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On Some Types of Faint Continuity

A.A. El-Atik

Department of Mathematics, Faculty of Science,
Tanta University, Tanta, Egypt
Department of Mathematics, Faculty of Science,
Shaqraa University, El-Dawadmi 11911 box 1040 KSU.
e-mail : aelatik55@yahoo.com

Abstract : Using the concept of pre-θ-open sets and semi-θ-open sets, we introduce and investigate two forms of faint continuity which are called faint pre-θ-continuity and faint semi-θ-continuity. We obtain their characterizations, their basic properties and their relationships with other forms of functions between topological spaces. Also some results of Noiri-Popa, El-Atik and Nasef are improved. Finally, we discuss the product of faintly pre-θ-continuous and faintly semi-θ-continuous functions.

Keywords : Faint pre-θ-continuity; Faint semi-θ-continuity.
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1 Introduction

In 1982, Long and Herrington [1] defined a weak form of continuity called faintly continuous by making use of θ-open sets. They obtained a large number of properties concerning such functions and among them, showed that every weakly continuous function is faintly continuous and faint continuity is equivalent to almost continuity in the sense of Signal [2] if the range is almost regular. Recently, Noiri and Popa [3] introduced and investigated three weaken forms of faint continuity which are called faint semicontinuity and faint precontinuity and faint β-continuity. In 2005, El Atik [4] defined pre-θ-open sets in topological spaces and investigate some of their properties. The purpose of the present paper is to introduce and investigate another form of faint continuity namely fain pre-θ-continuity. Some characterizations and basic properties of this new type of functions are obtained. Also, we investigate the relationships between new sorts and several weak forms of faint continuity.

2 Preliminaries

Throughout the present paper, (X,τ) and (Y,σ) denote topological spaces and $f:(X,\tau)\to(Y,\sigma)$ presents a (single valued) function from a topological space (X,τ) into a topological space (Y,σ). Let A be a subset of a space X. The closure (resp. interior) of A is denoted by $Cl(A)$ (resp. $Int(A)$), respectively. A subset A of X is said to be γ-open [5] or b-open [6] or sp-open [7] (resp. α-open [8], semi-open [9], preopen [10], β-open [11]) if $A\subset Cl(Int(A))\cup Int(Cl(A))$ (resp. $A\subset Int(Cl(Int(A)))$, $A\subset Cl(Int(A))$, $A\subset Int(Cl(A))$). The complement of a γ-open (resp. α-open, semi-open, preopen, β-open) set is called γ-closed [5] (resp. α-closed [8], semi-closed [12], preclosed [13], β-closed [11]). The family of all γ-open (resp. α-open, semi-open, preopen, β-open) sets of X is denoted by $\gamma O(X)$ (resp. $\tau^{\alpha}(X,\tau)$, $SO(X,\tau)$, $PO(X,\tau)$, $\beta O(X,\tau)$). The family of all γ-open (resp. α-open, semi-open, preopen) sets of X containing a point $x\in X$ is denoted by $\gamma(X,x)$ (resp. $\tau^{\alpha}(X,x)$, $S(X,x)$, $P(X,x)$, β(X,x)). The intersection of all γ-closed (resp. α-closed semi-closed, preclosed, β-closed) sets of X containing A is called γ-closure [5] (resp. α-closure [8], semi-closure [12], preclosure [13], β-closure [11]) of A and is denoted by $\gamma Cl(A)$ (resp.$\alpha Cl(A)$, $SCl(A)$, PCl(A), βCl(A)), respectively.

A point $x\in X$ is called a θ-cluster [14] (resp. semi-θ-cluster [15], pre-θ-cluster [4]) point of a subset A of X if $Cl(V)\cap A\neq\phi$ (resp. $s_\theta Cl(V)\cap A\neq\phi$, $p_\theta Cl(V)\cap A\neq\phi$), for every $V\in\tau$ (resp. $V\in SO(X)$, $V\in PO(X)$) respectively. The set of all θ-cluster (resp. semi-θ-cluster, pre-θ-cluster) points of A is called θ-closure [14](resp. semi-θ-closure [15], pre-θ-closure [4]) of A, respectively, and is denoted by $Cl_\theta(A)$(resp. $s_\theta Cl(A)$, $p_\theta Cl(A)$). If $A=Cl_\theta(A)$(resp. $A=s_\theta Cl(A)$, $A=p_\theta Cl(A)$), then A is said to be θ-closed [14] (resp. semi-θ-closed [15], pre-θ-closed [4]) respectively. The class of all θ-closed (resp. semi-θ-closed, pre-θ-closed) in X is denoted by $\theta C(X)$ (resp. $s_\theta C(X)$, $p_\theta C(X)$) respectively. The complement of a θ-closed (resp. semi-θ-closed, pre-θ-closed) set is said to be θ-open [14] (resp. semi-θ-open [15], pre-θ-open [4]) respectively. An equivalent definition of θ-open sets is given in [1]. A subset A is said to be θ-open if for each $x\in A$, there exists an open set U such that $x\in U\subset Cl(U)\subset A$. The family of all θ-open (resp. semi-θ-open, pre-θ-open) sets in X is denoted by X_θ(resp. $S_\theta(X)$, $P_\theta(X)$) respectively. The family of all θ-open (resp. semi-θ-open, pre-θ-open) sets of X containing a point x is denoted by (X_θ,x)(resp. $S_\theta(X,x)$, $P_\theta(X,x)$) respectively. It follows from [14, Lemma 3] that the collection of θ-open sets in a space (X,τ) forms a topology τ_θ of X. Therefore the space (X,τ_θ) will be simply denoted by X_θ. The union of all θ-open (resp. semi-θ-open, pre-θ-open) sets of X contained in A is called θ-interior [14] (resp. semi-θ-interior [15], pre-θ-interior [4]) and is denoted by $Int_\theta(A)$ (resp. $s_\theta Int(A)$, $p_\theta Int(A)$) respectively. Similarly, α Int(A) [8], γInt(A), $sInt(A)$ and $pInt(A)$ are defined in [16], [12] and [10], respectively. A subset A is a semi-regular [15] (preregular [17]) if it is both semi-open (preopen) and (semiclosed) preclosed. The family of all semi-regular (preregular) sets of X is denoted by $SR(X)$ ($PR(X)$) respectively. It is shown in [14] that $Cl_\theta(V) = Cl(V)$ for every open set V of X and $Cl_\theta(S)$ is closed in X for every subset S of X. It was shown that in [4] that $P_\theta O(X)$ a topology on X.

Definition 2.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be semicontinuous [9] (resp. precontinuous [10], α-continuous [8], γ-continuous [5], β-continuous [11]) if $f^{-1}(V) \in SO(X, \tau)$ (resp. $f^{-1}(V) \in PO(X, \tau)$, $f^{-1}(V) \in \alpha O(X, \tau)$, $f^{-1}(V) \in \gamma O(X, \tau)$, $f^{-1}(V) \in \beta O(X, \tau)$) for every open set V of Y.*

Definition 2.2. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be faintly continuous [1] (resp. faintly semi-continuous [3], faintly precontinuous [3] , faintly α-continuous [18, 19], faintly b-continuous [19], faintly β -continuous [3], faintly m-continuous [20], quasi θ-continuous[8]) if for each point $x \in X$ and each θ-open set V containing $f(x)$, there exists an open (resp. semi-open, preopen, α-open, b-open, β-open, m_X-open) set U containing x such that $f(U) \subset V$.*

3 Faintly Pre-θ(Semi-θ)-Continuous Functions

Definition 3.1. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be faintly pre-θ-continuous (resp. faintly semi-θ-continuous) if for each $x \in X$ and each θ-open set V of Y containing $f(x)$, there exits $U \in P_\theta(X, x)$ (resp. $U \in S_\theta(X, x)$) such that $f(U) \subset V$.*

The proofs of the following two theorems are not difficult and are thus omitted.

Theorem 3.1. *The following are equivalent for a function $f : (X, \tau) \to (Y, \sigma)$:*

(a) f is a faintly pre-θ-continuous,

(b) $f : (X, \tau) \to (Y, \sigma_\theta)$ is pre-θ-continuous,

(c) $f^{-1}(V) \in P_\theta(X, x)$ for every $V \in \sigma_\theta$,

(d) $f^{-1}(F)$ is pre-θ-closed in (X, τ) for every θ-closed subset F of (Y, σ),

(e) $p_\theta Cl(f^{-1}(B)) \subseteq f^{-1}(Cl_\theta(B))$ for every subset B of Y,

(f) $f^{-1}(Int_\theta(G)) \subseteq p_\theta Int(f^{-1}(G))$.

Theorem 3.2. *The following are equivalent for a function $f : (X, \tau) \to (Y, \sigma)$:*

(a) f is a faintly semi-θ-continuous,

(b) $f : (X, \tau) \to (Y, \sigma_\theta)$ is semi-θ-continuous,

(c) $f^{-1}(V) \in S_\theta(X,x)$ *for every* $V \in \sigma_\theta$,

(d) $f^{-1}(F)$ *is semi-θ-closed in* (X,τ) *for every θ-closed subset* F *of* (Y,σ),

(e) $s_\theta Cl(f^{-1}(B)) \subseteq f^{-1}(Cl_\theta(B))$ *for every subset* B *of* Y,

(f) $f^{-1}(Int_\theta(G)) \subseteq s_\theta Int(f^{-1}(G))$.

If (Y,σ) is a regular space, we have $\sigma = \sigma_\theta$ and the next theorem follows immediately from the definitions.

Definition 3.2. *A function* $f : (X,\tau) \to (Y,\sigma)$ *is said to be pre-θ-continuous (resp. semi-θ-continuous, strongly θ-continuous [21]) if* $f^{-1}(V) \in P_\theta O(X,\tau)$ *(resp.* $f^{-1}(V) \in S_\theta O(X,\tau)$, $f^{-1}(V) \in X_\theta$*) for every open set* V *of* Y.

Theorem 3.3. *Let* Y *be a regular space. Then a function* $f : X \to Y$ *is a pre-θ-continuous (resp. semi-θ-continuous) if and only if it is faintly pre-θ-continuous (resp. faintly semi-θ-continuous).*

Let $f : X \to Y$ be a function. A function $g : X \to X \times Y$ defined by $g(x) = (x, f(x))$ for every $x \in X$ is called the graph function of f.

Theorem 3.4. *A function* $f : (X,\tau) \to (Y,\sigma)$ *is faintly pre-θ-continuous (resp. faintly semi-θ-continuous) if the graph function* $g : X \to X \times Y$ *is faintly pre-θ-continuous (resp. faintly semi-θ-continuous).*

Proof. Let $x \in X$ and V be a θ-open set of Y containing $f(x)$. Then $X \times V$ is θ-open in $X \times Y$ ([1, Theorem 5]) and contains $g(x) = (x, f(x))$. Therefore there exits $U \in P_\theta(X,x)$(resp. $U \in S_\theta(X,x)$) such that $g(U) \subset X \times V$. This implies that $f(U) \subset V$. Thus f is faintly pre-θ-continuous (resp. faintly semi-θ-continuous). □

4 Comparisons

In this section, we investigate the relationships among several weak forms of continuity which are implied by weak continuity.

Theorem 4.1. *A function* $f : (X,\tau) \to (Y,\sigma)$ *is faintly pre-θ-continuous (resp. faintly semi-θ-continuous) if and only if* $f^{-1}(V) \in P_\theta(X,\tau)$ *(resp.* $f^{-1}(V) \in S_\theta(X,\tau)$*) for every θ-open set* V *of* Y.

Proof. This follows immediately from Definition 3.1. □

Definition 4.1. *A function* $f : (X,\tau) \to (Y,\sigma)$ *is said to be:*

(a) *weakly continuous [22] if* $f^{-1}(V) \subset Int(f^{-1}(Cl(V)))$, *for every* $V \in \sigma$.

(b) *almost weakly continuous [23] if* $f^{-1}(V) \subset Int(Cl(f^{-1}(Cl(V))))$, *for every* $V \in \sigma$.

Lemma 4.2. ([10]) *If $A \subset X$. Then $x \in pCl(A)$ if and only if for each $G \in PO(X)$, $x \in G$, $G \cap A \neq \phi$.*

Lemma 4.3. ([4])

(i) Let $A \in PO(X)$, then $pCl(A)$ is preregular and $pCl(A) = p_\theta Cl(A)$.

(ii) If A is pre-θ-open, then A is the union of preregular sets.

Lemma 4.4. ([4]) *The following hold for a subset A of a topological space (X, τ):*

(i) A is preregular if and only if A is pre-θ-clopen.

(ii) If A is a preopen set, then $pCl(A)$ is preregular.

(iii) A is preregular if and only if $A = pInt(pCl(A))$.

Lemma 4.5. ([24]) *For a subset A of a topological space X, the following properties hold:*

(i) If A is a semi-open set, then $sCl(A)$ is semi-regular,

(ii) If A is a semi-regular set, then it is semi-θ-open.

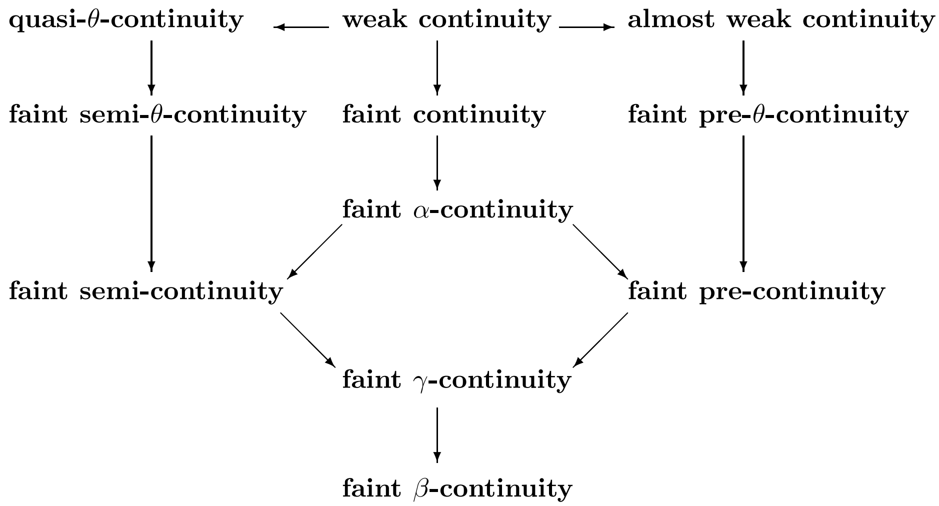
Theorem 4.6. *For a function $f : (X, \tau) \to (Y, \sigma)$ the following hold:*

(a) faintly pre-θ-continuity implies faint pre-continuity.

(b) faintly semi-θ-continuity implies faint semi-continuity.

Proof. This follows directly from Lemma 4.4 and Lemma 4.5. □

From the definitions and Theorems 3.1 and 3.2, we have the following relationships:



However the converses are not true in general by Examples 4.5, 4.6 of [3], Example 2 of [1] and the following examples.

Example 4.2. *Let* $X = Y = \{a, b, c, d\}$*,* $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ *and* $\sigma = \{Y, \phi, \{a, b\}, \{a, b, c\}\}$*. The function defined by* $f(a) = a$*,* $f(b) = d$*,* $f(c) = b$*,* $f(d) = d$ *is faint semi-θ-continuous but not faint quasi-θ-continuous because* $f^{-1}(\{d\}) = \{b, d\}$ *is pre-θ-open but not quasi-θ-open, for a θ-open set* $\{d\}$*.*

Example 4.3. *Let* (X, τ) *and* (Y, σ) *be define as in Example 4.2. The function defined by* $f(a) = d$*,* $f(b) = c$*,* $f(c) = b$*,* $f(d) = a$ *is faint pre-continuous but not faint pre-θ-continuous because* $f^{-1}(\{d\}) = \{a\}$ *is preopen but not pre-θ-open, for a θ-open set* $\{d\}$*.*

Example 4.4. *Let* $X = Y = \{a, b, c, d\}$*,* $\tau = \{Y, \phi, \{a, b\}, \{a, b, c\}\}$ *and* $\sigma = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$*. The function defined by* $f(a) = c$*,* $f(b) = d$*,* $f(c) = a$*,* $f(d) = b$ *is faint semi-continuous but not faint semi-θ-continuous because* $f^{-1}(\{c, d\}) = \{a, b\}$ *is semi-open but not semi-θ-open, for a θ-open set* $\{c, d\}$*.*

El-Atik [4] has shown that if each subset of a space X is preregular, then $PO(X, \tau) = P_\theta O(X, \tau)$. Also, Noiri and Popa [24] have shown that if each subset of a space X is semiregular, then $SO(X, \tau) = S_\theta O(X, \tau)$. Consequently we have the following result.

Theorem 4.7. *Let every subset in* (X, τ) *is preregular (resp. semi-regular). Then a function* $f : (X, \tau) \to (Y, \sigma)$ *is faintly pre-θ-continuous (resp. faintly semi-θ-continuous) if and only if faintly precontinuous (resp. faintly semicontinuous).*

Proof. It follows from Lemma 4.2, Lemma 4.3 and Lemma 4.4 (resp. by Lemma 4.5). □

A space X is said to be submaximal if each dense subset of X is open in X and extremely disconnected (ED for short) if the closure of each open set of X is open in X.

Theorem 4.8. *If (X, τ) is submaximal ED, then the following are equivalent for a function $f : (X, \tau) \to (Y, \sigma)$:*

(a) f is faintly pre-θ-continuous;

(b) f is faintly semi-θ-continuous;

(c) f is faintly precontinuous;

(d) f is faintly semicontinuous;

(e) f is faintly α-continuous;

(f) f is faintly γ-continuous;

(g) f is faintly β-continuous;

(h) f is faintly continuous.

Proof. This follows directly from the fact that if (X, τ) is submaximal ED, then $\tau = P_\theta O(X, \tau) = S_\theta O(X, \tau) = \tau^\alpha = \gamma O(X, \tau) = PO(X, \tau) = SO(X, \tau) = \beta O(X, \tau)$. □

Theorem 4.9. *Let either (X, τ) be a cofinite a cocountable topology or two point Sierpinski space. Then the following are equivalent for a function $f : (X, \tau) \to (Y, \sigma)$:*

(a) f is faintly semi-θ-continuous;

(b) f is faintly semi-continuous;

(c) f is faintly continuous.

Proof. It follows from Lemma 4.5 in [24] and Corollary 2 of [25]. □

Theorem 4.10. *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties hold:*

(a) If every subset X is preopen and closed, then each of faintly pre-θ-continuous and faintly precontinuous are equivalent.

(b) If every subset X is semiopen and closed, then each of faintly semi-θ-continuous and faintly semicontinuous are equivalent.

(c) If (X, τ) is an indiscrete space, then faintly pre-θ-continuous (resp. faintly semi-θ-continuous) and faintly precontinuous (resp. faintly semicontinuous) are equivalent.

Proof. This follows from Theorem 2.9 in [4] (resp. Lemma 4.5 in [24]). □

5 Some basic properties

In this section, using properties of $P_\theta O(X,\tau)$ (resp. $S_\theta O(X,\tau)$), we investigate restrictions, compositions products for faintly pre-θ-continuous (resp. faintly semi-θ-continuous) functions.

Theorem 5.1. *If $f:(X,\tau)\to(Y,\sigma)$ is faintly pre-θ-continuous (resp. faintly semi-θ-continuous) and $X_0\in P_\theta O(X,\tau)$ (resp. $X_0\in S_\theta O(X,\tau)$), then the restriction $f|X_0 : X_0\to Y$ is faintly pre-θ-continuous (resp. faintly semi-θ-continuous).*

Proof. We prove only the case of faint pre-θ-continuity since the order is shown similarly. Let V be any θ-open set of Y. By Theorem 3.1, we have $f^{-1}(V)\in P_\theta O(X,\tau)$ and hence by definition of $P_\theta O(X,\tau)$, $(f|X_0)^{-1}(V)=f^{-1}(V)\cap X_0\in P_\theta O(X_0)$. Therefore, it follows from Theorem 3.1 that $f|X_0$ is faintly pre-θ-continuous. □

Theorem 5.2. *If $f:X\to Y$ is faintly pre-θ-continuous (resp. faintly semi-θ-continuous) and $g:Y\to Z$ is quasi θ-continuous, then $g\circ f$ is faintly pre-θ-continuous (resp. faintly semi-θ-continuous).*

Proof. Let G be any open set in Z. Then $g^{-1}(G)$ is θ-open in Y and hence $(g\circ f)^{-1}(G)=f^{-1}(g^{-1}(G))$ is pre-θ-open (semi-θ-open) in X. Therefore $g\circ f$ is faintly pre-θ-continuous (resp. faintly semi-θ-continuous). □

From the implication between faint continuous function, we have the following result.

Theorem 5.3. *The following hold for functions $f:X\to Y$ and $g:Y\to Z$:*

(a) If f is faintly pre-θ-continuous (resp. faintly semi-θ-continuous) and g is strongly θ-continuous, then $g\circ f$ is faintly precontinuous (resp. faintly semi-continuous).

(b) If f is faintly precontinuous (resp. faintly semicontinuous) and g is faintly pre-θ-continuous (resp. faintly semi-θ-continuous), then $g\circ f$ is faintly pre-continuous (resp. faintly semicontinuous).

Noiri and Popa [3] studied the product theorems for faintly semicontinuous (resp. faintly precontinuous, faintly β-continuous) functions. Also, Nasef [19] studied the product theorems for faintly α-continuous (resp. faintly γ-continuous) functions. We investigate the product theorems for faintly pre-θ-continuous (resp. faintly semi-θ-continuous) functions.

Let $\{X_\lambda:\lambda\in\Lambda\}$ and $\{Y_\lambda:\lambda\in\Lambda\}$ be two families of spaces with the same index set Λ. For each $\lambda\in\Lambda$, let $f_\lambda : X_\lambda\to Y_\lambda$ be a function. The product space $\prod\{X_\lambda:\lambda\in\Lambda\}$ will be denoted by $\prod\{X_\lambda\}$ and the product function $\prod f_\lambda:\prod X_\lambda\to\prod Y_\lambda$ is simply denoted by $f:\prod X_\lambda\to\prod Y_\lambda$.

Lemma 5.4. ([26]) *Let $\{Y_\lambda:\lambda\in\Lambda\}$ be a family of spaces. Then $\prod(Y_\lambda)_\theta\subset(\prod Y_\lambda)_\theta$.*

Since the class of $P_\theta O(X,\tau)$ (resp. $S_\theta O(X,\tau)$ is a topology on X, then the product of pre-θ-open sets (resp. semi-θ-open sets) is pre-θ-open (resp. semi-θ-open). Then we have the following result.

Theorem 5.5. *Let $\{X_\lambda : \lambda \in \Lambda\}$ and $\{Y_\lambda : \lambda \in \Lambda\}$ be two families of spaces with the same index set Λ. For each $\lambda \in \Lambda$, let $f_\lambda : X_\lambda \to Y_\lambda$ be a function. Then a function $f : \prod X_\lambda \to \prod Y_\lambda$ defined by $f(x_\lambda) = (f_\lambda(x_\lambda))$ is pre-θ-continuous (semi-θ-continuous) if and only if f_λ is pre-θ-continuous (semi-θ-continuous) for each $\lambda \in \Lambda$.*

Proof. We prove only the case of pre-θ-continuity since the order is shown similarly. Let $V_\lambda \in \tau_{Y_\lambda}$. Then by pre-$\theta$-continuity of f_λ, we have $f^{-1}(\prod V_\lambda) = \prod f_\lambda^{-1}(V_\lambda) \in P_\theta O(\prod Y_\lambda)$. If $W \in \tau_{\prod Y_\lambda}$, then $W = \bigcup_{j\in\Lambda}(\prod V_{\lambda_j})$, where $V_{\lambda_j} \in P_\theta O(\prod Y_\lambda)$. Therefore $f^{-1}(W) = f^{-1}(\bigcup_{j\in\Lambda}(\prod V_{\lambda_j})) = \bigcup_{j\in\Lambda} f^{-1}(\prod V_{\lambda_j}) \in P_\theta O(\prod X_\lambda)$. Then f is pre-θ-continuous. □

Theorem 5.6. *If a function $f : X \to \prod Y_\lambda$ is faintly pre-θ-continuous (resp. faintly semi-θ-continuous) functions, then $P_\lambda \circ f : X \to Y_\lambda$ is faintly pre-θ-continuous (resp. faintly semi-θ-continuous) for each $\lambda \in \Lambda$, where P_λ is the projection of $\prod Y_\lambda$ onto Y_λ.*

Proof. Follows from the fact P_λ is continuous. □

Theorem 5.7. *If $f : \prod X_\lambda \to \prod Y_\lambda$ is faintly pre-θ-continuous (resp. faintly semi-θ-continuous) functions, then $f_\lambda : X_\lambda \to Y_\lambda$ is faintly pre-θ-continuous (resp. faintly semi-θ-continuous) for each $\lambda \in \Lambda$.*

Proof. Suppose that $f : \prod X_\lambda \to \prod Y_\lambda$ is faintly pre-θ-continuous (resp. faintly semi-θ-continuous). By Theorem 3.1 (resp. Theorem 3.2) and Lemma 5.4 , we have $f : \prod X_\lambda \to (\prod Y_\lambda)_\theta$ is pre-θ-continuous (resp. semi-θ-continuous). It follows from Theorem 5.5 that $f_\lambda : X_\theta \to (Y_\lambda)_\lambda$ is pre-θ-continuous (resp. semi-θ-continuous). Therefore by Theorem 3.1 (resp. Theorem 3.2) $f_\lambda : X_\lambda \to Y_\lambda$ is faintly pre-θ-continuous (resp. faintly semi-θ-continuous). □

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