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On the Sequence Spaces of Interval Numbers

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Abstract : In recent years, mathematical structures were altered with fuzzy numbers or interval numbers and these mathematical structures have been very popular in mathematics world so we have taken courage and hope from it, and we defined bounded and convergent sequences spaces of interval numbers. The purpose of this paper is to introduce the null, convergent and bounded sequence spaces of interval numbers c_0^i, c^i and ℓ_∞^i, respectively, consisting of all sequences $\overline{x} = (\overline{x}_k)$ such that $(\overline{x}_k)$ is a sequence of interval numbers. Also some new definitions and theorems about sequence spaces of the interval numbers were given in this paper.

Keywords : Sequence of interval numbers; Banach space; Interval numbers; Solid; Interval basis.
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1 Introduction

We know that many mathematical structures have been constructed with real or complex numbers. In recent years, these mathematical structures were replaced by fuzzy numbers or interval numbers and these mathematical structures have been very popular since 1965. Interval arithmetic was first suggested by P. S. Dwyer [2] in 1951. Development of interval arithmetic as a formal system and evidence of its value as a computational device was provided by R. E. Moore [7], [9] in 1959 and 1962. Furthemore, Moore and others [2], [3], [4], [8] have developed applications to differential equations.

Recently in [1] Chiao introduced sequence of interval numbers and defined usual convergence of sequences of interval numbers and we have taken courage from him/her we defined bounded and convergent sequences spaces of interval numbers. We show that this spaces are complete metric spaces. Also we computed basis of the spaces c_0^i and c^i.

2 Preliminaries

A set consisting of a closed interval of real numbers x such that $a \leq x \leq b$ is called an interval number. A real interval can also be considered as a set. Thus we can investigate some properties of interval numbers, for instance arithmetic properties or analysis properties. Let's denote the set of all real valued closed intervals by $\mathbf{I}\mathbb{R}$. Any elements of $\mathbf{I}\mathbb{R}$ is called a closed interval and it denoted by $\overline{x}$. That is $\overline{x} = \{x \in \mathbb{R} : a \leq x \leq b\}$. An interval number $\overline{x}$ is a closed subset of real numbers [1]. Let x_ℓ and x_r be first and last points of $\overline{x}$ interval number, respectively. For all $\overline{x}_1, \overline{x}_2 \in \mathbf{I}\mathbb{R}$ we have

$\overline{x}_1 = \overline{x}_2 \Leftrightarrow x_{1_\ell} = x_{2_\ell}$ and $x_{r_1} = x_{r_2}$, $\overline{x}_1 + \overline{x}_2 = \{x \in \mathbb{R} : x_{1_\ell} + x_{2_\ell} \leq x \leq x_{r_1} + x_{r_2}\}$, if $\alpha \geq 0$ then $\alpha\overline{x} = \{x \in \mathbb{R} : \alpha x_{1_\ell} \leq x \leq \alpha x_{1_r}\}$ and if $\alpha < 0$ then $\alpha\overline{x} = \{x \in \mathbb{R} : \alpha x_{r_1} \leq x \leq \alpha x_{1_\ell}\}$,

$$\overline{x}_1\overline{x}_2 = \{x \in \mathbb{R} : \min\{x_{1_\ell}x_{2_\ell}, x_{1_\ell}x_{r_2}, x_{r_1}x_{2_\ell}, x_{1_r}x_{r_2}\} \leq x \leq \max\{x_{1_\ell}x_{2_\ell}, x_{1_\ell}x_{r_2}, x_{r_1}x_{2_\ell}, x_{r_1}x_{r_2}\}\}.$$

The set of all interval numbers $\mathbf{I}\mathbb{R}$ is a metric space [7] defined by

$$d(\overline{x}_1, \overline{x}_2) = \max\{|x_{1_\ell} - x_{2_\ell}|, |x_{1_r} - x_{2_r}|\}. \tag{2.1}$$

Moreover it is known that $\mathbf{I}\mathbb{R}$ is a complete metric space. In the special case $\overline{x}_1 = [a, a]$ and $\overline{x}_2 = [b, b]$, we obtain usual metric of the $\mathbb{R}$ with

$$d(\overline{x}_1, \overline{x}_2) = |a - b|.$$

Let's define transformation f from $\mathbb{N}$ to $\mathbf{I}\mathbb{R}$ by $k \to f(k) = \overline{x}$, $\overline{x} = (\overline{x}_k)$. Then $(\overline{x}_k)$ is called sequence of interval numbers. The $\overline{x}_k$ is called k^{th} term of sequence $(\overline{x}_k)$.

Let us denote the set of all sequences of interval number with real terms by w^i.

Given two sequences of interval numbers in w^i, say $(\overline{x}_k)$ and $(\overline{y}_k)$, then the linear structure of w^i includes the addition of $(\overline{x}_k) + (\overline{y}_k)$ and scalar multiplication $(\alpha\overline{x}_k)$ in terms by $(\overline{x}_k) + (\overline{y}_k) = [x_{k_\ell} + y_{k_\ell}, x_{k_r} + y_{k_r}]$; if $\alpha \geq 0$ then $(\alpha\overline{x}_k) = [\alpha x_{k_\ell}, \alpha x_{k_r}]$ and if $\alpha < 0$ then $(\alpha\overline{x}_k) = [\alpha x_{k_r}, \alpha x_{k_\ell}]$.

Since the set of all intervals on $\mathbb{R}$ is quasivector space [6] the set w^i be regarded as a quasivector space and the following rules are clearly satisfied: $(\overline{x}_k) + (\overline{y}_k) = (\overline{y}_k) + (\overline{x}_k)$; $(\overline{x}_k) + ((\overline{y}_k) + (\overline{z}_k)) = ((\overline{x}_k) + (\overline{y}_k)) + (\overline{z}_k)$; $(\overline{x}_k) + (\overline{y}_k) = (\overline{x}_k) + (\overline{z}_k)$ implies $(\overline{y}_k) = (\overline{z}_k)$; $\alpha((\overline{x}_k) + (\overline{y}_k)) = \alpha(\overline{x}_k) + \alpha(\overline{y}_k)$; $(\alpha + \beta)(\overline{x}_k) = \alpha(\overline{x}_k) + \beta(\overline{x}_k)$, $(where \ \ \alpha\beta \geq 0)$; $\alpha(\beta(\overline{x}_k)) = (\alpha\beta)(\overline{x}_k)$; $(\overline{x}_k) = [1, 1](\overline{x}_k)$. The zero element of w^i is the sequence $\theta = (\theta_k) = ([0, 0])$ all terms of which are zero interval.

Definition 2.1. *[1] A sequence $\overline{x} = (\overline{x}_k)$ of interval numbers is said to be convergent to the interval number $\overline{x}_0$ if for each $\varepsilon > 0$ there exists a positive integer n_0 such that $d(\overline{x}_k, \overline{x}_0) < \varepsilon$ for all $k \geq n_0$, and we denote it by writing $\lim_k \overline{x}_k = \overline{x}_0$.*

Thus, $\lim\limits_{k\to\infty} \overline{x}_k = \overline{x}_0 \Leftrightarrow \lim\limits_{k\to\infty} x_{k_\ell} = x_{0_\ell}$ and $\lim\limits_{k\to\infty} x_{k_r} = x_{0_r}$.

3 Some Sequence Spaces of the Interval Numbers

In this section we define sequence spaces null, convergent and bounded of the interval numbers.

Let us denote the space of all null, convergent, bounded sequences of interval number by c_0^i, c^i and ℓ_∞^i respectively, that is

$$\begin{aligned} c_0^i =&\{\overline{x} = (\overline{x}_k) \in w^i : \lim_k \overline{x}_k = \theta, \quad \text{where} \quad \theta = [0,0]\}, \\ c^i =&\{\overline{x} = (\overline{x}_k) \in w^i : \lim_k \overline{x}_k = \overline{x}_0, \quad \overline{x}_0 \in \mathbf{I}\mathbb{R}\}, \\ \ell_\infty^i =&\{\overline{x} = (\overline{x}_k) \in w^i : \sup_k \{|x_{k_\ell}|, |x_{k_r}|\} < \infty\}. \end{aligned}$$

Clearly we see that the spaces c_0^i, c^i and ℓ_∞^i are subspaces of the space w^i. Besides, for all $(\overline{x}_k), (\overline{y}_k) \in c_0^i$ (*or* c^i, ℓ_∞^i) the $\tilde{d}$ defined by

$$\tilde{d}(\overline{x}_k, \overline{y}_k) = \sup_k \max\{|x_{k_\ell} - y_{k_\ell}|, |x_{k_r} - y_{k_r}|\} \tag{3.1}$$

satisfies metric axioms. Thus, $(c_0^i, \tilde{d})$ (*or* $(c^i, \tilde{d})$ *and* $(\ell_\infty^i, \tilde{d})$) is a metric space.

Definition 3.1. *Let's suppose that $\overline{y} \in w^i$, $\overline{y} = ([y_{k_\ell}, y_{k_r}])$. If $y_{k_\ell} = y_{k_r}$, for all $k \in \mathbb{N}$, then the sequence $\overline{y} = (\overline{y}_k)$ is called degenerate interval sequence .*

If $\overline{x} = (\overline{x}_k)$ and $\overline{y} = (\overline{y}_k)$ are degenerate interval sequences then the metric in (3.1) reduces on the classical sequence spaces (i.e., null, convergent and bounded of the real or complex numbers). In fact, we easily see that the space of all real valued sequences w is degenerate sequences space since every real number is a degenerate interval. Therefore, each subspace of w is called a degenerate sequence space. We shall write ℓ_∞, c and c_0 for the spaces of all degenerate bounded, degenerate convergent and degenerate null sequences, respectively.

Definition 3.2. *An interval sequence $\overline{x} = (\overline{x}_k) \in w^i$ is said to be interval Cauchy sequence if for every $\varepsilon > 0$ there exists a $k_0 \in \mathbb{N}$ such that $\tilde{d}(\overline{x}_n, \overline{x}_m) < \varepsilon$ whenever $n, m > k_0$.*

Based on the definitions above, we give a theorem on completeness.

Theorem 3.3. *$(c_0^i, \tilde{d})$, $(c^i, \tilde{d})$ and $(\ell_\infty^i, \tilde{d})$ are complete metric spaces with the metric defined by in (3.1).*

Proof. We only give the proof for $(c_0^i, \tilde{d})$.

Let $(\overline{x}^n) = (\overline{x}_k^n) = (\overline{x}_0^n, \overline{x}_1^n, \overline{x}_2^n, ...) \in c_0^i$ for each n and $(\overline{x}^n)$ be a Cauchy sequence. Then, for every $\varepsilon > 0$ there exist a $k_0 \in \mathbb{N}$ such that $\tilde{d}(\overline{x}_k^n, \overline{x}_k^m) < \varepsilon$ whenever $n, m \geq k_0$. Hence, we have

$$\sup_{n,m} \max\{|x_{k_\ell}^n - x_{k_\ell}^m|, |x_{k_r}^n - x_{k_r}^m|\} < \varepsilon,$$

thus we have $|x_{k_\ell}^n - x_{k_\ell}^m| < \varepsilon$ and $|x_{k_r}^n - x_{k_r}^m| < \varepsilon$. This means that $(\overline{x}_k^n)$ is a Cauchy sequence in $\mathbb{R}$. Since $\mathbb{R}$ is a Banach space, $(\overline{x}_k^n)$ is convergent.

Now, let $\lim_{n\to\infty} \overline{x}_k^n = \overline{x}_k$ for each $k \in \mathbb{N}$. Since $\tilde{d}(\overline{x}_k^n, \overline{x}_k^m) < \varepsilon$ for all $n, m \geq k_0$,

$$\lim_{m\to\infty} \tilde{d}(\overline{x}_k^n, \overline{x}_k^m) = \tilde{d}(\overline{x}_k^n, \lim_{m\to\infty} \overline{x}_k^m) = \tilde{d}(\overline{x}_k^n, \overline{x}_k) < \varepsilon.$$

This implies that $\overline{x}^n \to \overline{x}$, $(n \to \infty)$ for all $n \geq k_0$ in c_0^i. On the other hand, since

$$\begin{aligned} \tilde{d}(\overline{x}_k, \overline{x}_k^n - \overline{x}_k^n) &= \sup_k \max\{|\underline{x}_k - (\underline{x}_k^n - \underline{x}_k^n)|, |\overline{x}_k - (\overline{x}_k^n - \overline{x}_k^n)|\} \\ &\leq \sup_k \max\{|\underline{x}_k - \underline{x}_k^n| + |\underline{x}_k^n|, |\overline{x}_k - \overline{x}_k^n| + |\overline{x}_k^n|\} \\ &\leq \sup_k \max\{|\underline{x}_k - \underline{x}_k^n|, |\overline{x}_k - \overline{x}_k^n|\} + \sup_k \max\{|\underline{x}_k|, |\overline{x}_k|\}, \end{aligned}$$

this shows that $\overline{x} \in c_0^i$. □

The norm function on the classical sequence spaces can be extended to the sequence spaces of the interval numbers. Suppose that λ^i is a subset of w^i.

Definition 3.4. *[1] A norm on λ^i is a non-negative function $||\cdot||_{\lambda^i} = \lambda^i \to \mathbb{R}^+ \cup \{0\}$ that satisfies the following properties: $\forall \overline{x}, \overline{y} \in \lambda^i$ and $\forall \alpha \in \mathbb{R}$ $\forall \overline{x} \in \lambda^i - \{\theta\}$,*

N1. $||\overline{x}||_{\lambda^i} > 0$;

N2. $||\overline{x}||_{\lambda^i} = 0 \Leftrightarrow \overline{x} = \theta$;

N3. $||\overline{x} + \overline{y}||_{\lambda^i} \leq ||\overline{x}||_{\lambda^i} + ||\overline{x}||_{\lambda^i}$

N4. $||\alpha\overline{x}||_{\lambda^i} = |\alpha|||\overline{x}||_{\lambda^i}$.

As we know that the norm $||x||$ of x is the distance from x to 0 in the sequences space real numbers (see, [5]). Then this idea can be extended on the metric spaces $(c_0^i, \tilde{d})$, $(c^i, \tilde{d})$ and $(\ell_\infty^i, \tilde{d})$.

Let $\tilde{d}(\overline{x}_k, \theta) = \sup_k \max\{|x_{k_\ell}, \theta_{k_\ell}|, |x_{k_r}, \theta_{k_r}|\} = \sup_k \max\{|x_{k_\ell}|, |x_{k_r}|\}$ where θ is unit element of the spaces c_0^i, c^i and ℓ_∞^i. After these explanations, we have

Theorem 3.5. *The spaces c_0^i, c^i and ℓ_∞^i are normed interval spaces with the norm*

$$||\overline{x}|| = \sup_k \max\{|x_{k_\ell}|, |x_{k_r}|\}. \tag{3.2}$$

Proof. Let $\lambda^i = c_0^i(or \;\; c^i \;\; and \;\; \ell_\infty^i)$ and $\overline{x}, \overline{y} \in \lambda^i$.

N1. Since $||\overline{x}||_{\lambda^i} = \sup_k \max\{|x_{k_\ell}|, |x_{k_r}|\}$ we easily see that $||\overline{x}||_{\lambda^i} > 0$ for $\forall \overline{x} \in \lambda^i - \{\theta\}$.

N2. $||\overline{x}||_{\lambda^i} = 0 \Leftrightarrow \sup_k \max\{|x_{k_\ell}|, |x_{k_r}|\} = 0 \Leftrightarrow \overline{x} = \theta$,

N3.

$$\begin{aligned} ||\overline{x} + \overline{y}||_{\lambda^i} &= \sup_k \max\{|x_{k_\ell} + y_{k_\ell}|, |x_{k_r} + y_{k_r}|\} \\ &\leq \sup_k \max\{||x_{k_\ell}| + |y_{k_\ell}|, |x_{k_r}| + |y_{k_r}|\} \\ &= \sup_k \max\{(|x_{k_\ell}|, |x_{k_r}|) + (|y_{k_\ell}|, |y_{k_r}|)\} \\ &\leq \sup_k \max\{(|x_{k_\ell}|, |x_{k_r}|)\} + \sup_k \max\{(|y_{k_\ell}|, |y_{k_r}|)\} = ||\overline{x}||_{\lambda^i} + ||\overline{y}||_{\lambda^i}, \end{aligned}$$

N4. $||\alpha\overline{x}||_{\lambda^i} = \sup_k \max\{|\alpha x_{k_\ell}|, |\alpha x_{k_r}|\} = |\alpha| \sup_k \max\{|x_{k_\ell}|, |x_{k_r}|\} = |\alpha| ||\overline{x}||_{\lambda^i}$. So $||\overline{x}||_{\lambda^i}$ is a norm on λ^i.

□

Now let's give definition of interval base.

Definition 3.6. *Let λ^i be normed sequence space of the interval numbers. If λ^i contains an interval sequence $(\overline{y}_k)$ with the property that for every $\overline{x} \in \lambda^i$ there is a unique sequence of scalars (α_k) such that*

$$\lim_k ||\overline{x} - (\alpha_0 \overline{y}_0 + \alpha_1 \overline{y}_1 + ... + \alpha_k \overline{y}_k)|| = \theta$$

then $(\overline{y}_k)$ is called an interval basis for λ^i. The series $\sum_k \alpha_k \overline{y}_k$ which has the sum $\overline{x}$ is called the expansion of $\overline{x}$ with the respect to $(\overline{y}_k)$ and written as $\overline{x} = \sum_k \alpha_k \overline{y}_k$.

Let $\overline{\theta} = ([\theta_{k_\ell}, \theta_{k_r}]) = ([0,0])$, $\overline{x}' = ([x'_\ell, x'_r]) = ([0,1])$ and (E_k) be an intervals sequence whose k^{th} position is $\overline{x}'$ and others all θ. Let's suppose that

$$\min\{x_{1_\ell} x_{2_\ell}, x_{1_\ell} x_{r_2}, x_{r_1} x_{2_\ell}, x_{1_r} x_{r_2}\} \tag{3.3}$$

and

$$\max\{x_{1_\ell} x_{2_\ell}, x_{1_\ell} x_{r_2}, x_{r_1} x_{2_\ell}, x_{1_r} x_{r_2}\} \tag{3.4}$$

in multiplication of $\overline{x}_1$ and $\overline{x}_2$.

Now we may establish interval basis of the spaces c_0^i and c^i.

Theorem 3.7. *The set $\{E_k : k = 0, 1, 2, ...\}$ is an interval basis for c_0^i under norm defined by (3.2) and conditions (3.3) with (3.4).*

Proof. Let $\overline{x} = (x_{k_r}) \in c_0^i$ and $\lim_k x_{k_r} = \theta$. Then for every $\varepsilon > 0$ there exists a $n \in \mathbb{N}$ such that

$$||\overline{x}||_{c_0^i} = \tilde{d}(x_{k_r}, \theta) = \sup_k \max\{|x_{k_\ell}|, |x_{k_r}|\} < \varepsilon$$

whenever $k \geq n$. Now, since

$$\begin{aligned} K &= ||\overline{x} - \sum_{k=0}^{n} \overline{x} E_k||_{c_0^i} = ||\overline{x} - (\overline{x}_0 E_0 + \overline{x}_1 E_1 + \cdots + \overline{x}_n E_n)||_{c_0^i} \\ &= ||\overline{x} - (\underbrace{[x_{0_\ell}, x_{0_r}]([0,1],[0,0],\cdots)}_{1.position} + \underbrace{[x_{1_\ell}, x_{1_r}]([0,0],[0,1],\cdots)}_{2.position} + \ldots \\ &+ \underbrace{[x_{n_\ell}, x_{n_r}]([0,0],[0,0],\cdots,[0,1],\cdots)}_{n.position} ||_{c_0^i}. \end{aligned}$$

From (3.3) and (3.4), we see that 1^{th} position=$([x_{0_\ell}, x_{0_r}], [0,0], [0,0], \ldots)$, 2^{th} position=$([0,0], [x_{1_\ell}, x_{1_r}], [0,0], \ldots), \cdots$ and n^{th} position=$([0,0], \ldots, [0,0], [x_{n_\ell}, x_{n_r}])$. Thus,

$$\begin{aligned} K &= ||\theta, \theta, \ldots, [x_{(n+1)_\ell}, x_{(n+1)_r}], [x_{(n+2)_\ell}, x_{(n+2)_r}], \ldots||_{c_0^i} \\ &= \sup_{k \geq n+1} \max\{|x_{k_\ell}|, |x_{k_r}|\} \to \theta, \quad (n \to \infty) \end{aligned}$$

and we have

$$\overline{x} = \sum_k \overline{x} E_k. \tag{3.5}$$

Let us show that uniqueness of the representation for $\overline{x} \in c_0^i$ given by (3.5). On the contrary, suppose that there exists a representation $\overline{x} = \sum_k \overline{y} E_k$. Then,

$$\begin{aligned} &|| \sum_{k=0}^{n} (\overline{y}_k - \overline{x}_k) E_k|| = \tilde{d}((\overline{y} - \overline{x}), \theta) \\ &\qquad = \sup_{k \geq n+1} \max\{|(y_{k_\ell} - x_{k_\ell}) - 0|, |(y_{k_r} - x_{k_r}) - 0|\} \to \theta \end{aligned}$$

for $n \to \infty$. This shows that $|y_{(k \geq n+1)_\ell} - x_{(k \geq n+1)_\ell}| \to 0$ and $|y_{(k \geq n+1)_r} - x_{(k \geq n+1)_r}| \to 0$. In this case, we have, $y_{(k \geq n+1)_\ell} = x_{(k \geq n+1)_\ell}$ and $y_{(k \geq n+1)_r} = x_{(k \geq n+1)_r}$, i.e., $\overline{x} = \overline{y}$. □

Theorem 3.8. *The set $\{E, \ \ E_k : k = 0, 1, 2, \ldots\}$ is a degenerate interval basis for c^i under norm defined by (3.2) and conditions (3.3) with (3.4), where $E = (\overline{x}', \overline{x}', \ldots)$.*

Proof. Let $\overline{x} = (\overline{x}_k) \in c^i$ and $\lim_k \overline{x}_k = \overline{x}_0$. Then for every $\varepsilon > 0$ there exists a $n \in \mathbb{N}$ such that whenever $k \geq n$. Since,

$$||\overline{x} - \overline{x}_0 E - \sum_{k=0}^{n} (\overline{x}_k - \overline{x}_0) E_k|| = \sup_{k \geq n+1} \max\{|x_{k_\ell} - x_{0_\ell}|, |x_{k_r} - x_{0_r}|\} \to \theta, \quad (n \to \infty).$$

Then, we have $\overline{x} = \overline{x}_0 E + \sum_{k=0}^{n} (\overline{x}_k - \overline{x}_0) E_k$. It is easy to check that this representation for $\overline{x}$ is unique. □

Definition 3.9. *Let λ^i is a sequence space of the interval numbers. Then λ^i is called normal or solid if $\overline{y} \in \lambda^i$ whenever $||\overline{y}_k|| \leq ||\overline{x}_k||$, $(k \in \mathbb{N})$ for some $\overline{x} \in \lambda^i$.*

Theorem 3.10. *The spaces c_0^i and c^i are solid and monotone.*

Proof. We consider only c_0^i. Now, let $||\overline{y}_k|| \leq ||\overline{x}_k||$, for all $(k \in \mathbb{N})$ and for some $\overline{x} \in c_0^i$. Then we have, $\tilde{d}(\overline{y}_k, \theta) \leq \tilde{d}(\overline{x}_k, \theta)$, that is $\{|y_{k_\ell} - 0|, |y_{k_r} - 0|\} \leq \{|x_{k_\ell} - 0|, |x_{k_r} - 0|\}$. Thus we obtain $y_{k_\ell} \leq x_{k_\ell}$ and $y_{k_r} \leq x_{k_r}$ i.e., $\overline{y} \leq \overline{x}$. It is clear that $\overline{y} \in c_0^i$. Therefore c_0^i is solid or normal. □

Theorem 3.11. *The inclusion $w \subset w^i$ holds.*

Proof. The proof is clear since every element of w is a degenerate interval sequence, (see, Definition 3.1). Also, the inclusions $\ell_\infty \subset \ell_\infty^i$, $c \subset c^i$ and $c_0 \subset c_0^i$ holds. □

Theorem 3.12. *The inclusion $c_0^i \subset c^i$ holds.*

Proof. If we take any $\overline{x} \in c_0^i$ then we see that $\overline{x} \in c^i$ since $\tilde{d}(\overline{x}_k, \theta) = \sup_k \max\{|x_{k_\ell} - 0|, |x_{k_r} - 0|\} < \varepsilon$. Furthermore, the convergent sequence of the interval numbers $\overline{y} = ([1, 1 + \frac{1}{n}]) \in c^i$ but $\overline{y} \notin c_0^i$ since $\lim_n y_n^- = 1$ and $\lim_n y_n^+ = 1$. □

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