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Some Generalized Derivations on n-ary Multiplicative Semilattices

Pongsatorn Chotchaya and Utsanee Leerawat*

Department of Mathematics, Faculty of Science, Kasetsart University, Bangkok 10900, Thailand
e-mail : pongsatorn.cho@ku.th (P. Chotchaya); fsciutl@ku.ac.th (U. Leerawat)

Abstract In this paper, we introduce a generalization of derivation on n-ary multiplicative semilattices; namely permuting n-(f,g)-derivation, and investigate some related properties. Moreover, we study the notion of trace of permuting n-(f,g)-derivation in multiplicative semilattices and we also obtain some results concerning identities with traces and permuting n-(f,g)-derivation in multiplicative semilattices.

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1. Introduction

For any lattice $L := (L, \wedge, \vee)$, a derivation on L is a function $d : L \to L$ satisfying:

(i) $d(x \vee y) = d(x) \vee d(y)$ and
(ii) $d(x \wedge y) = (d(x) \wedge y) \vee (x \wedge d(y))$ for all $x, y \in L$.

The notion of derivations in lattices have been studied by Szasz[2], Ferrari[6] and Xin et al.[11]. They studied some properties of derivations, and characterized modular and distributive lattices by some special derivations. The concept of derivation in lattices has been generalized in several ways by various authors (see [5,8,9,12,13]). In 2018, Wang et al.[4] investigated related properties of some particular derivations and gave some characterizations of zero derivations in prime commutative multiplicative semilattices. In [7], Ozturk et al. introduced the permuting tri-derivations in lattices. Yazarli and Ozturk[3] generalized the permuting tri-derivations to permuting tri-f-derivations. Recently, Leerawat and Chotchaya[10] generalized the permuting tri-f-derivations to the permuting n-(f,g)-derivation, where n is a positive integer, and investigated some related properties. In this paper we study the notion of a permuting n-(f,g)-derivation on n-ary multiplicative semilattices, and investigate some results involving this derivations. Furthermore,

*Corresponding author.

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we study the notion of trace of permuting n-(f, g)-derivation in multiplicative semilattices and we also obtain some results concerning identities with traces and permuting n-(f, g)-derivation in multiplicative semilattices.

2. Preliminaries

In this section, we give some definitions of multiplicative semilattice and some properties about multiplicative semilattices gathered by Jayaram[1] and Wang et al.[3].
A *semilattice* is a nonempty set L with a binary operation $*$ such that for all x, y, and z in L, the following identities hold:

(i) $x * x = x$.
(ii) $x * y = y * x$.
(iii) $x * (y * z) = (x * y) * z$.

In other words, a semilattice is an idempotent commutative semigroup. If L is a semilattice with a binary operation $*$, then we write $(L, *)$.

Lemma 2.1. *In a semilattice $(L, *)$, define $x \leqslant y$ if and only if $x * y = x$. Then $(L, \leqslant)$ is a partially ordered set (poset) in which every pair of elements has a greatest lower bound or infimum. Conversely, given a partially ordered set P with that property, define $x * y = inf\{x, y\}$. Then $(P, *)$ is a semilattice.*

Proof. Let $(L, *)$ be a semilattice and define $\leqslant$ as above. First, we show that $\leqslant$ is a partial order. For any $x, y, z \in L$,

(i) $x * x = x$ implies $x \leqslant x$.
(ii) If $x \leqslant y$ and $y \leqslant x$, then $x = x * y = y * x = y$.
(iii) If $x \leqslant y$ and $y \leqslant z$, then $x * z = (x * y) * z = x * (y * z) = x * y = x$, so $x \leqslant z$.

Since $(x * y) * x = x * (y * x) = (x * x) * y = x * y$, we have $x * y \leqslant x$. Similarly, $x * y \leqslant y$. Thus $x * y$ is a lower bound for $\{x, y\}$. Let z be a lower bound for $\{x, y\}$. Then $z \leqslant x$ and $z \leqslant y$. Hence $z * (x * y) = (z * x) * y = z * y = z$. Thus, $z \leqslant x * y$. Therefore $x * y$ is the greatest lower bound for $\{x, y\}$. The proof of the converse is likewise a direct application of the definitions. ∎

Sometimes it is more natural to use the dual order, setting $x \leqslant y$ if and only if $x * y = y$. The following lemma can be proved similarly.

Lemma 2.2. *In a semilattice $(L, *)$, define $x \leqslant y$ if and only if $x * y = y$. Then $(L, \leqslant)$ is a partially ordered set (poset) in which every pair of elements has a least upper bound or supremum. Conversely, given a partially ordered set P with that property, define $x * y = sup\{x, y\}$. Then $(P, *)$ is a semilattice.*

Definition 2.3. Let L be a nonempty set with two binary operations $\vee$ and $\cdot$ on L. Then $(L, \vee, \cdot) := L$ is called a *multiplicative semilattice* if it satisfies the following conditions:

(i) $(L, \vee)$ is a semilattice.
(ii) $(L, \cdot)$ is a semigroup.
(iii) There exists an element 0 in L such that $0 \leqslant x$ and $x \cdot 0 = 0 = 0 \cdot x$ for all $x \in L$.
(iv) There exists an element 1 in L such that $x \leqslant 1$ and $x \cdot 1 = x = 1 \cdot x$ for all $x \in L$.

(v) $x \cdot (y \vee z) = (x \cdot y) \vee (x \cdot z)$ and $(x \vee y) \cdot z = (x \cdot z) \vee (y \cdot z)$ for all $x, y, z \in L$.

For convenience, we abbreviate $x \cdot y$ by xy for all $x, y \in L$. From Definition 2.3, $0 \vee x = x$ and $x \vee 1 = 1$ for all $x \in L$.
In a multiplicative semilattice L, by Lemma 2.2. we have $(L, \leqslant)$ is a partially ordered set, where $\leqslant$ is defined by $x \leqslant y$ if and only if $x \vee y = y$. Conversely, in a poset $(L, \leqslant)$, if a subset $\{x, y\}$ of L has a least upper bound or supremum, then this supremum is unique and denoted by $x \vee y$.

The following lemma is a basic property on a multiplicative semilattice. The proof is straightforward and hence omitted.

Lemma 2.4. *Let L be a multiplicative semilattice. If $y, z \in L$ such that $y \leqslant z$ then $xy \leqslant xz$ and $yx \leqslant zx$ for all $x \in L$.*

Definition 2.5. Let L be a multiplicative semilattice. Then L is called

(i) *idempotent* if $xx = x$ for all $x \in L$.
(ii) *commutative* if $xy = yx$ for all $x, y \in L$.
(iii) *prime* if $x\ L\ y = \{0\}$ where $x, y \in L$ implies either $x = 0$ or $y = 0$.

Definition 2.6. Let L be a multiplicative semilattice. A nonempty subset S of L is called a *subsemilattice* of L. If S is closed under the operation $\cdot$ and $\vee$ that is,

(i) $x \vee y \in S$ for all $x, y \in S$.
(ii) $xy \in S$ and $yx \in S$ for all $x, y \in S$.

Definition 2.7. Let L be a multiplicative semilattice and I be a nonempty subset of L. Then I is called an *ideal* of L if it satisfies the following conditions:

(i) $x \vee y \in I$ for all $x, y \in I$.
(ii) for $x, y \in L$, if $x \in I$ and $y \leqslant x$ then $y \in I$.
(iii) $xy \in I$ and $yx \in I$ for all $x \in I, y \in L$.

3. Main Results

In what follows, let $(L, \vee, \cdot)$ be a multiplicative semilattice and $f, g : L \to L$ be functions unless otherwise specified. Let n be a fixed positive integer and L^n denote $L \times L \times \cdots \times L$ (n terms).

Definition 3.1. A mapping $D : L^n \to L$ is said to be a *permuting* if the relation

$$D(x_1, x_2, \ldots, x_n) = D(x_{\pi(1)}, x_{\pi(2)}, \ldots, x_{\pi(n)})$$

holds for all $x_i \in L$ and for every permutation $\pi \in S_n$, where S_n is the permutation group on $\{1, 2, \ldots, n\}$.

Definition 3.2. A permuting mapping $D : L^n \to L$ is called a *permuting n-(f, g)-derivation of* L if it satisfies the following conditions:

$$D(x_1 \vee y, x_2, \ldots, x_n) = D(x_1, x_2, \ldots, x_n) \vee D(y, x_2, \ldots, x_n)$$
$$D(x_1 y, x_2, \ldots, x_n) = (D(x_1, x_2, \ldots, x_n) f(y)) \vee (g(x_1) D(y, x_2, \ldots, x_n))$$

for all $x_1, x_2, \ldots, x_n, y \in L$.

Example 3.3. Let $L = \{0, a, b, 1\}$. Define operations $\vee$ and $\cdot$ on L as follows:

$\vee$	0	a	b	1
0	0	a	b	1
a	a	a	b	1
b	b	b	b	1
1	1	1	1	1

and

$\cdot$	0	a	b	1
0	0	0	0	0
a	0	0	0	a
b	0	0	b	b
1	0	a	b	1

Then $(L, \vee, \cdot)$ is a multiplicative semilattice.

Define a function $D : L^3 \to L$ by $D(x_1, x_2, x_3) = x_1x_2x_3$ for all $x_1, x_2, x_3 \in L$.
Let $f, g : L \to L$ be defined respectively by

$$f(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } x = a \\ a & \text{if } x = b, 1 \end{cases} \quad \text{and} \quad g(x) = \begin{cases} 0 & \text{if } x = 0, a \\ b & \text{if } x = b \\ 1 & \text{if } x = 1 \end{cases}$$

Then it can be easily verified that D is a permuting 3-(f, g)-derivation on L.

Theorem 3.4. *Let D be a permuting n-(f, g)-derivation on L. Then the following statements hold for all $x_1, x_2, \ldots, x_n, y \in L$:*

(i) If $f(0) = 0 = g(0)$, then $D(0, x_2, \ldots, x_n) = 0$.
(ii) If $f(1) = 1$, then $g(x_1)D(1, x_2, \ldots, x_n) \leqslant D(x_1, x_2, \ldots, x_n)$.
(iii) If $g(1) = 1$, then $D(1, x_2, \ldots, x_n)f(x_1) \leqslant D(x_1, x_2, \ldots, x_n)$.
(iv) If $f(1) = 1 = g(1)$ and $D(1, x_2, \ldots, x_n) = 1$, then $g(x_1) \leqslant D(x_1, x_2, \ldots, x_n)$ and $f(x_1) \leqslant D(x_1, x_2, \ldots, x_n)$.
(v) If $x \leqslant y$, then $D(x, x_2, \ldots, x_n) \leqslant D(y, x_2, \ldots, x_n)$.
(vi) $D(x_1, x_2, \ldots, x_n)f(x_1) \leqslant f(x_1)$ and $D(x_1, x_2, \ldots, x_n)f(x_1) \leqslant D(x_1, x_2, \ldots, x_n)$.
(vii) $g(x_1)D(x_1, x_2, \ldots, x_n) \leqslant g(x_1)$ and $g(x_1)D(x_1, x_2, \ldots, x_n) \leqslant D(x_1, x_2, \ldots, x_n)$.
(viii) $D(x_1y, x_2, \ldots, x_n) \leqslant D(x_1, x_2, \ldots, x_n) \vee D(y, x_2, \ldots, x_n)$.
(ix) If L is an idempotent multiplicative semilattice then $D(x_1, x_2, \ldots, x_n) \leqslant f(x_1) \vee g(x_1)$.

Proof. Let $x_1, x_2, \ldots, x_n, y \in L$.

(i) Suppose $f(0) = 0 = g(0)$. Then
$D(0, x_2, \ldots, x_n) = (D(0, x_2, \ldots, x_n)0) \vee (0D(0, x_2, \ldots, x_n)) = 0$.

(ii) Suppose $f(1) = 1$. Then
$D(x_1, x_2, \ldots, x_n) = (D(x_1, x_2, \ldots, x_n)f(1)) \vee (g(x_1)D(1, x_2, \ldots, x_n))$
$= D(x_1, x_2, \ldots, x_n) \vee (g(x_1)D(1, x_2, \ldots, x_n))$.
Hence $g(x_1)D(1, x_2, \ldots, x_n) \leqslant D(x_1, x_2, \ldots, x_n)$.

(iii) Similar to the proof of (ii).

(iv) Follows from (ii) and (iii).

(v) Assume $x \leqslant y$, then we have $y = x \vee y$. Therefore
$D(y, x_2, \ldots, x_n) = D(x \vee y, x_2, \ldots, x_n) = D(x, x_2, \ldots, x_n) \vee D(y, x_2, \ldots, x_n)$.
Hence $D(x, x_2, \ldots, x_n) \leqslant D(y, x_2, \ldots, x_n)$.

(vi) Consider

$$\begin{aligned} D(x_1, x_2, \ldots, x_n)f(x_1) \vee f(x_1) &= [D(x_1, x_2, \ldots, x_n)f(x_1)] \vee [1f(x_1)] \\ &= [D(x_1, x_2, \ldots, x_n) \vee 1]f(x_1) \\ &= f(x_1). \end{aligned}$$

So $D(x_1, x_2, \ldots, x_n)f(x_1) \vee f(x_1) = f(x_1)$.
Therefore $D(x_1, x_2, \ldots, x_n)f(x_1) \leqslant f(x_1)$.
Similarly, $D(x_1, x_2, \ldots, x_n)f(x_1) \leqslant D(x_1, x_2, \ldots, x_n)$.

(vii) Similar to the proof of (vi).

(viii) Consider $D(x_1, x_2, \ldots, x_n)f(y) \vee D(x_1, x_2, \ldots, x_n)$

$$\begin{aligned} &= [D(x_1, x_2, \ldots, x_n)f(y)] \vee [D(x_1, x_2, \ldots, x_n)1] \\ &= D(x_1, x_2, \ldots, x_n)[f(y) \vee 1] \\ &= D(x_1, x_2, \ldots, x_n)1 \\ &= D(x_1, x_2, \ldots, x_n). \end{aligned}$$

Hence $D(x_1, x_2, \ldots, x_n)f(y) \leqslant D(x_1, x_2, \ldots, x_n)$.
Similarly, $g(x_1)D(y, x_2, \ldots, x_n) \leqslant D(y, x_2, \ldots, x_n)$.
Therefore $D(x_1y, x_2, ..., x_n) = (D(x_1, x_2, ..., x_n)f(y)) \vee (g(x_1)D(y, x_2, ..., x_n))$
$\leqslant D(x_1, x_2, \ldots, x_n) \vee D(y, x_2, \ldots, x_n)$.
Hence $D(x_1y, x_2, \ldots, x_n) \leqslant D(x_1, x_2, \ldots, x_n) \ \vee \ D(y, x_2, \ldots, x_n)$.

(ix) Assume that L is an idempotent.
$D(x_1, x_2, \ldots, x_n) = [D(x_1, x_2, \ldots, x_n)f(x_1)] \vee [g(x_1)D(x_1, x_2, \ldots, x_n)]$
$\leqslant f(x_1) \vee g(x_1)$.
Hence $D(x_1, x_2, \ldots, x_n) \leqslant f(x_1) \vee g(x_1)$.

∎

Let D be a permuting n-(f, g)-derivation on L. Define a set $Ker(D)$ by
$Ker(D) = \{x \in L \mid D(x, x_2, \ldots, x_n) = 0$ for all $x_2, \ldots, x_n \in L\ \}$.

Theorem 3.5. *Let D be a permuting n-(f, g)-derivation on L. If $f(0) = 0 = g(0)$, then $Ker(D)$ is a subsemilattice of L.*

Proof. Assume that $f(0) = 0 = g(0)$. Let $x_2, \ldots, x_n \in L$.
By Theorem 3.4 (i), we have $D(0, x_2, \ldots, x_n) = 0$. Thus $0 \in Ker(D)$.
Let $x, y \in Ker(D)$, so we have $D(x, x_2, \ldots, x_n) = 0$ and $D(y, x_2, \ldots, x_n) = 0$.
By Theorem 3.4 (viii), we have
$D(xy, x_2, \ldots, x_n) \leqslant D(x, x_2, \ldots, x_n) \vee D(y, x_2, \ldots, x_n) = 0 \vee 0 = 0$. So
$D(xy, x_2, \ldots, x_n) \leqslant 0$. Hence $D(xy, x_2, \ldots, x_n) = 0$. That is $xy \in Ker(D)$.
$D(x \vee y, x_2, \ldots, x_n) = D(x, x_2, \ldots, x_n) \vee D(y, x_2, \ldots, x_n) = 0 \vee 0 = 0$. Hence
$D(x \vee y, x_2, \ldots, x_n) = 0$. That is $x \vee y \in Ker(D)$.
Therefore $Ker(D)$ is a subsemilattice of L. ∎

Theorem 3.6. *Let L be an idempotent multiplicative semilattice and D be a permuting n-(f, g)-derivation on L. Assume that $f(0) = 0 = g(0)$, $f(1) = 1 = g(1)$, and $D(1, x_2, \ldots, x_n) = 1$ for all $x_2, \ldots, x_n \in L$. Then $Ker(D)$ is an ideal of L.*

Proof. By Theorem 3.5, $Ker(D)$ is a subsemilattice of L. Then $x \vee y \in Ker(D)$ for all $x, y \in Ker(D)$. Now, let $x, y \in L$ be such that $x \leqslant y$ and $y \in Ker(D)$.
By theorem 3.4 (v), we get $D(x, x_2, \ldots, x_n) \leqslant D(y, x_2, \ldots, x_n) = 0$ for all $x_2, \ldots, x_n \in L$.
Therefore $D(x, x_2, \ldots, x_n) = 0$ for all $x_2, \ldots, x_n \in L$, and so $x \in Ker(D)$. Next, let $x \in L$ and $y \in Ker(D)$. We now show that $D(xy, x_2, \ldots, x_n) \leqslant D(x, x_2, \ldots, x_n)D(y, x_2, \ldots, x_n)$ for all $x_2, \ldots, x_n \in L$. Let $x_2, \ldots, x_n \in L$.
By Theorem 3.4 (iv) and Lemma 2.4 we have
$D(x, x_2, \ldots, x_n)f(y) \leqslant D(x, x_2, \ldots, x_n)D(y, x_2, \ldots, x_n)$ and

$g(x)D(y, x_2, \ldots, x_n) \leqslant D(x, x_2, \ldots, x_n)D(y, x_2, \ldots, x_n)$. Therefore

$$\begin{aligned} D(xy, x_2, \ldots, x_n) &= (D(x, x_2, \ldots, x_n)f(y)) \vee (g(x)D(y, x_2, \ldots, x_n)) \\ &\leqslant D(x, x_2, \ldots, x_n)D(y, x_2, \ldots, x_n). \end{aligned}$$

Since $y \in Ker(D), D(y, x_2, \ldots, x_n) = 0$. Then
$D(xy, x_2, \ldots, x_n) \leqslant D(x, x_2, \ldots, x_n)D(y, x_2, \ldots, x_n) = 0$.
Thus $D(xy, x_2, \ldots, x_n) = 0$, it follows that $xy \in Ker(D)$.
In the similar way, one can prove that $yx \in Ker(D)$.
Therefore, xy and $yx \in Ker(D)$. Hence $Ker(D)$ is an ideal of L. ∎

Theorem 3.7. *Let L be a prime multiplicative semilattice. Let D be a permuting n-(f, g)-derivation on L with $g : L \to L$ is onto. If there exists $u \in L$ such that $uD(x_1, x_2, \ldots, x_n) = 0$ for all $x_1, x_2, \ldots, x_n \in L$, then $u = 0$ or $D(x_1, x_2, \ldots, x_n) = 0$ for all $x_1, x_2, \ldots, x_n \in L$.*

Proof. Assume that there exists $u \in L$ such that $uD(x_1, x_2, \ldots, x_n) = 0$ for all $x_1, x_2, \ldots, x_n \in L$. Let $x, x_1, x_2, \ldots, x_n \in L$. Then we get

$$\begin{aligned} 0 &= uD(xx_1, x_2, \ldots, x_n) \\ &= u[(D(x, x_2, \ldots, x_n)f(x_1)) \vee (g(x)D(x_1, x_2, \ldots, x_n))] \\ &= (uD(x, x_2, \ldots, x_n)f(x_1)) \vee (ug(x)D(x_1, x_2, \ldots, x_n)) \\ &= 0 \vee (ug(x)D(x_1, x_2, \ldots, x_n)) \\ &= ug(x)D(x_1, x_2, \ldots, x_n). \end{aligned}$$

Thus $ug(x)D(x_1, x_2, \ldots, x_n) = 0$ for all $x, x_1, x_2, \ldots, x_n \in L$.
Since g is onto, $uLD(x_1, x_2, \ldots, x_n) = \{0\}$ for all $x_1, x_2, \ldots, x_n \in L$. By the primeness of L, we have $u = 0$ or $D(x_1, x_2, \ldots, x_n) = 0$ for all $x_1, x_2, \ldots, x_n \in L$. ∎

Definition 3.8. Let D be a permuting n-(f, g)-derivation on L. A mapping $d : L \to L$ defined by $d(x) = D(x, x, \ldots, x)$ for all $x \in L$ is called the *trace* of D.

Theorem 3.9. *Let L be an idempotent multiplicative semilattice and D be a permuting n-(f, g)-derivation on L with trace d. Then $d(x) \leqslant f(x) \vee g(x)$ for all $x \in L$.*

Proof. The proof follows from Theorem 3.4(ix). ∎

For simplicity, we denote from now on $D(x^{(n-k)}, y^{(k)})$ by $D(\underbrace{x, x, \ldots, x}_{n-k \text{ copies}}, \underbrace{y, y, \ldots, y}_{k \text{ copies}})$, where $k = 1, 2, 3, \ldots, n-1$, $x, y \in L$, and D is a permuting n-(f, g)-derivation of L.

Theorem 3.10. *Let L be a multiplicative semilattice and D be a permuting n-(f, g)-derivation on L with trace d. Then*

$$d(x \vee y) = d(x) \vee [D(x^{(n-1)}, y) \vee D(x^{(n-2)}, y^{(2)}) \vee \cdots \vee D(x, y^{(n-1)})] \vee d(y),$$

for all $x, y \in L$.

Proof. Let $x, y \in L$, we have

$$\begin{aligned} d(x \vee y) &= D(x \vee y, x \vee y, \ldots, x \vee y) \\ &= D(x, (x \vee y)^{(n-1)}) \vee D(y, (x \vee y)^{(n-1)}) \\ &= D(x, x, (x \vee y)^{(n-2)}) \vee D(x, y, (x \vee y)^{(n-2)}) \vee D(y, y, (x \vee y)^{(n-2)}) \\ &= D(x, x, x, (x \vee y)^{(n-3)}) \vee D(x, x, y, (x \vee y)^{(n-3)}) \vee D(x, y, y, (x \vee y)^{(n-3)}) \\ &\quad \vee D(y, y, y, (x \vee y)^{(n-3)}) \\ &\quad \vdots \\ &= D(x, x, \ldots, x) \vee [D(x, y^{(n-1)}) \vee D(x^{(2)}, y^{(n-2)}) \vee \cdots \vee D(x^{(n-1)}, y)] \\ &\quad \vee D(y, y, \ldots, y) \\ &= d(x) \vee [D(x, y^{(n-1)}) \vee D(x^{(2)}, y^{(n-2)}) \vee \cdots \vee D(x^{(n-1)}, y)] \vee d(y). \end{aligned}$$

This completes the proof. ∎

The proof of the following theorem is similar to the proof of Theorem 3.10.

Theorem 3.11. *Let L be a commutative multiplicative semilattice and D be a permuting n-(f, g)-derivation on L with trace d. Then*
$d(xy) = (d(x)f(y)) \vee (g(x)d(y)) \vee [(g(x)f(y))[D(x^{(n-1)}, y) \vee D(x^{(n-2)}, y^{(2)}) \vee \ldots \vee D(x, y^{(n-1)})]$
for all $x, y \in L$.

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