successive approximations for nonexpansive mappings, Bull. Amer. Math. Soc. 73 (1967) 591–597.

[18] K.K. Tan, H.K. Xu, Aproximating fixed points of nonexpansive mappings by the Ishikawa iterative process, J. Math. Anal. Appl. 178 (1993) 301–308.

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