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Robust Stability of Discrete-time Linear Parameter Dependent System with Delay

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Abstract : In this paper, we consider the problem of robust stability for the discrete-time linear parameter dependent (LPD) system with delay. We use the parameter dependent Lyapunov function and derive stability conditions in terms of linear matrix inequalities (LMIs). The new stability condition is more general than some existing results. Numerical examples are presented to illustrate the effectiveness of the theoretical results.

Keywords : Discrete-time LPD system; Parameter dependent Lyapunov function; Robust stability.
2000 Mathematics Subject Classification : 47H09; 47H10.

1 Introduction

Robust stability of linear continuous-time and discrete-time systems subject to time-invariant parametric uncertainty has received considerable attention in the past few decades. An important class of linear time-invariant parametric uncertain system is linear parameter dependent (LPD) system in which the uncertain state matrices are in the polytope consisting of all convex combination of known matrices. Within this context, a fundamental problem is that of establishing whether a polytope of given matrices consists of only Hurwitz matrices (for continuous-time

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case) or Schur matrices (for discrete-time case). To address this problem, several results have been obtained in terms of sufficient (or necessary and sufficient) conditions, see [1]-[16] and references cited therein. Most of these conditions have been obtained via Lyapunov theory approaches in which parameter dependent Lyapunov functions have been employed. Moreover, these conditions are always expressed in terms of linear matrix inequalities (LMI) which can be solved numerically by using available tools such as LMI toolbox in MATLAB. However, a few results have been obtained for robust stability for LPD systems in which time-delay occur in state variable. In [3] and [9], sufficient conditions have been obtained for robust stability of continuous-time LPD system with delays. In this paper, we shall give sufficient conditions of robust stability for discrete-time LPD system with delay. These conditions will be expressed in terms of LMI and when there are no delays in the system we shall obtain main results in [11] and [16] as corollaries of our results.

The paper is organized as follows. In Section 2, we shall review main notations and present problem formulation. In Section 3, based on a combination of the LMI approach and the use of parameter dependent Lyapunov function, sufficient conditions of robust stability for discrete-time LPD system with delay will be given. Numerical examples will be presented to illustrate the effectiveness of the theoretical results. The paper ends with acknowledgments and cited references.

2 Preliminaries

We first introduce some notations.

R^+- the set of all non-negative real numbers;

Z^+- the set of all non-negative integers;

R^n- the n-dimensional Euclidean space;

$M^{n\times m}-$ the space of all $n \times m$ real matrices;

A^T- the transpose of the matrix A; A is symmetric if $A = A^T$

The matrix A is positive semi-definite (negative semi-definite), denoted by $A \geq 0$ $(A \leq 0)$, if $x^TAx \geq 0$ $(x^TAx \leq 0)$ for all $x \in R^n$. The matrix A is positive definite (negative definite), denoted by $A > 0$ $(A < 0)$, if $x^TAx > 0$ $(x^TAx < 0)$ for all $x \in R^n - \{0\}$.

Consider the discrete-time LPD system with delay of the form

$$\begin{cases} x(k+1) = A(\alpha)x(k) + B(\alpha)x(k-h), & \forall k \geq 0; \\ x(k) = \phi(k), & \forall k \in [-h, 0], \end{cases} \tag{2.1}$$

where $x(k) \in R^n, k \in Z^+, h \in Z^+ - \{0\}$ is the delay and $\phi(k)$ is a vector-valued initial condition on $[-h, 0]$. $A(\alpha)$ and $B(\alpha)$ are uncertain $M^{n\times n}$ polytope matrices of the form

$$\begin{aligned} &\big[A(\alpha), B(\alpha)\big] \\ = \ &\Big[\sum_{i=1}^{N}\alpha_i A_i, \sum_{i=1}^{N}\alpha_i B_i\Big], \sum_{i=1}^{N}\alpha_i = 1, \alpha_i \geq 0, A_i, B_i \in M^{n\times n}, i = 1, ..., N. \end{aligned}$$

Definition 2.1 A function $V(x) : R^n \to R$ is called positive (negative) definite if $V(0) = 0$ and $V(x) > 0$ ($V(x) < 0$) whenever $x \neq 0$.
Definition 2.2 The system (2.1) is said to be *robustly stable* if there exists a positive definite function $V(x) : R^n \to R^+$ such that

$$\Delta V(x(k)) = V(x(k+1)) - V(x(k)) < 0$$

along the solution of the system (2.1).

There are a number of conditions for robustly stable of system (2.1) when there is no delay (namely, $B(\alpha) = 0$) given in the literatures, see [4]-[5], [11]-[14], [16] for examples.

3 Main Results

In this section, we present our main results on the robust stability criteria for discrete-time LPD system with delay. We introduce the following notation for later use,

$$M_{i,j,l}[A,P,B] = \begin{bmatrix} A_i^T P_j A_l & A_i^T P_j B_l \\ B_i^T P_j A_l & B_i^T P_j B_l \end{bmatrix}, \quad N_i[P,Q] = \begin{bmatrix} -P_i + Q_i & 0 \\ 0 & -Q_i \end{bmatrix}.$$

Theorem 3.1. *The system* (2.1) *is robustly stable if there exist positive definite symmetric matrices* P_i, Q_i, $i = 1, 2, ..., N$, *such that the following LMIs are satisfied:*

$$\begin{aligned} (i). \quad & M_{i,i,i}[A,P,B] + N_i[P,Q] < -I, \quad i = 1, 2, ..., N, \\ (ii). \quad & M_{i,j,i}[A,P,B] + M_{i,i,j}[A,P,B] + M_{j,i,i}[A,P,B] + 2N_i[P,Q] \\ & +N_j[P,Q] < \frac{1}{(N-1)^2} I, \quad i = 1, 2, ..., N, i \neq j, j = 1, 2, ..., N, \\ (iii). \quad & M_{i,j,l}[A,P,B] + M_{i,l,j}[A,P,B] + M_{j,i,l}[A,P,B] + M_{j,l,i}[A,P,B] \\ & +M_{l,i,j}[A,P,B] + M_{l,j,i}[A,P,B] + 2N_i[P,Q] + 2N_j[P,Q] \\ & +2N_l[P,Q] < \frac{6}{(N-1)^2} I, \\ & i = 1, 2, ..., N-2, j = i+1, ..., N-1, l = j+1, ..., N. \end{aligned}$$

Proof. Consider a parameter dependent Lyapunov function of the form

$$V(x(k)) = x^T(k)P(\alpha)x(k) + \sum_{i=k-h}^{k-1} x^T(i)Q(\alpha)x(i)$$

where $P(\alpha) = \sum_{i=1}^{N} \alpha_i P_i$ and $Q(\alpha) = \sum_{i=1}^{N} \alpha_i Q_i$. The difference of parameter dependent Lyapunov function along a trajectory solution of system (2.1) is given by

$$\begin{aligned}\Delta V(x(k)) &= V(x(k+1)) - V(x(k)) \\ &= x^T A^T(\alpha) P(\alpha) A(\alpha) x + x_{-h}^T B^T(\alpha) P(\alpha) A(\alpha) x \\ &\quad + x^T A^T(\alpha) P(\alpha) B(\alpha) x_{-h} + x_{-h}^T B^T(\alpha) P(\alpha) B(\alpha) x_{-h} \\ &\quad + x^T Q(\alpha) x - x^T P(\alpha) x - x_{-h}^T Q(\alpha) x_{-h}.\end{aligned}$$

where, for simplicity, $x := x(k)$, $x_{-h} := x(k-h)$. By the definition of $A(\alpha), B(\alpha), P(\alpha)$ and $Q(\alpha)$, we obtain

$$\begin{aligned}\Delta V(x(k)) &= \sum_{i=1}^{N} \alpha_i \sum_{j=1}^{N} \alpha_j \sum_{l=1}^{N} \alpha_l \left[x^T \left(A_i^T P_j A_l - P_i + Q_i\right) x\right] \\ &\quad + \sum_{i=1}^{N} \alpha_i \sum_{j=1}^{N} \alpha_j \sum_{l=1}^{N} \alpha_l \left[x_{-h}^T \left(B_i^T P_j A_l\right) x\right] \\ &\quad + \sum_{i=1}^{N} \alpha_i \sum_{j=1}^{N} \alpha_j \sum_{l=1}^{N} \alpha_l \left[x^T \left(A_i^T P_j B_l\right) x_{-h}\right] \\ &\quad + \sum_{i=1}^{N} \alpha_i \sum_{j=1}^{N} \alpha_j \sum_{l=1}^{N} \alpha_l \left[x_{-h}^T \left(B_i^T P_j B_l - Q_i\right) x_{-h}\right] \\ &= \sum_{i=1}^{N} \alpha_i^3 Y^T \left[M_{i,i,i}[A,P,B] + N_i[P,Q]\right] Y \\ &\quad + \sum_{i=1}^{N} \sum_{i \neq j, j=1}^{N} \alpha_i^2 \alpha_j Y^T \begin{bmatrix} M_{i,j,i}[A,P,B] + M_{i,i,j}[A,P,B] \\ +M_{j,i,i}[A,P,B] + 2N_i[P,Q] \\ +N_j[P,Q] \end{bmatrix} Y \\ &\quad + \sum_{i=1}^{N-2} \sum_{j=i+1}^{N-1} \sum_{l=j+1}^{N} \alpha_i \alpha_j \alpha_l Y^T \left[\Delta_{i,j,l}(M,N)\right] Y,\end{aligned}$$

where

$$\Delta_{i,j,l}(M,N) = \begin{bmatrix} M_{i,j,l}[A,P,B] + M_{i,l,j}[A,P,B] \\ +M_{j,i,l}[A,P,B] + M_{j,l,i}[A,P,B] \\ +M_{l,i,j}[A,P,B] + M_{l,j,i}[A,P,B] \\ +2N_i[P,Q] + 2N_j[P,Q] + 2N_l[P,Q] \end{bmatrix},$$

$Y = [x(k) \quad x(k-h)]^T$. By conditions $(i)-(iii)$, we get

$$\begin{aligned}\Delta V(x(k)) \quad < \quad & -Y^T[\sum_{i=1}^{N}\alpha_i^3 I - \frac{1}{(N-1)^2}\sum_{i=1}^{N}\sum_{i\neq j, j=1}^{N}\alpha_i^2\alpha_j I \\ & -\frac{6}{(N-1)^2}\sum_{i=1}^{N-2}\sum_{j=i+1}^{N-1}\sum_{l=j+1}^{N}\alpha_i\alpha_j\alpha_l I]Y.\end{aligned}$$

We define Φ and Λ as

$$\Phi \equiv \sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_i(\alpha_i-\alpha_j)^2 = (N-1)\sum_{i=1}^{N}\alpha_i^3 - \sum_{i=1}^{N}\sum_{j\neq i;j=1}^{N}\alpha_i^2\alpha_j \geq 0,$$

$$\begin{aligned}\Lambda \quad \equiv \quad & \sum_{i=1}^{N}\sum_{j\neq i;j=1}^{N-1}\sum_{l\neq i;l=2}^{N}\alpha_i[\alpha_j-\alpha_j]^2 = (N-2)\sum_{i=1}^{N}\sum_{j\neq i;j=1}^{N-1}\alpha_i^2\alpha_j \\ & -6\sum_{i=1}^{N-2}\sum_{j=i+1}^{N-1}\sum_{l=j+1}^{N}\alpha_i\alpha_j\alpha_l \geq 0.\end{aligned}$$

From $(N-1)\Phi + \Lambda \geq 0$, we obtain

$$\sum_{i=1}^{N}\alpha_i^3 - \frac{1}{(N-1)^2}\sum_{i=1}^{N}\sum_{i\neq j,j=1}^{N}\alpha_i^2\alpha_j - \frac{6}{(N-1)^2}\sum_{i=1}^{N-2}\sum_{j=i+1}^{N-1}\sum_{l=j+1}^{N}\alpha_i\alpha_j\alpha_l \geq 0.$$

Therefore, $\Delta V(x(k)) < 0$ which means that the system (2.1) is robustly stable. The proof of theorem is complete. □

Theorem 3.2. *The system* (2.1) *is robustly stable if there exist positive definite symmetric matrices* P_i, Q_i, $i = 1, 2, ..., N; Z_{iii}, i = 1, 2, ..., N; Z_{iij} = Z_{jii}^T$, $Z_{iji}, i = 1, 2, ..., N, j \neq i, j = 1, 2, ..., N; Z_{ijl} = Z_{lji}^T, Z_{ilj} = Z_{jli}^T, Z_{jil} = Z_{lij}^T$, $i = 1, 2, ..., N-2, j = i+1, ..., N-1, l = j+1, ..., N$, *such that the following LMIs are satisfied:*

(i). $M_{i,i,i}[A, P, B] + N_i[P, Q] < Z_{iii}, \quad i = 1, 2, ..., N,$

(ii). $M_{i,j,i}[A, P, B] + M_{i,i,j}[A, P, B] + M_{j,i,i}[A, P, B] + 2N_i[P, Q]$
$+N_j[P, Q] < Z_{iij} + Z_{jii}^T + Z_{iji},$
$i = 1, 2, ..., N, i \neq j, j = 1, 2, ..., N,$

(iii). $M_{i,j,l}[A, P, B] + M_{i,l,j}[A, P, B] + M_{j,i,l}[A, P, B] + M_{j,l,i}[A, P, B]$
$+M_{l,i,j}[A, P, B] + M_{l,j,i}[A, P, B] + 2N_i[P, Q] + 2N_j[P, Q]$
$+2N_l[P, Q] < Z_{ijl} + Z_{lji}^T + Z_{ilj} + Z_{jli}^T + Z_{jil} + Z_{lij}^T,$
$i = 1, 2, ..., N-2, j = i+1, ..., N-1, l = j+1, ..., N,$

(iv). $$\begin{bmatrix} Z_{1i1} & Z_{1i2} & \cdots & Z_{1iN} \\ Z_{2i1} & Z_{2i2} & \cdots & Z_{2iN} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{Ni1} & Z_{Ni2} & \cdots & Z_{NiN} \end{bmatrix} \leq 0, \quad i = 1, 2, ..., N.$$

Proof. Consider a Lyapunov function of the form

$$V(x(k)) = x^T(k)P(\alpha)x(k) + \sum_{i=k-h}^{k-1} x^T(i)Q(\alpha)x(i)$$

where $P(\alpha) = \sum_{i=1}^{N} \alpha_i P_i$ and $Q(\alpha) = \sum_{i=1}^{N} \alpha_i Q_i$. The difference of Lyapunov function along a trajectory solution of (2.1) is given by

$$\begin{aligned}
\Delta V(x(k)) &= \sum_{i=1}^{N} \alpha_i^3 Y^T \left[M_{i,i,i}[A,P,B] + N_i[P,Q]\right] Y \\
&\quad + \sum_{i=1}^{N} \sum_{i\neq j, j=1}^{N} \alpha_i^2 \alpha_j Y^T \begin{bmatrix} M_{i,j,i}[A,P,B] + M_{i,i,j}[A,P,B] \\ +M_{j,i,i}[A,P,B] + 2N_i[P,Q] + N_j[P,Q] \end{bmatrix} Y \\
&\quad + \sum_{i=1}^{N-2} \sum_{j=i+1}^{N-1} \sum_{l=j+1}^{N} \alpha_i \alpha_j \alpha_l Y^T \begin{bmatrix} M_{i,j,l}[A,P,B] + M_{i,l,j}[A,P,B] \\ +M_{j,i,l}[A,P,B] + M_{j,l,i}[A,P,B] \\ +M_{l,i,j}[A,P,B] + M_{l,j,i}[A,P,B] \\ +2N_i[P,Q] + 2N_j[P,Q] + 2N_l[P,Q] \end{bmatrix} Y,
\end{aligned}$$

where $Y = [x(k) \quad x(k-h)]^T$ (see Theorem 3.1). By conditions $(i)-(iii)$, we get

$$\begin{aligned}
\Delta V(x(k)) &< \sum_{i=1}^{N} \alpha_i^3 Y^T Z_{iii} Y + \sum_{i=1}^{N} \sum_{i\neq j, j=1}^{N} \alpha_i^2 \alpha_j Y^T [Z_{iij} + Z_{jii}^T + Z_{iji}] Y \\
&\quad + \sum_{i=1}^{N-2} \sum_{j=i+1}^{N-1} \sum_{l=j+1}^{N} \alpha_i \alpha_j \alpha_l Y^T \begin{bmatrix} Z_{ijl} + Z_{lji}^T + Z_{ilj} + Z_{jli}^T \\ +Z_{jil} + Z_{lij}^T \end{bmatrix} Y \\
&< Y^T \alpha_1 \begin{bmatrix} \alpha_1 I \\ \alpha_2 I \\ \vdots \\ \alpha_N I \end{bmatrix}^T \begin{bmatrix} Z_{111} & Z_{112} & \cdots & Z_{11N} \\ Z_{211} & Z_{212} & \cdots & Z_{21N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N11} & Z_{N12} & \cdots & Z_{N1N} \end{bmatrix} \begin{bmatrix} \alpha_1 I \\ \alpha_2 I \\ \vdots \\ \alpha_N I \end{bmatrix} Y \\
&\quad + Y^T \alpha_2 \begin{bmatrix} \alpha_1 I \\ \alpha_2 I \\ \vdots \\ \alpha_N I \end{bmatrix}^T \begin{bmatrix} Z_{121} & Z_{122} & \cdots & Z_{12N} \\ Z_{221} & Z_{222} & \cdots & Z_{22N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N21} & Z_{N22} & \cdots & Z_{N2N} \end{bmatrix} \begin{bmatrix} \alpha_1 I \\ \alpha_2 I \\ \vdots \\ \alpha_N I \end{bmatrix} Y \\
&\quad + \cdots + Y^T \alpha_N \begin{bmatrix} \alpha_1 I \\ \alpha_2 I \\ \vdots \\ \alpha_N I \end{bmatrix}^T \begin{bmatrix} Z_{1N1} & Z_{1N2} & \cdots & Z_{1NN} \\ Z_{2N1} & Z_{2N2} & \cdots & Z_{2NN} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{NN1} & Z_{NN2} & \cdots & Z_{NNN} \end{bmatrix} \begin{bmatrix} \alpha_1 I \\ \alpha_2 I \\ \vdots \\ \alpha_N I \end{bmatrix} Y.
\end{aligned}$$

Thus, we have

$$\Delta V(x(k)) \quad < \quad Y^T \left(\begin{bmatrix} \alpha_1 I \\ \alpha_2 I \\ \vdots \\ \alpha_N I \end{bmatrix}^T \sum_{i=1}^N \alpha_i \begin{bmatrix} Z_{1i1} & Z_{1i2} & \cdots & Z_{1iN} \\ Z_{2i1} & Z_{2i2} & \cdots & Z_{2iN} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{Ni1} & Z_{Ni2} & \cdots & Z_{NiN} \end{bmatrix} \begin{bmatrix} \alpha_1 I \\ \alpha_2 I \\ \vdots \\ \alpha_N I \end{bmatrix} \right) Y.$$

By the condition (iv), we obtain $\Delta V(k) < 0$. Therefore, the system (2.1) is robustly stable. □

Example 3.3. *Consider the following discrete-time LPD system with delay of the form*

$$x(k+1) = A(\alpha)x(k) + B(\alpha)x(k-h), \quad k \in Z^+, \tag{3.1}$$

where h is any positive integer and

$$A(\alpha) = \alpha_1 A_1 + \alpha_2 A_2, \quad B(\alpha) = \alpha_1 B_1 + \alpha_2 B_2,$$

where

$$A_1 = \begin{bmatrix} 0.0233 & 0.4211 \\ -0.7472 & 0.1765 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.0198 & 0.5124 \\ -0.6389 & 0.2015 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.0634 & 0.0036 \\ -0.0054 & 0.0523 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.0701 & 0.0041 \\ -0.0065 & 0.0621 \end{bmatrix}.$$

By using LMI Toolbox in MATLAB, we use the condition $(i), (ii)$ and (iii) in Theorem 3.1 for this example. The solutions of LMI verify as follows of the form

$$P_1 = 10^3 \times \begin{bmatrix} 1.6967 & -0.1306 \\ -0.1306 & 2.3893 \end{bmatrix}, \quad P_2 = 10^3 \times \begin{bmatrix} 1.7014 & -0.1525 \\ -0.1525 & 2.3556 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 521.0819 & 17.3861 \\ 17.3861 & 547.6851 \end{bmatrix}, Q_2 = \begin{bmatrix} 528.0290 & 11.8215 \\ 11.8215 & 559.8311 \end{bmatrix}.$$

Therefore, the system (3.1) *is robustly stable.* □

Example 3.4. *Consider the following discrete-time LPD systems with time-delay of the form*

$$x(k+1) = A(\alpha)x(k) + B(\alpha)x(k-h), \quad k \in Z^+, \tag{3.2}$$

where h is any positive integer and

$$A(\alpha) = \alpha_1 \begin{bmatrix} -0.52130 & 0.34646 \\ -0.21218 & -0.72940 \end{bmatrix} + \alpha_2 \begin{bmatrix} -0.63410 & 0.26354 \\ -0.25410 & -0.71280 \end{bmatrix},$$

$$B(\alpha) = \alpha_1 \begin{bmatrix} 0.0318 & -0.00234 \\ -0.00456 & 0.0350 \end{bmatrix} + \alpha_2 \begin{bmatrix} 0.0354 & -0.00364 \\ -0.00605 & 0.0834 \end{bmatrix}.$$

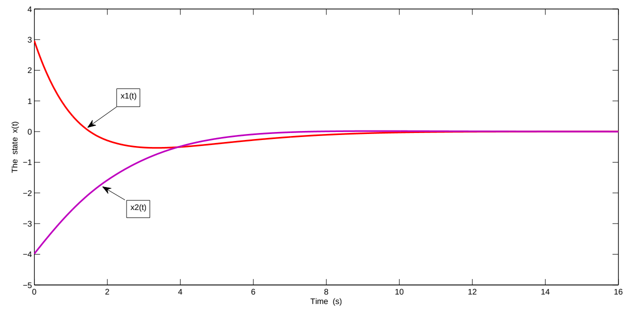

Figure 1: The Simulation solution of the states $x_1(k)$ and $x_2(k)$ in the discrete-time LPD delay system (3.1) where $h = 2, \alpha_1 = \alpha_2 = \frac{1}{2}$ with initial conditions $x_1(k) = 3$ and $x_2(k) = -4,\ k = -2, -1, 0$ by using method of Runge-Kutta order 4 with Matlab.

By using LMI Toolbox in MATLAB, we use the condition $(i), (ii), (iii)$ *and* (iv) *in Theorem 3.2 for this example. The solutions of LMI verify as follows of the form*

$$P_1 = 10^3 \times \begin{bmatrix} 3.8531 & 0.4311 \\ 0.4311 & 5.4663 \end{bmatrix}, \quad P_2 = 10^3 \times \begin{bmatrix} 4.4796 & 0.2519 \\ 0.2519 & 5.0000 \end{bmatrix},$$

$$Q_1 = 10^3 \times \begin{bmatrix} 1.3666 & 0.0713 \\ 0.0713 & 1.2885 \end{bmatrix}, \quad Q_2 = 10^3 \times \begin{bmatrix} 1.2795 & -0.0066 \\ -0.0066 & 1.2584 \end{bmatrix},$$

$$Z_{111} = 10^3 \times \begin{bmatrix} -2.9670 & 0 & 0 & 0 \\ 0 & -2.9670 & 0 & 0 \\ 0 & 0 & -2.9670 & 0 \\ 0 & 0 & 0 & -2.9670 \end{bmatrix},$$

$$Z_{222} = 10^3 \times \begin{bmatrix} -2.9770 & 0 & 0 & 0 \\ 0 & -2.9770 & 0 & 0 \\ 0 & 0 & -2.9770 & 0 \\ 0 & 0 & 0 & -2.9770 \end{bmatrix},$$

$$Z_{112} = Z_{121} = Z_{211} = 10^3 \times \begin{bmatrix} -0.4322 & 0 & 0 & 0 \\ 0 & -0.4322 & 0 & 0 \\ 0 & 0 & -0.4322 & 0 \\ 0 & 0 & 0 & -0.4322 \end{bmatrix},$$

$$Z_{212} = Z_{122} = Z_{221} = 10^3 \times \begin{bmatrix} -0.4162 & 0 & 0 & 0 \\ 0 & -0.4162 & 0 & 0 \\ 0 & 0 & -0.4162 & 0 \\ 0 & 0 & 0 & -0.4162 \end{bmatrix}.$$

Therefore, the system (3.2) *is robustly stable.* □

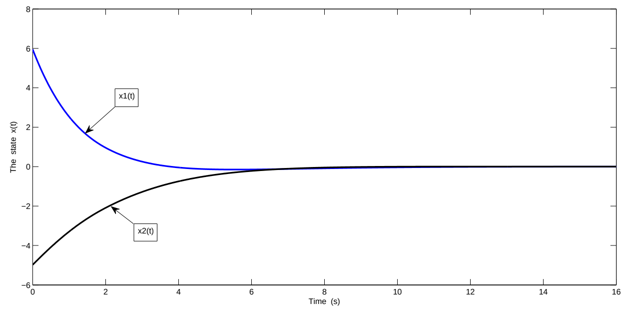

Figure 2: The Simulation solution of the states $x_1(k)$ and $x_2(k)$ in the discrete-time LPD delay system (3.2) where $h = 2, \alpha_1 = \alpha_2 = \frac{1}{2}$ with initial conditions $x_1(k) = 6$ and $x_2(k) = -5$, $k = -2, -1, 0$. by using method of Runge-Kutta order 4 with Matlab.

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