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Approximation of Common Fixed Points for a Finite Family of Uniformly Quasi-Lipschitzian Mappings in Banach Spaces

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Abstract : The purpose of this paper is to study the sufficient and necessary condition on the strong convergence of the multistep iterative sequences with errors for a finite family of uniformly quasi-Lipschitzian mappings in Banach spaces. Our results extend and improve some recent results in the literature.

Keywords : Uniformly quasi-Lipschitzian mapping ; Asymptotically nonexpansive mapping ; Asymptotically quasi-nonexpansive mapping ; Common fixed point
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1 Introduction and preliminaries

Throughout the paper, we assume that K is a nonempty subset of a Banach space E. $F(T)$ denotes the fixed point set of T, i.e., $F(T) = \{x \in K : Tx = x\}$.

Definition 1.1. *A mapping $T : K \to K$ is said to be*

(i) asymptotically nonexpansive, if there exists $k_n \in [1, \infty)$, $\lim_{n\to\infty} k_n = 1$, such that

$$\|T^n x - T^n y\| \leq k_n \|x - y\|, \quad \forall x, y \in K, n \geq 1.$$

(ii) asymptotically quasi-nonexpansive, if $F(T) \neq \emptyset$ and there exists $k_n \in [1, \infty)$, $\lim_{n\to\infty} k_n = 1$, such that

$$\|T^n x - p\| \leq k_n \|x - p\|, \quad \forall x \in K, p \in F(T), n \geq 1.$$

(iii) uniformly quasi-Lipschitzian, if $F(T) \neq \emptyset$ and there exists a constant $L > 0$, such that

$$\|T^n x - p\| \leq L\|x - p\|, \quad \forall x \in K, p \in F(T), n \geq 1.$$

Remark 1.2. *It is easy to see that the asymptotically nonexpansive mapping is asymptotically quasi-nonexpansive mapping, and the latter is uniformly quasi-Lipschitzian mapping with* $L = \sup_{n\geq 1}\{k_n\} < +\infty$. *However, the converse doesn't hold in general.*

Definition 1.3. *([1]) Let* $\{T_i : i = 1, 2, \cdots, k\} : K \to K$ *be a finite family of mappings. Let* $x_1 \in K$*, then the sequence* $\{x_n\}$ *is defined by*

$$\begin{aligned} x_{n+1} &= a_{kn}x_n + b_{kn}T_k^n y_{(k-1)n} + c_{kn}u_{kn}, \\ y_{(k-1)n} &= a_{(k-1)n}x_n + b_{(k-1)n}T_{k-1}^n y_{(k-2)n} + c_{(k-1)n}u_{(k-1)n}, \\ y_{(k-2)n} &= a_{(k-2)n}x_n + b_{(k-2)n}T_{k-2}^n y_{(k-3)n} + c_{(k-2)n}u_{(k-2)n}, \\ &\vdots \\ y_{2n} &= a_{2n}x_n + b_{2n}T_2^n y_{1n} + c_{2n}u_{2n}, \\ y_{1n} &= a_{1n}x_n + b_{1n}T_1^n x_n + c_{1n}u_{1n}, \quad n \geq 1, \end{aligned} \tag{1.1}$$

where $\{a_{in}\}, \{b_{in}\}, \{c_{in}\}$ *are sequences in* $[0, 1]$ *with* $a_{in} + b_{in} + c_{in} = 1$ *for all* $i = 1, 2, \cdots, k$ *and* $n \geq 1$, $\{u_{in}, i = 1, 2, \cdots, k, n \geq 1\}$ *are bounded sequences in* K*, is called multistep iterative sequences with errors.*

Remark 1.4. *The multistep iterative sequences with errors contains many well-known iterations as special case. Such as, the modified Mann iteration(see, Schu [2]), the modified Ishikawa iteration(see, Tan and Xu [3]), the three-step iteration(see, Xu and Noor [4]), the multistep iteration(see, Khan et al. [5]) etc.*

In 1972, Goebel and Kirk [6] introduced the conception of asymptotically nonexpansive mapping, they proved that if K is a nonempty closed convex bounded subset of a uniformly convex Banach space E, then every asymptotically nonexpansive mapping of K has a fixed point. Iterative techniques for approximating fixed points of asymptotically nonexpansive and asymptotically quasi-nonexpansive mappings in Banach spaces have been studied by many authors; See, [6–16, 18, 19, 21]and the references therein. Related work can be found in [22–27, 29, 30] and others.

Recently, Shahzad and Udomene [20] established several necessary and sufficient conditions for the modified Ishikawa iteration process for two asymptotically quasi-nonexpansive mappings in Banach spaces. Quan et al. [1] discarded Condition 2.1 in Theorem 2.1 of Chang et al. [17], and they studied sufficient and necessary conditions on the strong convergence of the multistep iterative sequences with errors for a finite family of asymptotically quasi-nonexpansive and type mappings in Banach spaces. Khan et al. [5] proved a necessary and sufficient conditions for the multistep iterative processes to converge strongly to a common fixed points of a finite family of asymptotically quasi-nonexpansive mappings in Banach spaces.

In this paper, we give a sufficient and necessary condition on the strong convergence of the iteration process (1.1) for a finite family of uniformly quasi-Lipschitzian mappings in Banach spaces. Furthermore, our results generalize and improve the corresponding results of Quan et al. [1], Khan et al. [5], Shahzad and Udomene [20], Chang et al. [17] and Wang and Liu [28] and many others.

In the sequel, we shall need the following lemma.

Lemma 1.5. *([10]) Let $\{a_n\}, \{b_n\}, \{\lambda_n\}$ be sequences of nonnegative real numbers satisfying the inequality*

$$a_{n+1} \leq (1 + \lambda_n)a_n + b_n, \qquad \forall n \geq 1.$$

If $\sum_{n=1}^{\infty} \lambda_n < +\infty, \sum_{n=1}^{\infty} b_n < +\infty$, we have (i) $\lim_{n\to\infty} a_n$ exists. (ii) In particular, if $\liminf_{n\to\infty} a_n = 0$, then $\lim_{n\to\infty} a_n = 0$.

2 Main Results

The following lemma plays an important role in this paper.

Lemma 2.1. *Let K be a nonempty closed convex subset of a Banach space E and $\{T_i : i = 1, 2, \cdots, k\} : K \to K$ be a finite family of uniformly quasi-Lipschitzian mappings, i.e., $\|T_i^n x - p_i\| \leq L_i\|x - p_i\|$ for all $x \in K$ and $p_i \in F(T_i)$, $i = 1, 2, \cdots, k, n \geq 1$. The sequence $\{x_n\}$ is defined by (1.1) satisfying: $\sum_{n=1}^{\infty}(b_{in} + c_{in}) < +\infty$, for all $i = 1, 2, \cdots, k$. If $F := \bigcap_{i=1}^{k} F(T_i) \neq \emptyset$, then*

(i) there exists a sequence $\{\alpha_n\}$ in $[0, +\infty)$, such that $\sum_{n=1}^{\infty} \alpha_n < +\infty$ and two constants L , $M > 0$, such that

$$\|x_{n+1} - p\| \leq \left(1 + \sum_{i=1}^{k} \alpha_n^i L^i\right) \|x_n - p\| + \sum_{i=1}^{k} \alpha_n^i L^{i-1} M, \quad \forall p \in F, n \geq 1. \quad (2.1)$$

(ii) there exists a constant $M_1 > 0$, such that

$$\|x_{n+m} - p\| \leq M_1\|x_n - p\| + M_1 \sum_{i=1}^{k} \sum_{j=n}^{n+m-1} \alpha_j^i L^{i-1} M, \quad \forall p \in F, n, m \geq 1. \quad (2.2)$$

Proof. (i)Let $p \in F$ and $L = \max_{1\leq i\leq k} L_i$. Assume $\alpha_n = \max_{1\leq i\leq k}(b_{in}+c_{in}), n \geq 1$. Since $\sum_{i=1}^{n}(b_{in} + c_{in}) < +\infty, i = 1, 2, \cdots, k$, so $\sum_{n=1}^{\infty} \alpha_n < +\infty$. Notice that $\{u_{in} : i = 1, 2, \cdots, k\}$ are bounded sequences in K, therefore there exists a $M > 0$, such that

$$M = max \left\{ \sup_{n\geq 1} \|u_{in} - p\|, i = 1, 2, \cdots, k \right\}.$$

Using iterative sequence (1.1), we have

$$\begin{aligned} \|y_{1n} - p\| &= \|a_{1n}(x_n - p) + b_{1n}(T_1^n x_n - p) + c_{1n}(u_{1n} - p)\| \\ &\leq a_{1n}\|x_n - p\| + b_{1n}L\|x_n - p\| + c_{1n}\|u_{1n} - p\| \\ &\leq (a_{1n} + b_{1n}L)\|x_n - p\| + \alpha_n M \\ &\leq (1 + \alpha_n L)\|x_n - p\| + \alpha_n M. \end{aligned} \quad (2.3)$$

Furthermore, by inequality (2.3), we obtain

$$\begin{aligned}\|y_{2n}-p\| &= \|a_{2n}(x_n-p)+b_{2n}\,(T_2^n y_{1n}-p)+c_{2n}(u_{2n}-p)\|\\ &\le a_{2n}\|x_n-p\|+b_{2n}L\|y_{1n}-p\|+c_{2n}M\\ &= (a_{2n}+b_{2n}L(1+\alpha_nL))\|x_n-p\|+b_{2n}L\alpha_nM+\alpha_nM\\ &\le (1+\alpha_nL+\alpha_n^2L^2)\|x_n-p\|+\alpha_n^2LM+\alpha_nM. \qquad (2.4)\end{aligned}$$

Repeatedly, we have

$$\|y_{jn}-p\| \le \left(1+\sum_{i=1}^{j}\alpha_n^iL^i\right)\|x_n-p\|+\sum_{i=1}^{j}\alpha_n^iL^{i-1}M, \quad j=1,2,\cdots,k-1. \quad (2.5)$$

In fact, (2.5) holds for $j=1$ via inequality (2.3). By using induction, suppose that (2.5) holds for j, then for $j+1$, we have

$$\begin{aligned}\|y_{(j+1)n}-p\| &= \|a_{(j+1)n}(x_n-p)+b_{(j+1)n}(T_{j+1}^n y_{jn}-p)+c_{(j+1)n}(u_{(j+1)n}-p)\|\\ &\le a_{(j+1)n}\|x_n-p\|+b_{(j+1)n}L\|y_{jn}-p\|+c_{(j+1)n}M\\ &\le a_{(j+1)n}\|x_n-p\|+b_{(j+1)n}L\left(1+\sum_{i=1}^{j}\alpha_n^iL^i\right)\|x_n-p\|\\ &\quad + b_{(j+1)n}L\sum_{i=1}^{j}\alpha_n^iL^{i-1}M+c_{(j+1)n}M\\ &\le \left[1+\alpha_nL\left(1+\sum_{i=1}^{j}\alpha_n^iL^i\right)\right]\|x_n-p\|+\alpha_nL\sum_{i=1}^{j}\alpha_n^iL^{i-1}M+\alpha_nM\\ &= \left(1+\sum_{i=1}^{j+1}\alpha_n^iL^i\right)\|x_n-p\|+\sum_{i=1}^{j+1}\alpha_n^iL^{i-1}M.\end{aligned}$$

Hence (2.5) holds. It follows from (1.1) and (2.5) that

$$\begin{aligned}\|x_{n+1}-p\| &= \|a_{kn}(x_n-p)+b_{kn}(T_k^n y_{(k-1)n}-p)+c_{kn}(u_{kn}-p)\|\\ &\le a_{kn}\|x_n-p\|+b_{kn}L\|y_{(k-1)n}-p\|+c_{kn}M\\ &\le \left[a_{kn}+b_{kn}L\left(1+\sum_{i=1}^{k-1}\alpha_n^iL^i\right)\right]\|x_n-p\|+b_{kn}L\sum_{i=1}^{k-1}\alpha_n^iL^{i-1}M+\alpha_nM\\ &\le \left(1+\sum_{i=1}^{k}\alpha_n^iL^i\right)\|x_n-p\|+\sum_{i=1}^{k}\alpha_n^iL^{i-1}M.\end{aligned}$$

This completes the proof of part(i).

(ii) If $x \geq 0$, then $1 + x \leq e^x$. For any integer $n, m \geq 1$ and from part(i), we have

$$\begin{aligned}
&\|x_{n+m} - p\| \\
&\leq \left(1 + \sum_{i=1}^{k} \alpha_{n+m-1}^{i} L^{i}\right) \|x_{n+m-1} - p\| + \sum_{i=1}^{k} \alpha_{n+m-1}^{i} L^{i-1} M \\
&\leq e^{\sum_{i=1}^{k} \alpha_{n+m-1}^{i} L^{i}} \|x_{n+m-1} - p\| + \sum_{i=1}^{k} \alpha_{n+m-1}^{i} L^{i-1} M \\
&\leq e^{\sum_{i=1}^{k} \alpha_{n+m-1}^{i} L^{i}} e^{\sum_{i=1}^{k} \alpha_{n+m-2}^{i} L^{i}} \|x_{n+m-2} - p\| + e^{\sum_{i=1}^{k} \alpha_{n+m-1}^{i} L^{i}} \sum_{i=1}^{k} \alpha_{n+m-2}^{i} L^{i-1} M \\
&\qquad + \sum_{i=1}^{k} \alpha_{n+m-1}^{i} L^{i-1} M \\
&\cdots \\
&\leq e^{\sum_{i=1}^{k} \sum_{j=1}^{n+m-1} \alpha_{j}^{i} L^{i}} \|x_{n} - p\| + e^{\sum_{i=1}^{k} \sum_{j=1}^{n+m-1} \alpha_{j}^{i} L^{i}} \sum_{i=1}^{k} \sum_{j=n}^{n+m-1} \alpha_{j}^{i} L^{i-1} M \\
&\leq e^{\sum_{i=1}^{k} \sum_{j=1}^{\infty} \alpha_{j}^{i} L^{i}} \|x_{n} - p\| + e^{\sum_{i=1}^{k} \sum_{j=1}^{\infty} \alpha_{j}^{i} L^{i}} \sum_{i=1}^{k} \sum_{j=n}^{n+m-1} \alpha_{j}^{i} L^{i-1} M \\
&= M_1 \|x_n - p\| + M_1 \sum_{i=1}^{k} \sum_{j=n}^{n+m-1} \alpha_{j}^{i} L^{i-1} M,
\end{aligned}$$

where $M_1 = e^{\sum_{i=1}^{k} \sum_{j=1}^{\infty} \alpha_{j}^{i} L^{i}}$. This completes the proof of part(ii). □

Theorem 2.2. *Let K be a nonempty closed convex subset of a Banach space E. Let $\{T_i : i = 1, 2, \cdots, k\} : K \to K$ be a finite family of uniformly quasi-Lipschitzian mappings, i.e., $\|T_i^n x - p_i\| \leq L_i \|x - p_i\|$ for all $x \in K$ and $p_i \in F(T_i)$, $i = 1, 2, \cdots, k, n \geq 1$. Suppose that $F := \bigcap_{i=1}^{k} F(T_i) \neq \emptyset$. The sequence $\{x_n\}$ is defined by (1.1) satisfying: $\sum_{n=1}^{\infty}(b_{in} + c_{in}) < +\infty, i = 1, 2, \cdots, k$. Then $\{x_n\}$ converges strongly to a common fixed point of the family of mappings $\{T_i : i = 1, 2, \cdots, k\}$ if and only if $\liminf_{n\to\infty} d(x_n, F) = 0$, where $d(x, F) = \inf_{p\in F} \|x - p\|$.*

Proof. The necessity of Theorem 2.2 is obvious. Thus, we will only prove the sufficiency. From Lemma 2.1(i), we have

$$\|x_{n+1} - p\| \leq \left(1 + \sum_{i=1}^{k} \alpha_{n}^{i} L^{i}\right) \|x_n - p\| + \sum_{i=1}^{k} \alpha_{n}^{i} L^{i-1} M, \quad \forall p \in F, n \geq 1.$$

Therefore,

$$d(x_{n+1} - p) \leq \left(1 + \sum_{i=1}^{k} \alpha_{n}^{i} L^{i}\right) d(x_n - p) + \sum_{i=1}^{k} \alpha_{n}^{i} L^{i-1} M.$$

Since $\sum_{n=1}^{\infty}\alpha_n<+\infty$, so $\sum_{i=1}^{k}\alpha_n^iL^i<+\infty$ and $\sum_{i=1}^{k}\alpha_n^iL^{i-1}M<+\infty$. With the help of Lemma 1.5 and $\liminf_{n\to\infty}d(x_n,F)=0$. We know $\lim_{n\to\infty}d(x_n,F)=0$.

Next, we prove that $\{x_n\}$ is a Cauchy sequence. For each $\varepsilon>0$, there exists a natural number n_1, such that

$$d(x_n,F)\le\frac{\varepsilon}{12M_1},\qquad \text{for all } n\ge n_1. \tag{2.6}$$

Hence, there exists $p_1\in F$ and a constant $n_2>n_1$, such that

$$\|x_{n_2}-p_1\|\le\frac{\varepsilon}{4M_1},\qquad \sum_{i=1}^{k}\sum_{j=n_2}^{\infty}\alpha_j^iL^{i-1}<\frac{\varepsilon}{2M_1M}. \tag{2.7}$$

By Lemma 2.1(ii), (2.6) and (2.7), for all $n\ge n_2$, we have

$$\begin{aligned}\|x_{n+m}-x_n\|&\le\|x_{n+m}-p_1\|+\|p_1-x_n\|\\&\le M_1\|x_{n_2}-p_1\|+M_1\sum_{i=1}^{k}\sum_{j=n_2}^{n+m-1}\alpha_j^iL^{i-1}M+\|p_1-x_{n_2}\|\\&\le 2M_1\|x_{n_2}-p_1\|+M_1\sum_{i=1}^{k}\sum_{j=n_2}^{n+m-1}\alpha_j^iL^{i-1}M\\&\le 2M_1\frac{\varepsilon}{4M_1}+M_1\frac{\varepsilon}{2M_1M}M=\varepsilon.\end{aligned}$$

Then $\{x_n\}$ is a Cauchy sequence. Because K is nonempty closed convex subset of E, so there exists a $q\in K$, such that $x_n\to q$ as $n\to\infty$. Finally, we prove $q\in F$.

In fact, since $d(q,F)=0$. So, for any $\varepsilon_1>0$, there exists $p_2\in F$, such that $\|p_2-q\|<\varepsilon_1$. Then we have

$$\begin{aligned}\|T_iq-q\|&\le\|T_iq-p_2\|+\|p_2-q\|\\&\le(1+L)\varepsilon_1.\end{aligned}$$

By the arbitrary of ε_1, we know that $\|T_iq-q\|=0, i=1,2,\cdots,k$, i.e., $q\in F$. □

Remark 2.3. *Theorem 2.2 improves Theorem 2.3 of Quan et al. [1], Theorem 2.2 of Khan et al. [5] and Theorem 3.2 of Shahzad and Udomene [20] from the asymptotically quasi-nonexpansive mappings to the uniformly quasi-Lipschitzian mappings. At the same time, it extends the iterative sequences in Khan et al. [5] and Wang and Liu [28] to multistep iterative sequences with errors.*

By Remark 1.2, we have the following Corollaries immediately.

Corollary 2.4. *Let K be a nonempty closed convex subset of a Banach space E and $\{T_i: i=1,2,\cdots,k\}: K\to K$ be a finite family of asymptotically quasi-nonexpansive mappings, such that $F:=\bigcap_{i=1}^{k}F(T_i)\ne\emptyset$. The sequence $\{x_n\}$ is defined by (1.1) satisfying: $\sum_{n=1}^{\infty}(b_{in}+c_{in})<+\infty, i=1,2,\cdots,k$. Then $\{x_n\}$ converges strongly to a common fixed point of $\{T_i: i=1,2,\cdots,k\}$ if and only if $\liminf_{n\to\infty}d(x_n,F)=0$, where $d(x,F)=\inf_{p\in F}\|x-p\|$.*

Corollary 2.5. *Let K be a nonempty closed convex subset of a Banach space E and $\{T_i : i = 1, 2, \cdots, k\} : K \to K$ be a finite family of asymptotically nonexpansive mappings, such that $F := \bigcap_{i=1}^{k} F(T_i) \neq \emptyset$. The sequence $\{x_n\}$ is defined by (1.1) satisfying: $\sum_{n=1}^{\infty}(b_{in} + c_{in}) < +\infty, i = 1, 2, \cdots, k$. Then $\{x_n\}$ converges strongly to a common fixed point of $\{T_i : i = 1, 2, \cdots, k\}$ if and only if $\liminf_{n\to\infty} d(x_n, F) = 0$, where $d(x, F) = \inf_{p\in F} \|x - p\|$.*

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References

[1] J. Quan, S. S. Chang, X. J. Long, Approximation common fixed point of asymptotically quasi-nonexpansive-type mappings by the finite steps iterative sequences, Fixed Point Theory Appl., (2006), Article ID70830, 8PP.

[2] J. Schu, Weak and strong convergence to fixed points of asymptotically nonexpansive mappings, Bull. Austral. Math. Soc., 43 (1991), 153-159.

[3] K.K. Tan, H.K. Xu, Fixed point iteration process for asymptotically nonexpansive mappings, Proc. Amer. Math. Soc., 122(3)(1994), 733-739.

[4] B. Xu, M.A. Noor, Fixed-point iterations for asymptotically nonexpansive mappings in Banach spaces, J. Math. Anal. Appl., 267(2002), 444-453.

[5] A.R. Khan, A.A. Domlo, H. Fukhar-ud-din, Common fixed points Noor iteration for a finite family of asymptotically quasi-nonexpansive mappings in Banach spaces, J. Math. Anal. Appl., 341(2008), 1-11.

[6] K. Goebel,W.A. Kirk, A fixed point theorem for asymptotically nonexpansive mappings, Proc. Amer. Math. Soc., 35(1972), 171-174.

[7] S.C. Bose, Weak convergence to the fixed point of an asymptotically nonexpansive map, Proc. Amer. Math. Soc., 68(1978), 305-308.

[8] B.E. Rhoades, Fixed point iterations for certain nonlinear mappings, J. Math. Anal. Appl., 183(1994), 118-120.

[9] Z. Huang, Mann and Ishikawa iterations with errors for asymptotically nonexpansive mappings, Comput. Math. Appl., 37(1999), 1-7.

[10] M.O. Osilike, S.C. Aniagbosor, Weak and Strong convergence theorems for fixed points of asymptotically nonexpansive mappings, Math. Comput. Model., 32(2000), 1181-1191.

[11] S.H. Khan, W. Takahashi, Approximating common fixed points of two asymptotically nonexpansive mappings, Sci. Math. Jpn., 53(2001), 133-138.

[12] Q.H. Liu, Iterative sequences for asymptotically quasi-nonexpansive mappings, J. Math. Anal. Appl., 259(2001), 1-7.

[13] Q.H. Liu, Iterative sequences for asymptotically quai-nonexpansive mappings with error member, J. Math. Anal. Appl., 259(1)(2001), 18-24.

[14] Q.H. Liu, Iteration sequences for asymptotically quasi-nonexpansive mapping with an error member of uniform convex Banach space, J. Math. Anal. Appl., 266(2002), 468-471.

[15] Z.Q. Liu, S.M. Kang, Weak and strong convergence for fixed points of asymptotically nonexpansive mappings, Acta. Math. Sin.(Engl. Ser)., 20(2004), 1009-1018.

[16] L.C. Zeng, Modified Ishikawa and Mann iterative processes with errors for asymptotically nonexpansive mappings in Banach spaces, Acta. Math. Appl. Sinica(in chinese)., 27(4)(2004), 674-681.

[17] S.S. Chang, J.K. Kim, S.M. Kang, Approximating fixed points of asymptotically quasi-nonexpansive type mappings by the Ishikawa iterative sequences with mixed errors, Dynamic Systems Appl., 13(2)(2004), 179-186.

[18] S.Plubtieng, R.Wangkeeree, Strong convergence theorems asymptotically nonexpansive nonselfmappings, Thai J.Math., 3(1)(2005), 71-81.

[19] J. Quan, Three-step iterative sequences with errors for uniformly quasi-Lipschitzian mappings, Numer. Math. J. Chinese Univ.(English Ser)., 15(4)(2006), 306-311.

[20] N. Shahzad, A. Udomene, Approximating common fixed points of two asymptotically quasi-nonexpansive mappings in Banach spaces, Fixed Point Theory Appl., (2006), Article ID18909, 10 pp.

[21] K. Nammanee, M.A. Noor, S. Suantai, Convergence criteria of modified Noor iterations with errors for asymptotically nonexpansive mappings, J. Math. Anal. Appl., 314(2006), 320-334.

[22] T.H. Kim, H.K. Xu, Strong convergence of modified Mann iterations for asymptotically nonexpansive mappings and semigroups, Nonlinear Anal.TMA 64(2006), 1140-1152.

[23] L.P. Yang, Modified multistep iterative process for some common fixed point of a finite family of nonself asymptotically nonexpansive mappings, Math. Comput Model., 45(2007), 1157-1169.

[24] A. Banerjee, Weak and strong convergence of iterates of non-lipschitzian asymptotically quasinonexpansive type mapping, Thai J. Math., 5(1)(2007), 151-161.

[25] C.E. Chidume, B. Ali, Approximation of common fixed points for finite families of nonself asymptotically nonexpansive mappings in Banach spaces, J. Math. Anal. Appl., 326(2007), 960-973.

[26] S.H. Khan, N. Hussain, Convergence theorems for nonself asymptotically nonexpansive mappings, Comput Math. Appl., 55(2008), 2544-2553.

[27] J.Ayaragarnchanakul, A common fixed point iterative process with errors for quasi-nonexpansive nonself-mappings in Banach spaces, Thai J. Math., 6(2)(2008), 323-330.

[28] C. Wang, L.W. Liu, Convergence theorems for fixed points of uniformly quasi-Lipschitzian mappings in convex metric spaces, Nonlinear Anal.TMA 70(2009), 2067-2071.

[29] J.Z. Xiao, J. Sun, X. Huang, Approximating common fixed points of asymptotically quasi-nonexpansive mappings by a $k+1$ step iterative scheme with error terms, J. Comput Appl. Math., (2009), doi:10.1016/j.cam.2009.09.039.

[30] N. Onjai-uea, S. Suantai, Common fixed point of modified Noor iterations with errors for non-lipschitzian mappings in Banach spaces, Thai J. Math., 7(1)(2009), 115-132.

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