# Analysis of Greens Function and Influence Function in Fluid Saturated Porous Medium Underlying a Inviscid Fluid Layer

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**Abstract :** The article reports on a methodology to synthesize the response of Greens function and influence function in a fluid saturated porous half space underlying a inviscid fluid layer .Closed form solutions for displacements, stresses, pore pressure and acoustic pressure are obtained by applying the Laplace transform on time and Fourier transform on the space variables. Concentrated and distributed sources are taken to illustrate the utility of the approach. Numerical inversion technique has been applied to obtain the solutions in physical domain. Effect of pore pressure on displacement and stress components are shown graphically for a particular model. Some special case of interest has been deduced.

**Keywords :** Porous, fluid layer, Laplace and Fourier transform.

## 1 Introduction

Most of the modern engineering structures are generally made up of multiphase porous continuum, the classical theory, which represents a fluid saturated porous medium as a single phase material, is inadequate to represent the mechanical behavior of such materials especially when the pores are filled with liquid. In this context the solid and liquid phases have different motions.Due to these different motions, the different material properties and the complicated geometry of pore structures, the mechanical behavior of a fluid saturated porous medium is very complex and difficult. So from time to time, researchers have tried to overcome this difficulty. For more details and for the historical review on the subject of the multiphase continuum mechanics, the reader is referred to the work of de Boer and Ehlers [7] or to the recently published monograph de Boer [6].

Based on the work of von Terzaghi [19,20], Biot [1]proposed a general theory of three- dimensional deformations of fluid saturated porous solid.Then the wave propagation and dynamic extensions were done by Boit [2-4]. Boit theory is based on the assumption of compressible constituents and till recently, some of his results have been taken as standard references and basis for subsequent analysis in acoustic, geophysics and other such fields.

Based on the work of Fillunger model [12] ( which is further based on the concept of volume fractions combined with surface porosity coefficients) , Bowen [5] and de Boer Ehlers [8,9] developed and used another interesting theory in which all the constituents of a porous medium are assumed to be incompressible . There are reasonable grounds for the assumption that the constituents of many fluid saturated porous media are incompressible. For example, taking the composition of soil, the solid constituents are incompressible and liquid constituents, which are generally water or oils are also incompressible. Moreover in an empty porous solid as a case of classical theory, the change in the volume is due to the changes in porosity during the propagation of a longitudinal wave. The assumption of incompressible constituents does not only meet the properties appearing in many branches of engineering practice, but it also avoids the introduction of many complicated material parameters as considered in the Biot theory. So this model meets the requirements of further scientific developments. Based on this theory de Boer and Ehlers [10] and Recently, Kumar and Hundal [14-18] studied some problems of wave propagation in fluid saturated incompressible porous media. However, no attempt has been made to study the response of source problem in inviscid fluid layer lying over a fluid saturated incompressible porous media.

The present investigation seeks to determine the components of displacement, stress, pore pressure and acoustic pressure due to various types of sources acting on a inviscid fluid layer lying over a fluid saturated incompressible porous half space. Solutions are obtained by employing integral transform techniques. Integral transforms are inverted using a numerical method .The results of the problem may be applied to a wide class of geophysical problems. The physical applications are encountered in the context of problems such as ground explosions and oil industries. This problem is also useful in the field of geomechanics where the interests in various phenomenons occurring in earthquakes and measurement of displacement, stresses, pore pressure due to presence of certain sources.

## 2    Problem Formulation

We consider a normal point load acting over the fluid layer (depth H) and lying over the homogenous fluid saturated incompressible porous half space $z \geq 0$ of a rectangular Cartesian coordinate system (x, y, z) having origin on the surface z =0 and z axis pointing vertically into the medium. (Fig.1)

Following de Boer and Ehlers [9], the equations governing the deformation of an incompressible porous medium saturated with non-viscous fluid in the absence

of body forces are

$$\nabla.(\eta^S \dot{\mathbf{u}}_S + \eta^F \dot{\mathbf{u}}_F) = 0, \tag{2.1}$$

$$(\lambda^S + \mu^S)\nabla(\nabla.\mathbf{u_S}) + \mu^S\nabla^2\mathbf{u_S} - \eta^S\nabla p - \rho^S\ddot{\mathbf{u}}_\mathbf{S} + \mathbf{s_v}(\dot{\mathbf{u}}_\mathbf{F} - \dot{\mathbf{u}}_\mathbf{S}) = 0, \tag{2.2}$$

$$\eta^F\nabla p + \rho^F\ddot{\mathbf{u}}_\mathbf{F} + \mathbf{s_v}(\dot{\mathbf{u}}_\mathbf{F} - \dot{\mathbf{u}}_\mathbf{S}) = 0, \tag{2.3}$$

$$\mathbf{T^s_E} = 2\mu^S\mathbf{E_S} + \lambda^S(\mathbf{E_S}.\mathbf{I})\mathbf{I}, \tag{2.4}$$

$$\mathbf{E_S} = \frac{1}{2}(grad\mathbf{u_S} + grad^T\mathbf{u_S}), \tag{2.5}$$

where

$\mathbf{u_i}, \dot{\mathbf{u}}_\mathbf{i}, \ddot{\mathbf{u}}_\mathbf{i}$, i = S,F denote the displacement, velocities and acceleration of solid and fluid phases respectively and p is the effective pore pressure of the incompressible pore fluid. $\rho^S$ And $\rho^F$ are the densities of the solid and fluid respectively.$\mathbf{T^s_E}$ is the stress in the solid phase and $\mathbf{E}_S$ is the linearized langrangian strain tensor. $\lambda^S$ and $\mu^S$ are the macroscopic Lame's parameters of the porous solid and $\eta^S$ and $\eta^F$ are the volume fraction s satisfying $\eta^S + \eta^F = 1$.

The case of isotropic permeability, the tensor $\mathbf{S}_V$ describing the coupled interaction between the solid and fluid is given by de Boer and Ehlers [9] as

$\mathbf{S}_V = (\frac{(\eta^F)^2\gamma^{FR}}{K^F})\mathbf{I} = S_V\mathbf{I}$,

where $\gamma^{FR}$ is the effective specific weight of the fluid and $K^F$ is the Darcy's permeabilitbility coefficient of the porous medium.

The equation which govern governs the motion of fluid is given Ewing et al[11] as

$$\lambda^L\nabla(\nabla.\mathbf{u_L}) = \rho^L\frac{\partial^2\mathbf{u_L}}{\partial t^2}, \tag{2.6}$$

The stress and displacement relations are given by

$$T^L_{mn} = \lambda^L\nabla\mathbf{u_L}\delta_{mn}, m, n = 1, 2, 3, \tag{2.7}$$

where $\rho^L$ is density of fluid and $\lambda^L$ is lame's constant.

For two dimensional problem, we assume the displacement vector $\mathbf{u_i}$ (i=F, S, L) as

$$\mathbf{u_i} = (u^i, 0, w^i), where, i = F, S, L. \tag{2.8}$$

The field equations for a fluid saturated incompressible medium are

$$(\lambda^S + \mu^S)\frac{\partial\theta^S}{\partial x} + \mu^S\nabla^2 u^S - \eta^S\frac{\partial p}{\partial x} - \rho^S\frac{\partial^2 u^S}{\partial t^2} + S_V[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}] = 0, \tag{2.9}$$

$$(\lambda^S + \mu^S)\frac{\partial\theta^S}{\partial z} + \mu^S\nabla^2 w^S - \eta^S\frac{\partial p}{\partial z} - \rho^S\frac{\partial^2 w^S}{\partial t^2} + S_V[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}] = 0, \tag{2.10}$$

$$\eta^F\frac{\partial p}{\partial x} + \rho^F\frac{\partial^2 u^F}{\partial t^2} + S_V[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}] = 0, \tag{2.11}$$

$$\eta^F\frac{\partial p}{\partial z} + \rho^F\frac{\partial^2 w^F}{\partial t^2} + S_V[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}] = 0, \tag{2.12}$$

$$\eta^S\left[\frac{\partial^2 u^S}{\partial x \partial t} + \frac{\partial^2 w^S}{\partial z \partial x}\right] + \eta^F\left[\frac{\partial^2 u^F}{\partial x \partial t} + \frac{\partial^2 w^F}{\partial z \partial x}\right] = 0, \tag{2.13}$$

where
$\theta^S = \frac{\partial u^S}{\partial x} + \frac{\partial w^S}{\partial z}$.

Using (8) in (6) and (7) yields

$$\lambda^L \frac{\partial}{\partial x}\left[\frac{\partial u^L}{\partial x} + \frac{\partial w^L}{\partial z}\right] = \rho^L \frac{\partial^2 u^L}{\partial t^2}, \tag{2.14}$$

$$T_{33}^L = T_{11}^L = \lambda^L\left[\frac{\partial u^L}{\partial x} + \frac{\partial w^L}{\partial z}\right]. \tag{2.15}$$

For further consideration it is convenient to introduce in equations (9)-(15) the dimensionless quantities defined as:

$$x' = \frac{\omega^*}{C_1}x, z' = \frac{\omega^*}{C_1}z, t' = \omega^* t, u'^S = \left[\frac{\lambda^S + 2\mu^S}{E}\right]\frac{\omega^*}{C_1}u^S, w'^S = \left[\frac{\lambda^S + 2\mu^S}{E}\right]\frac{\omega^*}{C_1}w^S,$$

$$u'^F = \left[\frac{\lambda^S + 2\mu^S}{E}\right]\frac{\omega^*}{C_1}u^F, w'^F = \left[\frac{\lambda^S + 2\mu^S}{E}\right]\frac{\omega^*}{C_1}w^F, u'^L = \left[\frac{\lambda^S + 2\mu^S}{E}\right]\frac{\omega^*}{C_1}u^L,$$

$$w'^L = \left[\frac{\lambda^S + 2\mu^S}{E}\right]\frac{\omega^*}{C_1}w^L, T_{33}' = \frac{T_{33}}{E}, T_{31}' = \frac{T_{31}}{E}, T_{33}'^L = \frac{T_{33}^L}{E}, p' = \frac{p}{E}. \tag{2.16}$$

In these relations E is the Young's modulus of the solid phase, $\omega^*$ is a constant having the dimensions of frequency, $C_1$ is the velocity of a longitudinal wave propagating in a fluid saturated incompressible porous medium and is given by

$$C_1 = \sqrt{\frac{(\eta^F)^2(\lambda^S + 2\mu^S)}{(\eta^F)^2\rho^S + (\eta^S)^2\rho^F}}. \tag{2.17}$$

If pore is absent or gas is filled in the pores then is very small as compare to and can be neglected so the relation reduce to

$$C_0 = \sqrt{\frac{(\lambda^S + 2\mu^S)}{\rho^S}}. \tag{2.18}$$

This gives the velocity of the longitudinal wave propagating in an incompressible empty porous solid where the change in volume is due to the change in porosity and well known result of the classical theory of elasticity. In an incompressible non porous solid $\eta^F \to 0$, then (17) becomes $C_1 = 0$ and physically acceptable as longitudinal wave cannot propagate in an incompressible medium.

The displacement components $u^i$ and $w^i$ are related to the non dimensional potentials $\phi^i$ and $\psi^i$ as

$$u^i = \frac{\partial \phi^i}{\partial x} + \frac{\partial \psi^i}{\partial z}, w^i = \frac{\partial \phi^i}{\partial z} - \frac{\partial \psi^i}{\partial x}, (i = F, S) \tag{2.19}$$

$$u^L = \frac{\partial \phi^L}{\partial x}, w^L = \frac{\partial \phi^L}{\partial z}. \tag{2.20}$$

Making use of non dimensional quantities given by (16) in equations (9) -(15) and with the help of (19)and (20), we obtain the following equations determining $\phi^S, \phi^F, p, \psi^S, \psi^F, \phi^L$ as :

$$\nabla^2 \phi^S - \frac{\partial^2 \phi^S}{\partial t^2} - \frac{\delta_2}{(\eta^F)^2} \frac{\partial \phi^S}{\partial t} = 0, \tag{2.21}$$

$$\phi^F = -\frac{\eta^S}{\eta^F} \phi^S, \tag{2.22}$$

$$(\eta^F)^2 p - \eta^S \delta_1^2 \frac{\rho^F}{\rho^S} \frac{\partial^2 \phi^S}{\partial t^2} - \delta_2 \frac{\partial \phi^S}{\partial t} = 0, \tag{2.23}$$

$$\delta^2 \nabla^2 \psi^S - \delta_1^2 \frac{\partial^2 \psi^S}{\partial t^2} + \delta_2 [\frac{\partial \psi^F}{\partial t} - \frac{\partial \psi^S}{\partial t}] = 0, \tag{2.24}$$

$$\delta_1^2 \frac{\rho^F}{\rho^S} \frac{\partial^2 \psi^F}{\partial t^2} + \delta_2 [\frac{\partial \psi^F}{\partial t} - \frac{\partial \psi^S}{\partial t}] = 0, \tag{2.25}$$

$$\nabla^2 \phi^L = \delta_5^2 \frac{\partial^2 \phi^L}{\partial t^2}, \tag{2.26}$$

where
$\delta_1 = \frac{C_1}{C_0}, \delta = \frac{\beta_0}{C_0}, \beta_0 = \sqrt{\frac{\mu^S}{\rho^S}}, \delta_2 = \frac{S_V C_1^2}{\omega^* \rho^S C_0^2}, \delta_5^2 = \frac{C_1^2}{C_L^2}, C_L^2 = \frac{\lambda^L}{\rho^L}.$

Applying the Laplace transform with respect to time t defined by

$$\{\bar{\phi}^i, \bar{\psi}^i\}(x, y, s) = \int_0^\infty \{\phi^i, \psi^i\}(x, y, t) e^{-st} dt, (i = F, S, L) \tag{2.27}$$

and then Fourier transform with respect to x defined by

$$\{\tilde{\bar{\phi}}^i, \tilde{\bar{\psi}}^i\}(\xi, y, s) = \int_{-\infty}^\infty \{\bar{\phi}^i, \bar{\psi}^i\}(x, y, s) e^{ix\xi} dx, (i = F, S, L) \tag{2.28}$$

on equations (21) -( 26), yield

$$[\frac{d^2}{dz^2} - a^2] \tilde{\bar{\phi}}^S = 0, \tag{2.29}$$

$$\tilde{\bar{\phi}}^F = M_1 \tilde{\bar{\phi}}^S, \tag{2.30}$$

$$\tilde{\bar{p}} = N_1 \tilde{\bar{\phi}}^S, \tag{2.31}$$

$$[\frac{d^2}{dz^2} - b^2] \tilde{\bar{\psi}}^S = 0, \tag{2.32}$$

$$\tilde{\bar{\psi}}^F = M_2 \tilde{\bar{\psi}}^S, \tag{2.33}$$

$$[\frac{d^2}{dz^2} - r^2]\tilde{\bar{\phi}}^L = 0, \tag{2.34}$$

where

$$a^2 = \xi^2 + R_1^2, b^2 = \xi^2 + R_2^2, M_1 = -\frac{\eta^S}{\eta^F}, M_2 = \frac{\rho^S \delta_2}{s\rho^F \delta_1^2 + \rho^S \delta_2}, r^2 = s^2\delta_5^2 + \xi^2,$$

$$N_1 = \frac{s\delta_2\rho^S + s^2\eta^S\rho^F\delta_1^2}{(\eta^F)^2\rho^S}, R_1^2 = \frac{s\delta_2 + s^2(\eta^F)^2}{(\eta^F)^2}, R_2^2 = \frac{\frac{\rho^F}{\rho^S}\delta_1^4 s^3 + \delta_2 s^2\delta_1^2 + \frac{\rho^F}{\rho^S}\delta_1^2 s^2\delta_2}{[\frac{\rho^F}{\rho^S}\delta_1^2 s + \delta_2]\delta^2}.$$

The solution of these equations (29) - (33) satisfying the radiation conditions that $\tilde{\bar{\phi}}^S, \tilde{\bar{\phi}}^F, \tilde{\bar{\psi}}^S, \tilde{\bar{\psi}}^F \to 0$ as $z \to 0$ are given by

$$\tilde{\bar{\phi}}^S = A_1 e^{-az}, \tag{2.35}$$

$$\tilde{\bar{\phi}}^F = M_1 A_1 e^{-az}, \tag{2.36}$$

$$\tilde{\bar{p}} = N_1 A_1 e^{-az}, \tag{2.37}$$

$$\tilde{\bar{\psi}}^S = A_2 e^{-bz}, \tag{2.38}$$

$$\tilde{\bar{\psi}}^F = M_2 A_2 e^{-bz}. \tag{2.39}$$

Also, the solution of equation (34) is given by

$$\tilde{\bar{\phi}}^L = A_3 e^{-rz} + A_4 e^{rz}. \tag{2.40}$$

With the aids of (4), (5), (8), (15), (16), (19), (20), (27), (28) and (35)-(40), we obtain the displacement and stress components as:

$$\tilde{\bar{u}}^S = -\iota\xi A_1 e^{-az} - bA_2 e^{-bz}, \tag{2.41}$$

$$\tilde{\bar{w}}^S = -aA_1 e^{-az} + \iota\xi A_2 e^{-bz}, \tag{2.42}$$

$$\tilde{\bar{u}}^F = -\iota\xi M_1 A_1 e^{-az} - bM_2 A_2 e^{-bz}, \tag{2.43}$$

$$\tilde{\bar{w}}^F = -aM_1 A_1 e^{-az} + \iota\xi M_2 A_2 e^{-bz}, \tag{2.44}$$

$$\tilde{\bar{w}}^L = -rA_3 e^{-rz} + rA_4 e^{rz} \tag{2.45}$$

$$\tilde{\bar{T}}_{33}^S = Ld_1 A_1 e^{-az} - 2\iota\xi bd_2 A_2 e^{-bz}, \tag{2.46}$$

$$\tilde{\bar{T}}_{31}^S = 2a\iota d_1 A_1 e^{-az} + (\xi^2 + b^2)d_1 A_2 e^{-bz}, \tag{2.47}$$

$$\tilde{\bar{T}}_{33}^L = d_3 A_3 e^{-rz} + d_3 A_4 e^{rz}, \tag{2.48}$$

where

$$d_1 = \frac{\lambda^S}{\lambda^S + 2\mu^S}, d_2 = \frac{\mu^S}{\lambda^S + 2\mu^S}, d_3 = (r^2 - \xi^2)\delta_6^2, \delta_6^2 = \frac{\lambda^L}{\lambda^S + 2\mu^S},$$

$$L = \frac{-\lambda^S \xi^2 + a^2\lambda^S + 2a^2\mu^S}{\lambda^S}.$$

## 3   Boundary Conditions

We consider the normal point source acting on the fluid layer at the free plane boundary, continuity of the components of normal stress and displacement and vanishing of the tangential stress component at the interface. Mathematically these can be written as:

$$T_{33}^L = -RF(x,t), at z = -H, \tag{3.1}$$

$$T_{33}^S - p = T_{33}^L, at z = 0, \tag{3.2}$$

$$w^S = w^L, at z = 0, \tag{3.3}$$

$$T_{31}^S = 0, at z = 0, \tag{3.4}$$

where R is the magnitude of the constant pressure applied on the boundary and F(x,t) is a known function of x and t.

Making use of dimensionless quantities defined by (16) and considering $R' = \frac{R}{E}$ and applying the Laplace and Fourier transform defined by (27) and (28) on the boundary conditions given by (49) - (52), we obtain,

$$\tilde{\tilde{T}}_{33}^L = -R\tilde{\tilde{F}}(\xi, s), at z = -H, \tag{3.5}$$

$$\tilde{\tilde{T}}_{33}^S - \tilde{\tilde{p}} = \tilde{\tilde{T}}_{33}^L, at z = 0, \tag{3.6}$$

$$\tilde{\tilde{w}}^S = \tilde{\tilde{w}}^L, at z = 0, \tag{3.7}$$

$$\tilde{\tilde{T}}_{31}^S = 0, at z = 0, \tag{3.8}$$

Substituting values of $\tilde{\tilde{p}}, \tilde{\tilde{w}}^S, \tilde{\tilde{w}}^L, \tilde{\tilde{T}}_{33}^S, \tilde{\tilde{T}}_{31}^S, \tilde{\tilde{T}}_{33}^L$ from equations (37),(42),(45)-(48) in equations (53) - (56), we obtain ,

$$d_3 A_3 e^{rH} + d_3 A_4 e^{-rH} = -R\tilde{\tilde{F}}(\xi, s), \tag{3.9}$$

$$(d_1 L - N_1)A_1 - 2\iota\xi b d_2 A_2 - d_3 A_3 - d_3 A_4 = 0, \tag{3.10}$$

$$- aA_1 + \iota\xi A_2 + rA_3 - rA_4 = 0, \tag{3.11}$$

$$2a\iota\xi d_1 A_1 + (\xi^2 + b^2)d_1 A_2 = 0, \tag{3.12}$$

Equations (57) - (60) yield the values of $A_1, A_2, A_3, A_4$ and substituting these values in equations (37) and (41) - (48),we obtain the components of displacements, stress and pore pressure as:

$$\tilde{u}^{\tilde{S}} = -\iota\xi F_1 - bF_2, \tag{3.13}$$

$$\tilde{w}^{\tilde{S}} = -aF_1 + \iota\xi F_2, \tag{3.14}$$

$$\tilde{u}^{\tilde{F}} = -\iota\xi M_1 F_1 - bM_2 F_2, \tag{3.15}$$

$$\tilde{w}^{\tilde{F}} = -aM_1 F_1 + \iota\xi M_2 F_2, \tag{3.16}$$

$$\tilde{w}^{\tilde{L}} = -rF_3 + rF_4, \tag{3.17}$$

$$\tilde{T}_{33}^{\tilde{S}} = Ld_1 F_1 - 2\iota\xi b d_2 F_2, \tag{3.18}$$

$$\tilde{T}_{31}^{\tilde{S}} = 2a\iota d_1 F_1 + (\xi^2 + b^2)d_1 F_2, \tag{3.19}$$

$$\tilde{T}_{33}^{\tilde{L}} = d_3 F_3 + d_3 F_4, \tag{3.20}$$

$$\tilde{p} = N_1 F_1, \tag{3.21}$$

where

$$F_1 = \frac{1}{\triangle}[2r(\xi^2 + b^2)d_1 d_3]R\tilde{\tilde{F}}(\xi, s)e^{-az}, F_2 = \frac{1}{\triangle}[-4ar\iota d_1 d_3]R\tilde{\tilde{F}}(\xi, s)e^{-bz},$$

$$F_3 = \frac{1}{\triangle}[-4abr\xi d_1 d_2 - 2a\xi d_1 d_3 + r(\xi^2+b^2)d_1(d_1 L - N_1) + a(\xi^2+b^2)d_1 d_3]R\tilde{\tilde{F}}(\xi, s)e^{-rz},$$

$$F_4 = \frac{1}{\triangle}[-4abr\xi d_1 d_2 + 2a\xi d_1 d_3 + r(\xi^2+b^2)d_1(d_1 L - N_1) - a(\xi^2+b^2)d_1 d_3]R\tilde{\tilde{F}}(\xi, s)e^{rz},$$

,

$$\triangle = d_3 e^{rH}[4abr\xi d_1 d_2 + 2a\xi d_1 d_3 - r(\xi^2 + b^2)d_1(d_1 L - N_1) - a(\xi^2 + b^2)d_1 d_3] -$$

$$d_3 e^{-rH}[-4abr\xi d_1 d_2 + 2a\xi d_1 d_3 + r(\xi^2 + b^2)d_1(d_1 L - N_1) - a(\xi^2 + b^2)d_1 d_3].$$

# 4 Applications

**Case 1.Green's function:**

The general solutions for displacement, stress and pore pressure presented in eqs. (61)-(69) will be used to yield the response of a fluid layer over the half space subjected to a concentrated source as

$$F(x, t) = \delta(x)\delta(t), \tag{4.1}$$

where, $\delta()$ is the Dirac - delta function .

Applying the Laplace and Fourier transforms defined by (27) and (28) on (70), yield

$$\tilde{\bar{F}}(\xi, s) = 1. \tag{4.2}$$

**Case 2: Influence functions :**

$$Here F(x,t) = \psi(x)\delta(t), \tag{4.3}$$

where $\psi(x)$ is a known function and takes two types of values
　　1. Uniformly distributed source

$$\psi(x) = \left\{ \begin{array}{l} 1, |x| \le a \\ 0, |x| > a, \end{array} \right., \tag{4.4}$$

where 2a is non - dimensional width of the strip.
　　Applying Laplace and Fourier transforms defined by (27) and (28) on (72) and (73), we obtain,

$$\tilde{\bar{F}}(\xi, s) = \frac{2 sin \xi a}{\xi} \tag{4.5}$$

2. Linearly distributed source

$$\psi(x) = \left\{ \begin{array}{l} 1 - \frac{|x|}{a}, |x| \le a \\ 0, |x| > a, \end{array} \right., \tag{4.6}$$

where 2a is non - dimensional width of the strip.
　　Applying Laplace and Fourier transforms defined by (27) and (28) on (72) and (75), we obtain ,

$$\tilde{\bar{F}}(\xi, s) = \frac{2[1 - cos(a\xi)]}{a\xi^2}. \tag{4.7}$$

The expressions for displacements, stresses and pore pressure can be obtained for concentrated, uniformly and linearly distributed source by replacing from (71), (74) and (76) in (61)-(69).

## 5　Particular case

If the pore liquid is absent, we obtain the corresponding expressions for the components of displacement and stress in empty porous elastic half space underlying a acoustic fluid layer as:

$$\tilde{\bar{u}}^S = -\iota\xi F_{11} - b_1 F_{22}, \tag{5.1}$$

$$\tilde{\bar{w}}^S = -a_1 F_{11} + \iota\xi F_{22}, \tag{5.2}$$

$$\tilde{\tilde{w}}^L = -r_1 F_{33} + r_1 F_{44}, \tag{5.3}$$

$$\tilde{\tilde{T}}_{33}^S = L_1 d_1 F_{11} - 2\iota\xi b_1 d_2 F_{22}, \tag{5.4}$$

$$\tilde{\tilde{T}}_{31}^S = 2a_1\iota d_1 F_{11} + (\xi^2 + b_1^2)d_1 F_{22}, \tag{5.5}$$

$$\tilde{\tilde{T}}_{33}^L = d_3' F_{33} + d_3' F_{44}, \tag{5.6}$$

where

$$F_{11} = \frac{1}{\triangle_0}[2r_1(\xi^2 + b_1^2)d_1 d_3']R\tilde{\tilde{F}}(\xi, s)e^{-a_1 z}, F_{22} = \frac{1}{\triangle_0}[-4a_1 r_1 \iota d_1 d_3']R\tilde{\tilde{F}}(\xi, s)e^{-b_1 z},$$

$$F_{33} = \frac{1}{\triangle_0}[-4a_1 b_1 r_1 \xi d_1 d_2 - 2a_1 \xi d_1 d_3' + r_1(\xi^2 + b_1^2)d_1(d_1 L_1) + a_1(\xi^2 + b_1^2)d_1 d_3']R\tilde{\tilde{F}}(\xi, s)e^{-r_1 z},$$

$$F_{44} = \frac{1}{\triangle_0}[-4a_1 b_1 r_1 \xi d_1 d_2 + 2a_1 \xi d_1 d_3' + r_1(\xi^2 + b_1^2)d_1(d_1 L_1) - a_1(\xi^2 + b_1^2)d_1 d_3']R\tilde{\tilde{F}}(\xi, s)e^{r_1 z},$$

$$\triangle_0 = d_3' e^{r_1 H}[4a_1 b_1 r_1 \xi d_1 d_2 + 2a_1 \xi d_1 d_3' - r_1(\xi^2 + b_1^2)d_1(d_1 L_1) - a_1(\xi^2 + b_1^2)d_1 d_3'] -$$

$$d_3' e^{-r_1 H}[-4a_1 b_1 r_1 \xi d_1 d_2 + 2a_1 \xi d_1 d_3' + r_1(\xi^2 + b_1^2)d_1(d_1 L_1) - a_1(\xi^2 + b_1^2)d_1 d_3'], a_1^2 = \xi^2 + s^2,$$

$$b_1^2 = \xi^2 + \frac{s^2}{\delta^2}, r_1^2 = s^2\delta_7^2 + \xi^2, d_3' = (r_1^2 - \xi^2)\delta_6^2,, \delta_7^2 = \frac{C_0^2}{C_L^2}, L_1 = \frac{-\lambda^S \xi^2 + a_1^2 \lambda^S + 2a_1^2 \mu^S}{\lambda^S}$$

# 6   Inversion of the transform

The transformed displacements, stresses and pore pressures are functions of the parameters of Laplace and Fourier transforms s and $\xi$ respectively and hence are of the form $\tilde{\tilde{f}}(\xi, z, s)$ . To obtain the solution of the problem in the physical domain, we must invert the Laplace and Fourier transform by using the method applied by Kumar et.al [13].

# 7   Numerical results and discussion

With the view of illustrating the theoretical results and for numerical discussion we take a model for which the values of the various physical parameters are taken from de Boer and Ehlers [10]

$$\eta^S = .67, \eta^F = .33, \rho^S = 1.34 Mg/m^3, \rho^F = .33 Mg/m^3, \rho^L = 1.0 gm/cm^3, R = 1,$$

$$\lambda^S = 5.5833 MN/m^2, \gamma^{KF} = 10.00 KN/m^3, K^F = .01 m/s, \mu^S = 8.3750 MN/m^2,$$

$$\lambda^L = 2.14 dyne/cm^2.$$

The values of normal solid displacement component $w^S$, normal fluid displacement component $w^F$, normal displacement component $w^L$ (in fluid layer), normal stress component $t_{33}^S$, horizontal stress component $t_{31}^S$, normal stress component $t_{33}^L$ (in fluid layer) and pore pressure p for fluid saturated incompressible porous half space (FS) and empty porous elastic half space (ES) are shown due to concentrated source (CS), uniformly distributed source (UDS), linearly distributed source (LDS) at t= 0.1.The variations of these components with distance x are shown by

(1)The solid lines with and without central symbols to represent the case when CS is applied for ES and FS respectively.

(2)The long dashed lines with and without central symbols to represent the case when UDS is applied for ES and FS respectively.

(3)The small dashed lines with and without central symbols to represent the case when LDS is applied for ES and FS respectively.

These variations are shown in figs. 2-8. The computations are carried out for z =1 in the range $0 \leq x \leq 10$ , a=1.

Fig.2. Shows the variations of normal solid displacement component $w^S$ with distance x for FS, ES. The values of $w^S$ decrease in the range $0 \leq x \leq 2.8$, then oscillates for all the values of x for both FS and ES due to all three sources (CS,UDS, LDS).

Fig.3.Shows the variations of the normal fluid displacement component $w^F$ with distance x for FS. The values of $w^F$ decrease in the range $0 \leq x \leq 2.1$ then oscillates for all the values of x due to all the sources (CS, UDS, LDS).

Fig.4. Depicts the variations of normal solid stress component $t_{33}^S$ with distance x for both FS, ES. Near the point of application of source values of the $t_{33}^S$ increase in the range $0 \leq x \leq 2.5$ for FS and decrease for ES due to all the three applied sources and as x increases further its values oscillates for both FS and ES . It is evident that values of $t_{33}^S$ for FS as compared to the values of $t_{33}^S$ for ES are less in the range $0 \leq x \leq 2$ , more in the range $2 \leq x \leq 3.1$ , less in the range $3.1 \leq x \leq 4.2$ then more as x increases further.

Behaviour of horizontal solid stress component $t_{31}^S$ for both FS and ES is shown in the figure 5.The values of $t_{31}^S$ for FS starts with sharp increase, whereas for ES it starts with sharp decrease and then their behaviour is opposite oscillatory with each other .

Fig.6.Depicts the variations of the pore pressure p with distance x for FS. The values of pore pressure p decrease sharply in the range $0 \leq x \leq 2.2$ , then oscillates for the remaining values of x due to all the sources (CS, UDS, LDS).The values of p are greater due to UDS in comparison to CS and LDS.

Fig.7. Shows the variations of normal stress component $t_{33}^L$ with distance x for FS, ES. The values of$t_{33}^L$ for FS first decrease sharply in the range $0 \leq x \leq 2.5$ , increase in the range $2.5 \leq x \leq 4.1$ , decrease in the range $4.1 \leq x \leq 5.2$ then increase as x increases further, whereas for ES , the values of $t_{33}^L$ decrease in the range $0 \leq x \leq 2.2$ then oscillates for the remaining values of x. Near the point of application of source values of $t_{33}^L$ for ES are less as compared to the values for FS and then reverse behaviour is observed.

Fig.8. Shows the variations of the normal displacement component $w^L$ with distance x for FS, ES. Values of $w^L$ increase in the range $0 \leq x \leq 2$ , then oscillates for the remaining values of x for both FS and ES. Near the point of application of source values of $w^L$ for FS are less as compared to the values for ES.

# 8   Conclusion

1.  The Laplace and Fourier transforms are used to derive the components of normal solid displacement, normal fluid displacement, normal stress, horizontal stress and pore pressure.

2. Values of displacement components, stress components, and pore pressure are close to each other due to CS and UDS, LDS in both the media.

3. Behaviour of variation of fluid displacement component $w^F$ and pore pressure p is observed similar for all types of sources.

4. Near the point of application of source, the porosity effect decrease the values of $w^L, t_{33}^S, t_{31}^S$ and increase the values of $w^S, t_{33}^L$ .

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R    Z = -H

H    Liquid layer of thickness H    Z = 0

0    x

Incompressible porous elastic half space

z

Fig.1. Geometry of the problem
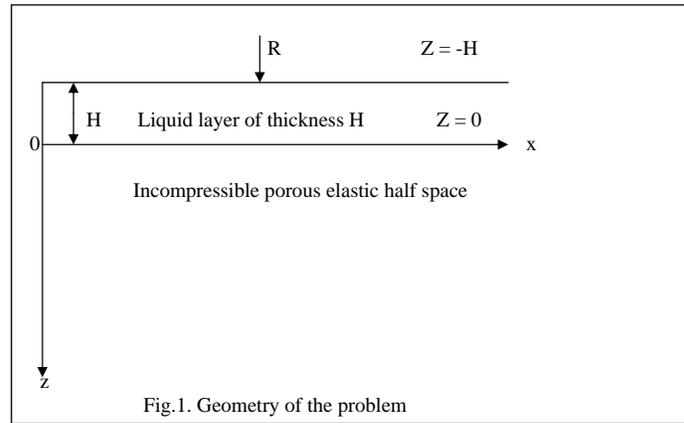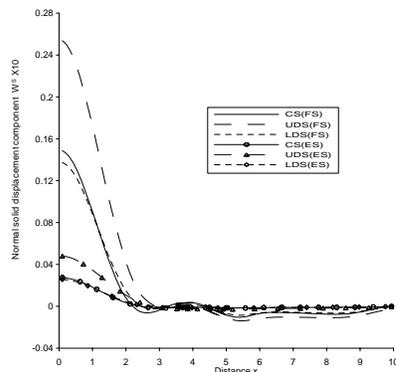
Figure 2. Variation of normal solid displacement component $W^s$ with horizontal distance x.
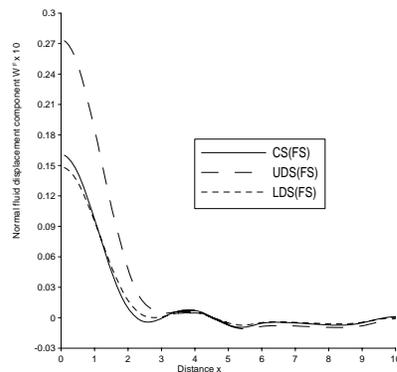
Figure 3. Variation of normal fluid displacement component $W^F$ with horizontal distance x
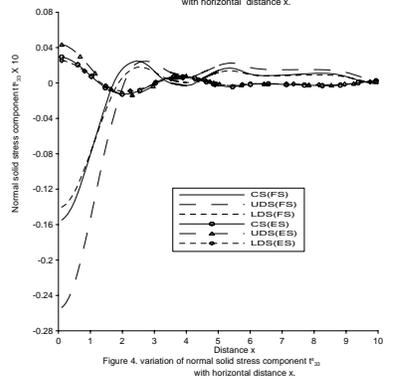
Figure 4. variation of normal solid stress component $t^s_{33}$ with horizontal distance x.
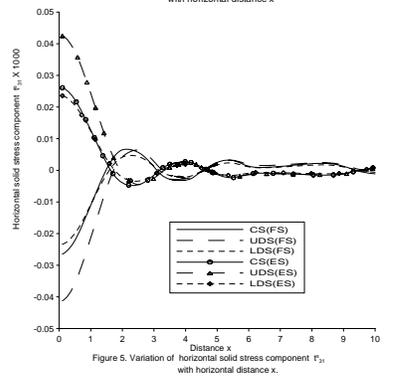
Figure 5. Variation of horizontal solid stress component $t^s_{31}$ with horizontal distance x.
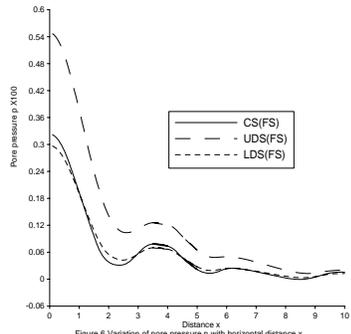
Figure 6. Variation of pore pressure p with horizontal distance x.
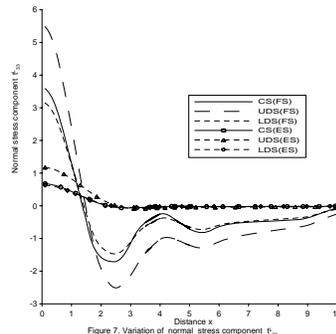


Figure 7. Variation of normal stress component $t^L_{33}$ with horizontal distance x.
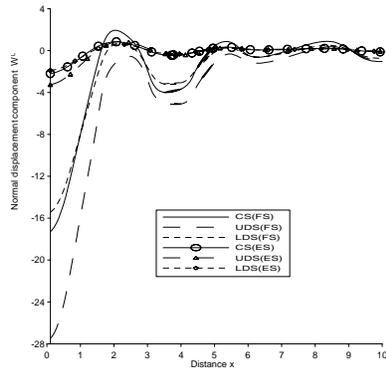


Figure 8. Variation of normal displacement component $W^L$ with horizontal distance x