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# Forgotten Index and Forgotten Coindex of Graphs Using Degree Sequence

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**Abstract** The Forgotten index is one of the degree based topological index which was introduced by Furtula and Gutman in 2015, $F(G) = \sum_{u\in V(G)} d_u^3$ where $d_u$, degree of vertex $u$ in $G$. The paper focuses on some general results and inequalities of the forgotten index and forgotten coindex of graphs using degree sequence.

**MSC:** 05C05; 05C07; 05C09
**Keywords:** Forgotten index; Forgotten coindex; complement

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## 1. Introduction

Chemical Graph theory is one of the interesting area in mathematics, it is the combination of graph theory and chemistry which in turn used to model the physical properties of alkane chemical compounds [1]. Alkanes are a series of saturated hydrocarbon atoms with a single bond. A molecular descriptor is a structural property of a molecule. Topological indices are numerical values that are related with a chemical contribution for the correlation of chemical structure with various physical and chemical properties.

The molecular graph is a topological representation of the structure of a chemical compound. In a molecular graph, these nodes and edges are named vertices and edges respectively in terms of graph theory.

The degree of a vertex $v$ of $G$ represented by $d(v)$, is the number of edges incident on a vertex $v \in G$. The maximum and minimum degree of a graph $G$ are represented by $\triangle(G)$ and $\delta(G)$ respectively.The degree sum formula states that the sum of the degree of vertices in any undirected graph is two times the number of edges.

The complement of $G$ is a graph $\overline{G}$ with the same number of vertices but there is an edge between the vertices $x$ and $y$ in $G$, iff there is no edge between that $x$ and $y$ vertices in $\overline{G}$.

The degree sequence of a graph $G = (V, E)$ is nothing but a limit of the degree of each vertex in $V$. It is denoted by $d_i$, $i = 1, 2, ..., n$.

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The first Zagreb index ($M_1(G)$) and second Zagreb index ($M_2(G)$) are considered as the first degree-based topological index of adjacent vertices (historically) [2] which came into existence during the study of total Π-electron energy of conjugated systems which was introduced in 1972 by Gutman and Trinajtic [3] which is defined as

$$M_1(G) = \sum_{u \in V(G)} d_u^2 \tag{1.1}$$

$$M_2(G) = \sum_{u,v \in V(G)} d_u d_v \tag{1.2}$$

and [4]

$$M_1(\overline{G}) = \sum_{u \in V(\overline{G})} \overline{d}_u^2. \tag{1.3}$$

The Forgotten index $F(G)$ of $G$ was introduced by Furtula and Gutman in 2015 [5]. It is defined as

$$F(G) = \sum_{u \in V(G)} d_u^3. \tag{1.4}$$

Nilanjan De et al. [6] introduced a Forgotten coindex of $G$ in 2016. It is defined as [7]

$$\overline{F}(G) = \sum_{u \in V(G)} \overline{d}_u d_u^2 \tag{1.5}$$

where $\overline{d}_u$ - degree of vertex $u$ in $\overline{G}$.

The Forgotten Coindex of $\overline{G}$ is defined as

$$\overline{F}(\overline{G}) = \sum_{u \in V(G)} d_u \overline{d}_u^2. \tag{1.6}$$

Sharma et al. [8] introduced the eccentric connectivity index $\xi^c(G)$ in 1997. It is defined as

$$\xi^c(G) = \sum_{u \in V(G)} \varepsilon(u) d(u) \tag{1.7}$$

where, $\varepsilon(u) = \max\{d(u)\}$.

## 2. Main Results

In this part, we prove the Forgotten index for some graph families [9, 10].

**Theorem 2.1.** *Consider a connected graph $G(n, m)$ with $n$ verties and $m$ edges. Adding $n$ new edges which connect $n$ verties of $G$ with new vertex $x$ is termed as $G + x$ then the Forgotten index of $G + x$ is*

$$F(G) + 3M_1(G) + 6m + n(n^2 + 1).$$

*Proof.* Let $G\big(\{v_1, v_2, ..., v_n\}, \{e_1, e_2, ..., e_m\}\big) + x$ contains $n+1$ vertices and $n+m$ edges.

By (1.4) of Forgotten index of $G + x$ is

$$\begin{aligned} F(G+x) &= \sum_{i=1}^{n}(d_i+1)^3 + n^3 \\ &= \sum_{i=1}^{n} d_i^3 + 3\sum_{i=1}^{n} d_i^2 + 3\sum_{i=1}^{n} d_i + \sum_{i=1}^{n} 1 + n^3 \\ F(G+x) &= F(G) + 3M_1(G) + 6m + n(n^2+1). \end{aligned}$$

■

**Corollary 2.2.** $\overline{F}(G+x) = \overline{F}(G) - 2M_1(G) + n(n-1) - 6m + 4nm.$

*Proof.* By (1.5) of a Forgotten coindex of $G + x$ is

$$\begin{aligned} \overline{F}(G+x) &= \sum_{i=1}^{n}((d_i+1)^2\overline{d_i}) + n^2.0 \\ &= \sum_{i=1}^{n} d_i^2.\overline{d_i} + 2\sum_{i=1}^{n} d_i\overline{d_i} + \sum_{i=1}^{n}\overline{d_i} \\ &= \overline{F}(G) + 2\big(2m(n-1) - M_1(G)\big) + n(n-1) - 2m \\ \overline{F}(G+x) &= \overline{F}(G) - 2M_1(G) + n(n-1) - 6m + 4nm. \end{aligned}$$

■

**Corollary 2.3.** $\overline{F}(\overline{G+x}) = \overline{F}(\overline{G}) + M_1(\overline{G}).$

*Proof.* By (1.6) of a Forgotten coindex of $\overline{G+x}$ is

$$\begin{aligned} \overline{F}(\overline{G+x}) &= \sum_{i=1}^{n}(d_i+1)\overline{d_i}^2 + n.0 \\ \overline{F}(\overline{G+x}) &= \overline{F}(\overline{G}) + M_1(\overline{G}). \end{aligned}$$

■

**Theorem 2.4.** *Assume a connected graph $G$ having at least one vertex $x$ with $d(x) = n-1$ and no pendant vertices. The Forgotten index of $G - x$ is*

$$F(G) - 3M_1(G) - n(n^2 - 6n + 13) + 6m + 8$$

*where the graph $G - x$ is obtained by removing a vertex $x$ from $G$.*

*Proof.* A graph with no pendant vertices and having atleast one vertex $x$ with $d(x) = n-1$. Removal of a vertex $x$ from $G$ leads to $G - x$ which contains $n-1$ vertices and $m-n+1$ edges.

By (1.4) of Forgotten index of $G - x$ is

$$\begin{aligned}
F(G-x) &= \sum_{i=1}^{n-1}(d_i-1)^3 \\
&= \sum_{i=1}^{n-1} d_i^3 - \sum_{i=1}^{n-1} 1 - 3\sum_{i=1}^{n-1} d_i^2 + 3\sum_{i=1}^{n-1} d_i \\
&= F(G) - (n^3-3n^2+3n-1) - n + 1 - 3M_1(G) + 3(n^2+1-2n) \\
&\quad + 6m - 6n + 6 \\
F(G-x) &= F(G) - 3M_1(G) - n(n^2-6n+13) + 6m + 8.
\end{aligned}$$

■

**Corollary 2.5.** $\overline{F}(G-x) = \overline{F}(G) + 2M_1(G) + n(n-1) - 2m(2n-1)$.

*Proof.*

$$\begin{aligned}
\overline{F}(G-x) &= \sum_{i=1}^{n-1}(d_i-1)^2\overline{d_i} \\
&= \sum_{i=1}^{n-1} d_i^2\overline{d_i} + \sum_{i=1}^{n-1}\overline{d_i} - 2\sum_{i=1}^{n-1} d_i\overline{d_i} \\
&= \overline{F}(G) + (n-1)^2 - n + 1 - 2(m-n+1) - 2\big(\sum_{i=1}^{n-1} d_i(n-1-d_i)\big) \\
\overline{F}(G-x) &= \overline{F}(G) + 2M_1(G) + n(n-1) - 2m(2n-1).
\end{aligned}$$

■

**Corollary 2.6.** $\overline{F}(\overline{G-x}) = \overline{F}(\overline{G}) - M_1(\overline{G})$.

*Proof.*

$$\begin{aligned}
\overline{F}(\overline{G-x}) &= \sum_{i=1}^{n-1}(d_i-1)\overline{d_i}^2 \\
&= \overline{F}(\overline{G}) - (n-1).0 - M_1(\overline{G}) \\
\overline{F}(\overline{G-x}) &= \overline{F}(\overline{G}) - M_1(\overline{G}).
\end{aligned}$$

■

**Theorem 2.7.** *Consider a connected graph $G(n,m)$ with $n$ vertices and $m$ edges which connects every vertex of $G$ to every vertex of $wK_1$, where $wK_1$ contains $w$ isolated vertices which is termed as $G + wK_1$ then*

$$F(G+wK_1) = F(G) + 3wM_1(G) + nw(n^2+w^2) + 6mw^2, w \geq 1.$$

*Proof.* Let the vertex set and edge set of $G + wK_1$ is defined as

$$\begin{aligned}
&V(G+wK_1) = \{v_1, v_2, ...v_n, x_1, x_2, ..., x_w\} \text{ and} \\
&E(G+wK_1) = E(G) \cup \{v_ix_j : 1 < i < n, 1 < j < w\}.
\end{aligned}$$

$$\begin{aligned}
F(G+wK_1) &= \sum_{i=1}^{n}(d_i+w)^3 + wn^3 \\
&= \sum_{i=1}^{n} d_i^3 + \sum_{i=1}^{n} w^3 + 3w\sum_{i=1}^{n} d_i^2 + 3w^2\sum_{i=1}^{n} d_i + wn^3 \\
&= F(G) + nw^3 + 3wM_1(G) + 3w^2(2m) + wn^3 \\
F(G+wK_1) &= F(G) + 3wM_1(G) + nw(n^2+w^2) + 6mw^2.
\end{aligned}$$

∎

**Corollary 2.8.**

$$\overline{F}(G+wK_1) = \overline{F}(G) + w\big((n-1)(wn+4m) - 2(wm+M_1(G)) + n^2(w-1)\big),\ \ w \geq 1.$$

*Proof.*

$$\begin{aligned}
\overline{F}(G+wK_1) &= \sum_{i=1}^{n}(d_i+w)^2\overline{d_i} + wn^2(w-1) \\
&= \sum_{i=1}^{n} d_i^2\overline{d_i} + \sum_{i=1}^{n} w^2\overline{d_i} + 2w\sum_{i=1}^{n} d_i\overline{d_i} + wn^2(w-1) \\
&= \overline{F}(G) + w^2(n(n-1)-2m) + 2w(2m(n-1) - M_1(G)) \\
&\quad + wn^2(w-1) \\
\overline{F}(G+wK_1) &= \overline{F}(G) + w\big((n-1)(wn+4m) - 2(wm+M_1(G)) + n^2(w-1)\big).
\end{aligned}$$

∎

**Corollary 2.9.** $\overline{F}(\overline{G+wK_1}) = \overline{F}(\overline{G}) + w\big[M_1(\overline{G}) + n(w-1)^2\big]$.

*Proof.*

$$\begin{aligned}
\overline{F}(\overline{G+wK_1}) &= \sum_{i=1}^{n}(d_i+w)\overline{d_i}^2 + wn(w-1)^2 \\
\overline{F}(\overline{G+wK_1}) &= \overline{F}(\overline{G}) + w\big[M_1(\overline{G}) + n(w-1)^2\big].
\end{aligned}$$

∎

**Theorem 2.10.** *Let $G'(n,m) = (\cup_{j=1}^{k} G_j, x)$ is obtained by connecting all the vertices of $G_j, j = 1, 2, ..., k$ to newly added vertex $x$ then*

$$F(G') = \sum_{j=1}^{k} F(G_j) + 3\sum_{j=1}^{k} M_1(G_j) + \sum_{j=1}^{k} n_j\Big(\big(\sum_{j=1}^{k} n_j\big)^2 + 1\Big) + 6\sum_{j=1}^{k} m_j.$$

*Proof.* Let $G_1, G_2, ..., G_k$ be the components of graph $G$ which contains $n_1, n_2, ..., n_k$ vertices and $m_1, m_2, ..., m_k$ edges respectively. The graph $G'$ is obtained by connecting all the vertices of $G_j, j = 1, 2, ..., k$ to newly added vertex $x$ which contains $n_1+n_2+...+n_k+1$ vertices and $m_1 + m_2 + ... + m_k + k$ edges denoted by $G'(n,m) = (\cup_{j=1}^{k} G_j, x)$.

$$\begin{aligned}
F(G') &= F(G_1 + G_2 + ... + G_k, x)\\
&= \big(\sum_{j=1}^{n_1}(d_j+1)^3 + \sum_{j=1}^{n_2}(d_j+1)^3 + ... + \sum_{i=1}^{n_k}(d_j+1)^3\big) + \big(\sum_{j=1}^{k} n_j\big)^3\\
&= \big(F(G_1) + 3M_1(G_1) + 6m_1 + n_1\big) + \big(F(G_2) + 3M_1(G_2) + 6m_2 + n_2\big)\\
&\quad + ... + \big(F(G_k) + 3M_1(G_k) + 6m_k + n_k\big) + \big(\sum_{j=1}^{k} n_j\big)^3\\
F(G') &= \sum_{j=1}^{k} F(G_j) + 3\sum_{j=1}^{k} M_1(G_j) + \sum_{j=1}^{k} n_j\big(\big(\sum_{j=1}^{k} n_j\big)^2 + 1\big) + 6\sum_{j=1}^{k} m_j.
\end{aligned}$$ ∎

**Corollary 2.11.**

$$\overline{F}(\cup_{i=1}^{k} G_i, x) = \sum_{i=1}^{k} \overline{F}(G_i) - 2\sum_{i=1}^{k} M_1(G_i) + \sum_{i=1}^{k}(4m_i + n_i)(n_i - 1) - 2\sum_{i=1}^{k} m_i.$$

*Proof.*

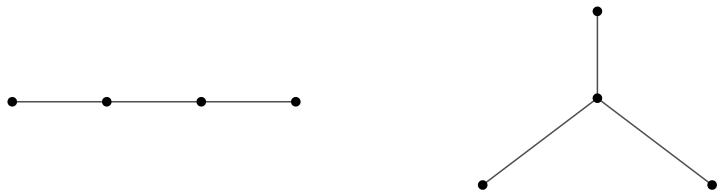
$$\begin{aligned}
\overline{F}(\cup_{i=1}^{k} G_i, x) &= \overline{F}(G_1 + G_2 + ... + G_k, x)\\
&= \big(\sum_{i=1}^{n_1}(d_i+1)^2\overline{d_i} + \sum_{i=1}^{n_2}(d_i+1)^2\overline{d_i} + ... + \sum_{i=1}^{n_k}(d_i+1)^2\overline{d_i}\big) + \big(\sum_{i=1}^{k} n_i\big)^2.0\\
&= \sum_{i=1}^{k}\overline{F}(G_i) + \sum_{i=1}^{k}(n_i(n_i-1) - 2m_i) + 4\sum_{i=1}^{k}(n_i-1)m_i - 2\sum_{i=1}^{k} M_1(G_i)\\
\overline{F}(\cup_{i=1}^{k} G_i, x) &= \sum_{i=1}^{k}\overline{F}(G_i) - 2\sum_{i=1}^{k} M_1(G_i) + \sum_{i=1}^{k}(4m_i + n_i)(n_i - 1) - 2\sum_{i=1}^{k} m_i.
\end{aligned}$$ ∎

**Corollary 2.12.** $\overline{F}(\overline{\cup_{i=1}^{k} G_i, x}) = \sum_{i=1}^{k}\overline{F}(\overline{G}_i) + \sum_{i=1}^{k} M_1(\overline{G_i})$.

*Proof.*

$$\begin{aligned}
\overline{F}(\overline{\cup_{i=1}^{k} G_i, x}) &= \overline{F}(\overline{G_1 + G_2 + ... + G_k, x})\\
&= \sum_{i=1}^{n_1}(d_i+1)\overline{d_i}^2 + \sum_{i=1}^{n_2}(d_i+1)\overline{d_i}^2 + ... + \sum_{i=1}^{n_k}(d_i+1)\overline{d_i}^2 + \big(\sum_{i=1}^{k} n_i\big).0\\
&= \overline{F}(\overline{G}_1) + M_1(\overline{G_1}) + \overline{F}(\overline{G}_2) + M_1(\overline{G_2}) + ... + \overline{F}(\overline{G}_k) + M_1(\overline{G_k})\\
\overline{F}(\overline{\cup_{i=1}^{k} G_i, x}) &= \sum_{i=1}^{k}\overline{F}(\overline{G}_i) + \sum_{i=1}^{k} M_1(\overline{G_i}).
\end{aligned}$$ ∎

The following theorem is proved for $n \geq 5$. For $n = 2$, the Forgotten coindex and its complement is zero and when $n = 3$, the Forgotten coindex and its complement is 2. The possible trees with $n = 4$ (see Figure 1) is as follows:



(A) $T_1$ (B) $T_2$

FIGURE 1. Possible trees with $n = 4$

It is clear that for $T_1$, $\overline{F}(T_1) = \overline{F}(\overline{T}_1) = 12$, $\overline{F}(T_2)$ is 6 and $\overline{F}(\overline{T}_2)$ is 12 which follows $\overline{F}(\overline{T}_2) > \overline{F}(T_2)$.

**Theorem 2.13.** *Let $T_n$ be a tree with $n$ vertices for $n \geq 5$, then*

$$\overline{F}(\overline{T}_n) > \overline{F}(T_n).$$

*Proof.* By the definition of (1.5) and (1.6), we have

$$\begin{aligned}
\overline{F}(\overline{T}_n) - \overline{F}(T_n) &= \sum_{i=1}^{n} \overline{d}_i^2 d_i - \sum_{i=1}^{n} d_i^2 \overline{d}_i \\
&= \sum_{i=1}^{n} \big(n-1-d_i\big)^2 d_i - \sum_{i=1}^{n} d_i^2 \big(n-1-d_i\big) \\
&= 2\sum_{i=1}^{n} d_i^3 - 3(n-1)\sum_{i=1}^{n} d_i^2 + 2(n-1)^3 \\
&= 2\sum_{i=1}^{n} d_i^3 - 3(n-1)\sum_{i=1}^{n} d_i^2 + 2(n-1)^3 + (n-1)\sum_{i=1}^{n} d_i^2 - (n-1)\sum_{i=1}^{n} d_i^2 \\
&+ \sum_{i=1}^{n} d_i^3 - \sum_{i=1}^{n} d_i^3
\end{aligned}$$

$$\overline{F}(\overline{T}_n) - \overline{F}(T_n) = \overline{F}(T_n) + 2(n-1)^3 + 3F(G) - 4(n-1)M_1(G) > 0.$$

Therefore, $\overline{F}(\overline{T}_n) - \overline{F}(T_n) > 0$. ∎

**Theorem 2.14.** *Let $T_n \in T_n^3$, $n \geq 6$ then*

$$2n^3 - 14n^2 + 34n - 28 > \overline{F}(\overline{T}_n) > 2n^3 - 16n^2 + 46n - 44 + (2n-8)\left(\frac{1-(-1)^n}{2}\right). \tag{2.1}$$

*The left hand side holds iff $T_n$ is $P_n$ and the right hand side holds iff*
(i) *any tree except degree 2 when $n \equiv 0$ mod 2*
(ii) *any tree with exactly one vertex of degree 2 when $n \equiv 1$ mod 2.*

*Proof.* Consider a tree $T_n \in T_n^{\triangle}$, where $\triangle = 3$. Let $t$ denotes the number of vertex degree 3, $l = t + 2$ denotes the number of leaves and number of vertex degree 2 is $n - 2t - 2$. As the graph structure starts with the star graph $K_{1,3} = T_4$ then it will extend with 2 leaves on one leaf to maintain the vertex degree 3.

The Forgotten coindex of a complement of path graph is

$$\overline{F}(\overline{P}_n) = 2n^3 - 14n^2 + 34n - 28. \tag{2.2}$$

Calculating the $\overline{F}(\overline{T}_n)$ for maximum vertex degree 3, we have

$$\overline{F}(\overline{T}_n) = 2n^3 - 14n^2 + 34n - 28 - 4(n-4)t. \tag{2.3}$$

From eqn. (2.2) and eqn. (2.3), we get

$$\overline{F}(\overline{T}_n) - \overline{F}(\overline{P}_n) = -4(n-4)t. \tag{2.4}$$

The eqn. (2.4) solved the left inequality directly and it shows that the right inequality is a decreasing function of $t$. So, minimum value of equation (2.1) is obtained.
The maximum number of vertex of degree 3, for

$$t = \begin{cases} \frac{n-2}{2}, & n \equiv 0 \; modulo \; 2 \\ \frac{n-3}{2}, & n \equiv 1 \; modulo \; 2. \end{cases} \tag{2.5}$$

For $n \equiv 0$ *modulo* 2,

$$\overline{F}(\overline{T}_n) = 2n^3 - 16n^2 + 46n - 44 \tag{2.6}$$

which is obtained by substituting the $t$ value in the eqn. (2.3). Similarly, we obtain

$$\overline{F}(\overline{T}_n) = 2n^3 - 16n^2 + 48n - 52, \tag{2.7}$$

when $n \equiv 1$ *modulo* 2.
In general of eqn. (2.6) and (2.7), we get

$$\overline{F}(\overline{T}_n) = 2n^3 - 16n^2 + 46n - 44 + (2n-8)\left(\frac{1-(-1)^n}{2}\right). \tag{2.8}$$

Eqn. (2.8) is the minimum value of $\overline{F}(\overline{T}_n)$.

Finally, the comparison of equations (2.2) and (2.8) leads to

$$2n^3 - 14n^2 + 34n - 28 > \overline{F}(\overline{T}_n) > 2n^3 - 16n^2 + 46n - 44 + (2n-8)\left(\frac{1-(-1)^n}{2}\right).$$

■

**Theorem 2.15.** *Let $T_n \in T_n^5$, $n \geq 14$ then*

$$2n^3 - 14n^2 + 34n - 28 > \overline{F}(\overline{T}_n) > 2n^3 - 20n^2 + O(n).$$

*The left hand side holds iff $T_n$ is $P_n$ and the right hand side holds for any tree with $n$ vertices having a maximum number of vertex degree 5.*

*Proof.* For any graph $T_n \in T_n^5$ $(n \geq 14)$. Let vertex degrees 5, 4 and 3 be named as $g$, $f$ and $t$ respectively. The number of leaves denotes $l = 3g + 2f + t + 2$ and number of vertex degree 2 is $n - 4g - 3f - 2t - 2$.

Calculating the $\overline{F}(\overline{T}_n)$ for maximum vertex degree 3, then

$$\overline{F}(\overline{T}_n) = 2n^3 - 14n^2 + 34n - 28 - 24(n-5)g - 6(2n-5)f - 4(n-4)t. \tag{2.9}$$

Substituting (2.2) in (2.9), we get

$$\overline{F}(\overline{T}_n) - \overline{F}(\overline{P}_n) = -24(n-5)g - 6(2n-5)f - 4(n-4)t. \tag{2.10}$$

The eqn. (2.10) immediatly solved the left inequality and it is an increasing function of $g$,$f$ and $t$. The right inequality of a tree with $n$ vertices having a maximum number of vertex degree 5 starts from 10, 11, 12 and 13, which are built by adding 4 leaves on any one leaf.

The maximum number of vertex degree 5 for varies $n$ is as follows,

$$g = \begin{cases} \frac{n-2}{4}, & n \equiv 2 \ modulo\ 4 \\ \frac{n-3}{4}, & n \equiv 3 \ modulo\ 4 \\ \frac{n-4}{4}, & n \equiv 0 \ modulo\ 4 \\ \frac{n-5}{4}, & n \equiv 1 \ modulo\ 4. \end{cases} \tag{2.11}$$

After substituting all g values in eqn. (2.10), we get

$$\begin{aligned} \overline{F}(\overline{T}_n) - \overline{F}(\overline{P}_n) &= -6n^2 + O(n) \\ &= -\left[\frac{(2(\triangle-1)^2 + 2(\triangle-1)}{\triangle - 1}\right] n^2 + O(n). \end{aligned}$$

Again substituting all $g$ values in eqn. (2.9), we get

$$\overline{F}(\overline{T}_n) = 2n^3 - 20n^2 + O(n) \tag{2.12}$$

$$= 2n^3 - \left[\frac{2(\triangle-1)^2 + 2(\triangle-1)}{\triangle-1} + 14\right] n^2 + O(n). \tag{2.13}$$

Thus, the comparison of inequalities in eqn. (2.2) and eqn. (2.12), we have

$$2n^3 - 14n^2 + 34n - 28 > \overline{F}(\overline{T}_n) > 2n^3 - 20n^2 + O(n).$$

In general for $T_n^{\triangle}$, where $\triangle$ is all the maximum degree. Then the inequalities we have

$$2n^3 - 14n^2 + 34n - 28 > \overline{F}(\overline{T}_n) > 2n^3 - \left[\frac{2(\triangle-1)^2 + 2(\triangle-1)}{\triangle-1} + 14\right] n^2 + O(n).$$

■

To prove Theorem 2.16, the following result is used which is taken from [11]:

"Let $G$ be a nontrivial connected graph of order $n$. For each vertex $u \in V(G)$, $\varepsilon_G(u) \leq n - d_G(u)$, with equality if and only if $G \cong P_4$ or $G \cong K_n - iK_2$, $0 \leq i \leq \lfloor \frac{n}{2} \rfloor$, where $K_n - iK_2$ denotes the graph obtained from the complete graph $K_n$ by removing $i$ independent edges ".

**Theorem 2.16.** *Let $G$ be any graph,*

$$\overline{F}(\overline{G}) \geq 2m - \xi^c(G)$$

*with equality holds iff $G$ is a complete graph.*

*Proof.* By definition (1.6) and (1.7), we get

$$\begin{aligned}
\overline{F}(\overline{G}) &= \sum_{u\in V(G)} \overline{d}_u^2 d_u \\
&= \sum_{u\in V(G)} (n-1-d_u)^2 d_u \\
&\geq \sum_{u\in V(G)} (\varepsilon_u - 1)^2 d_u \\
&= \sum_{u\in V(G)} \varepsilon_u^2 d_u - 2\sum_{u\in V(G)} \varepsilon_u d_u + \sum_{u\in V(G)} d_u \\
&\geq \sum_{u\in V(G)} \varepsilon_u d_u - 2\sum_{u\in V(G)} \varepsilon_u d_u + \sum_{u\in V(G)} d_u \\
\overline{F}(\overline{G}) &= 2m - \xi^c(G).
\end{aligned}$$

■

## 3. Conclusion

In this paper, we proved some general results and inequalities on Forgotten index and Forgotten coindex of $G$ using degree sequence. Finding the extreme bounds for chemical compounds using graph structures is our future work.

## References

[1] D. Bonchev, Chemical Graph Theory: Introduction and Fundamentals, CRC Press, 1991.

[2] I. Gutman, Degree-based topological indices, Croatica chemica acta 86 (4) (2013) 351–361.

[3] I. Gutman, N. Trinajstic, Graph theory and molecular orbitals. Total $\varphi$-electron energy of alternant hydrocarbons, Chem. Phys. Lett. 17 (1972) 535–538.

[4] A.R. Ashrafi, T. Došlić, A. Hamzeh, The Zagreb coindices of graph operations, Discrete Applied Mathematics 158 (15) (2010) 1571–1578.

[5] B. Furtula, I. Gutman, A forgotten topological index, J. Math. Chem. 53 (2015) 1184–1190.

[6] N. De, S.M.A. Nayeem, A. Pal, The F-coindex of some graph operations, Springer Plus 5 (1) (2016) 1–13.

[7] A. Ghalavand, A.R. Ashrafi, On forgotten coindex of chemical graphs, MATCH Commun. Math. Comput. Chem 83 (2020) 221–232.

[8] V. Sharma, R. Goswami, A.K. Madan, Eccentric connectivity index: A novel highly discriminating topological descriptor for structure–property and structure–activity studies, J. Chem. Inf. Comput. Sci. 37 (1997) 273–282.

[9] J.B. Babujee, J. Senbagamalar, Wiener index of graphs using degree sequence, Applied Mathematical Sciences 6 (88) (2012) 4387–4395.

[10] D. Vukicevic, Q. Li, J. Sedlar, T. Došlic, Lanzhou index, MATCH Commun. Math. Comput. Chem 80 (2018) 863–876.

[11] M. Azari, F. Falahati-Nezhed, Some results on forgotten topological coindex, Iranian Journal of Mathematical Chemistry 10 (4) (2019) 307–318.