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Soft Semigraphs and Different Types of Degrees, Graphs and Matrices Associated with Them

Bobin George[1,*], Rajesh K. Thumbakara[2] and Jinta Jose[3]

[1] *Department of Mathematics, Pavanatma College, Murickassery, India*
e-mail : bobingeorge@pavanatmacollege.org

[2] *Department of Mathematics, Mar Athanasius College(Autonomous), Kothamangalam, India*
e-mail : rthumbakara@macollege.in

[3] *Department of Science and Humanities, Viswajyothi College of Engineering and Technology, Vazhakulam, India*
e-mail : jinta@vjcet.org

Abstract This is an introductory paper on soft semigraphs. Semigraph is a generalization of graph introduced by E. Sampathkumar which is different from hypergraph. In 1999, D. Molodtsov initiated the novel concept of soft set theory. This is an approach for modelling vagueness and uncertainty. It is a classification of elements of the universe with respect to some given set of parameters. The concept of soft graph introduced by Rajesh K. Thumbakara and Bobin George is used to provide a parameterized point of view for graphs. The theory of soft graphs is a fast developing area in graph theory due to its capability to deal with the parameterization tool. In this paper, we introduce soft semigraph by applying the concept of soft set in semigraph. Also, we introduce different types of degrees, graphs and matrices associated with a soft semigraph and investigate some of their properties.

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Keywords: semigraph; soft semigraph

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1. Introduction

The notion of semigraph introduced by E. Sampathkumar [9] is a generalization of that of a graph. This generalization is different from hypergraphs. In semigraphs, the vertices contained in each edge are arranged in an order. More contributions to semigraph came from C. M. Deshpande, B. Y. Bam, L. Pushpalatha, V. Swaminathan [9], N. Murugesan [7], A. Paneerselvam and T. B. Rani[8]. D. Molodtsov [6] presented the innovative concept of soft set theory in 1999. This is a technique in mathematics for dealing with uncertainties. Many practical problems can be tackled using soft set theory.

*Corresponding author.

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Authors like R. Biswas, P. K. Maji and A. R. Roy [4], [5] have delved deeper into the idea of soft sets and applied it to various decision-making situations. The notion of soft graph was first developed in 2014 by R. K. Thumbakara and B. George [13]. They[14] also discussed soft graph operations and introduced notions such as soft tree, soft subgraph and soft complete graph. M. Akram and S. Nawas [1] updated R. K. Thumbakara and B. George's notion of soft graph in 2015. They [2] also defined many ideas in soft graphs, such as soft trees, soft bridges, soft cut vertex, soft cycle etc. More contributions to soft graph came from J. D. Thenge, R. S. Jain and B. S. Reddy[10],[11],[12]. In this paper, we introduce soft semigraph by applying the concept of soft set in semigraph. Also, we introduce different types of degrees, graphs and matrices associated with a soft semigraph and investigate some of their properties.

2. Preliminaries

2.1. Semigraph

A *semigraph* G^* is a pair (V, X) where V is a nonempty set whose elements are called vertices of G^*, and X is a set of n-tuples, called edges of G, of distinct vertices, for various $n \geq 2$, satisfying the following conditions.

(1) Any two edges have at most one vertex in common
(2) Two edges $(u_1, u_2, \ldots, u_n)$ and $(v_1, v_2, \ldots, v_m)$ are considered to be equal if and only if
  (a) $m = n$ and
  (b) either $u_i = v_i$ for $1 \leq i \leq n$, or $u_i = v_{n-i+1}$ for $1 \leq i \leq n$.

The idea of a semigraph is a generalisation of the idea of a graph. Semigraphs are more closely related to graphs than hypergraphs in terms of definition. Edges in a hypergraph are sets. However, just like in graphs, we can represent the vertices of semigraphs as points and their edges as lines. Figure 1 shows an example for a semigraph $G^* = (V, X)$. The vertex set of this semigraph G^* is $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$ and the edge set is $X = \{(v_1, v_2, v_3), (v_1, v_4, v_6), (v_3, v_6), (v_4, v_5), (v_6, v_7, v_8)\}$.

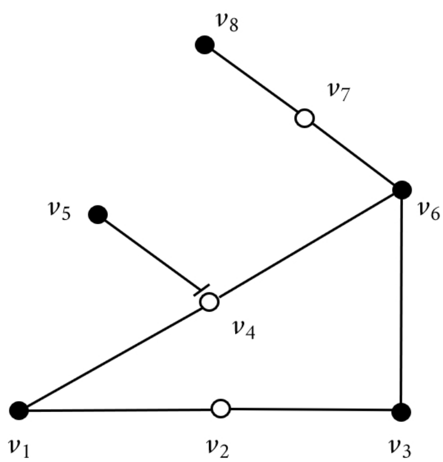


Figure 1. Semigraph $G^* = (V, X)$

Let $G^* = (V, X)$ be a semigraph and $E = (v_1, v_2, \ldots, v_n)$ be an edge of G^*. Then v_1 and v_n are the *end vertices* of E and $v_i, 2 \leq i \leq n-1$ are the *middle vertices(or m-vertices)* of E. If a vertex v of a semigraph G appears only as an end vertex then it is called an *end vertex*. If a vertex v is only a middle vertex then it is a *middle vertex* or *m-vertex* while a vertex v is called *middle-cum-end vertex or* (m, e)*-vertex* if it is a middle vertex of some edge and an end vertex of some other edge. A *subedge* of an edge $E = (v_1, v_2, \ldots, v_n)$ is a k-tuple $E' = (v_{i_1}, v_{i_2}, \ldots, v_{i_k})$, where $1 \leq i_1 < i_2 < \cdots < i_k \leq n$ or $1 \leq i_k < i_{k-1} < \cdots < i_1 \leq n$. We say that the subedge E' is *induced* by the set of vertices $\{v_{i_1}, v_{i_2}, \ldots v_{i_k}\}$. A *partial edge* of $E = (v_1, v_2, \ldots, v_n)$ is a $(j-i+1)$-tuple $E(v_i, v_j) = (v_i, v_{i+1}, \ldots, v_j)$, where $1 \leq i < j \leq n$. $G'^* = (V', X')$ is a *partial semigraph* of a semigraph G^* if the edges of G'^* are partial edges of G^*. Two vertices u and v in a semigraph G^* are said to be *adjacent* if they belong to the same edge. If u and v are adjacent and consecutive in order then they are said to be *consecutively adjacent*. u and v are said to be *e-adjacent* if they are the end vertices of an edge and *1e-adjacent* if both the vertices u and v belong to the same edge and at least one of them is an end vertex of that edge. We can associate three different graphs with a semigraph $G^* = (V, X)$ each having the same vertex set V of G^*. The *end vertex graph* G^*_e is a graph having vertex set V and two vertices in G^*_e are adjacent if and only if, they are end vertices of an edge in G^*. The *adjacency graph* G^*_a is a graph having vertex set V and two vertices in G^*_a are adjacent if and only if, they are adjacent in G^*. The *consecutive adjacency graph* G^*_{ca} is a graph having vertex set V and two vertices in G^*_{ca} are adjacent if and only if, they are consecutively adjacent vertices in G^*. We can define various types of degrees for a vertex v in a semigraph G^*. *Degree* of v, denoted by $deg\ v$ is the number of edges having v as an end vertex. *Edge degree* of v is the number of edges containing v, and is denoted by $deg_e v$. *Adjacent degree* of v, denoted by $deg_a v$ is the number of vertices adjacent to v. *Consecutive adjacent degree* of v, denoted by $deg_{ca} v$ is the number of vertices which are consecutively adjacent to v.

2.2. Soft Set

In 1999 D. Molodtsov [7] initiated the concept of soft sets. Let U be an initial universe set and let E be a set of parameters. A pair (F, E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U. That is, $F : E \to \mathcal{P}(U)$.

3. Soft Semigraph

Definition 3.1. Let V be the vertex set of a semigraph G^*. Consider a subset V_1 of V. Then a partial edge formed by some or all vertices of V_1 is said to be a *maximum partial edge* or *mp edge* if it is not a partial edge of any other partial edge formed by some or all vertices of V_1.

Definition 3.2. Let $G^* = (V, X)$ be a semigraph having vertex set V and edge set X. Let X_p be the collection of all partial edges of the semigraph G^* and A be a nonempty set. Let a subset R of $A \times V$ be an arbitrary relation from A to V. We define a mapping Q from A to $\mathcal{P}(V)$ by $Q(x) = \{y \in V | xRy\}$, for all x in A, where $\mathcal{P}(V)$ denotes the power set of V. Also define a mapping W from A to $\mathcal{P}(X_p)$ by $W(x) = \{mp\ edges < Q(x) >\}$, where $\{mp\ edges < Q(x) >\}$ denotes the set of all mp edges that can be formed by some or all vertices of $Q(x)$ and $\mathcal{P}(X_p)$ denotes the power set of X_p. Then we define a soft semigraph as follows: The 4-tuple $G = (G^*, Q, W, A)$ is called a *soft semigraph* of G^* if the following conditions are satisfied:

(1) $G^* = (V, X)$ is a semigraph having vertex set V and edge set X,
(2) A is the set of parameters which is nonempty,
(3) (Q, A) is a soft set over V,
(4) (W, A) is a soft set over X_p,
(5) $H(a) = (Q(a), W(a))$ is a partial semigraph of G^*, for all a in A.

Example 3.3. We know that the best way to represent the relationship among the members of a number of families is by a semigraph, where the parents are represented by the end vertices of the edges and their children are represented by the m-vertices of edges. Let $G^* = (V, X)$ given below is a semigraph representing a family relationship.

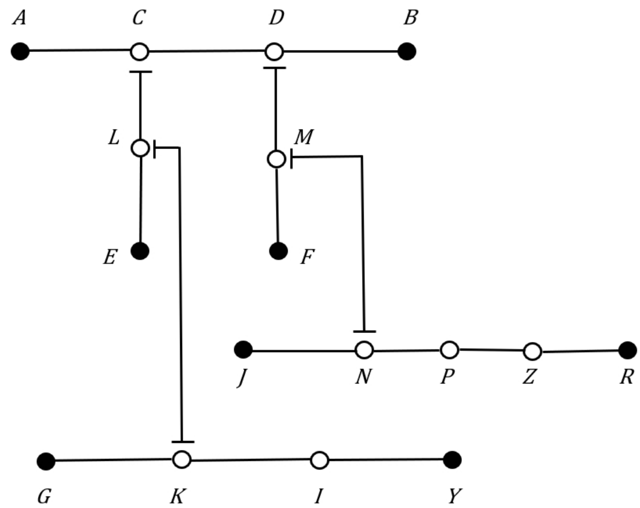


FIGURE 2. Semigraph $G^* = (V, X)$

Sometimes we may get the semigraph representing a family relationship as a more complicated structure than this. We can separate this into small family structures by using the concept of soft semigraph.
For example consider the semigraph $G^* = (V, X)$ given in Figure 2. Let $A = \{C, M\} \subseteq V$ be a parameter set. Define Q from A to $\mathcal{P}(V)$ by $Q(x) = \{y \in V | xRy \Leftrightarrow x = y$ or x and y are adjacent$\}$, for all x in A and W from A to $\mathcal{P}(X_p)$ by $W(x) = \{$*mp edges* $< Q(x) >\}$, for all x in A. That is, $Q(C) = \{A, C, D, B, L, E\}$ and $Q(M) = \{D, M, F, N\}$. Also $W(C) = \{(A, C, D, B), (C, L, E)\}$ and $W(M) = \{(D, M, F), (M, N)\}$. Then $H(C) = (Q(C), W(C))$ and $H(M) = (Q(M), W(M))$ are partial semigraphs of G^* as shown below in Figure 3. Hence, $G = \{H(C), H(M)\}$ is a soft semigraph of G^*.

Here, the soft semigraph reduced family structures to the small families of C and M. The partial semigraph $H(C)$ contains the parents, sibling, partner and child of the person C. The partial semigraph $H(M)$ contains the parents and partner of the person M. Also the relationships are clear from these partial semigraphs.

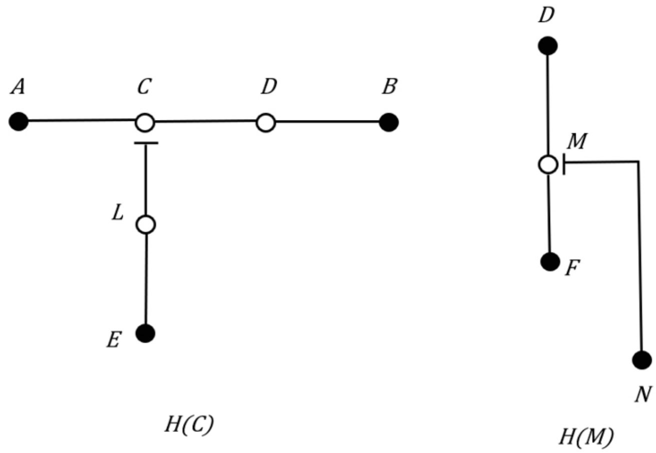


FIGURE 3. Soft Semigraph $G = \{H(C), H(M)\}$

Definition 3.4. Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* which is also given by $\{H(x) : x \in A\}$. Then, the partial semigraph $H(x)$ corresponding to any parameter x in A is called a *p-part* of the soft semigraph G.

Definition 3.5. An edge present in a soft semigraph G of G^* is called an *f-edge* . It may be a partial edge of some edge in G^* or an edge in G^*.

Definition 3.6. If the vertex v, coming at the end of an f-edge E is also an end vertex of the corresponding edge E' in G^* whose partial edge is E, then v is called an *end vertex* of E. Otherwise it is called a *partial end vertex.*

Definition 3.7. A partial edge of any f-edge of the soft semigraph G is called a *p-edge* of G. An f-edge is a p-edge of itself.

Definition 3.8. An edge is called an *fp-edge* of the soft semigraph G, if it is an f-edge or a p-edge of G.

4. Graphs Associated with a Soft Semigraph

We define various types of graphs associated with a soft semigraph as follows.

4.1. Adjacency Graph of a Soft Semigraph

Definition 4.1. Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* given by $\{H(x) : x \in A\}$. Then, the *adjacency graph* G_a of the soft semigraph G is given by $G_a = \{H(x)_a : x \in A\}$ where $H(x)_a$ is a graph having vertex set $Q(x)$ and two vertices in $H(x)_a$ are adjacent if they are adjacent in the p-part $H(x)$. $H(x)_a$ is called *p-part adjacency graph* of $H(x)$.

Example 4.2. Let $G^* = (V, X)$ be a semigraph given in Figure 4 having vertex set $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and the edge set $X = \{(v_1, v_2, v_3, v_4), (v_4, v_5, v_6, v_7), (v_3, v_6)\}$.

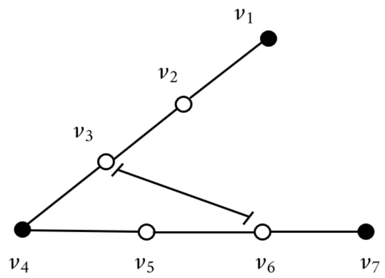


Figure 4. Semigraph $G^* = (V, X)$

Let the parameter set be $A = \{v_2, v_6\} \subseteq V$. Define $Q : A \to \mathcal{P}(V)$ by $Q(x) = \{y \in V | xRy \Leftrightarrow x = y$ or x and y are adjacent$\}$, for all x in A and $W : A \to \mathcal{P}(X_p)$ by $W(x) = \{mp\ edges < Q(x) >\}$, for all x in A. That is, $Q(v_2) = \{v_1, v_2, v_3, v_4\}$ and $Q(v_6) = \{v_3, v_4, v_5, v_6, v_7\}$. Also $W(v_2) = \{(v_1, v_2, v_3, v_4)\}$ and $W(v_6) = \{(v_3, v_6), (v_4, v_3), (v_4, v_5, v_6, v_7)\}$. Then $H(v_2) = (Q(v_2), W(v_2))$ and $H(v_6) = (Q(v_6), W(v_6))$ are partial semigraphs of G^* as shown below in Figure 5. Hence $G = \{H(v_2), H(v_6)\}$ is a soft semigraph of G^*.

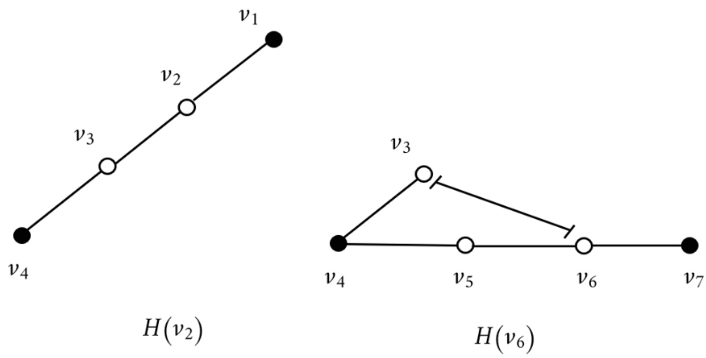


Figure 5. Soft Semigraph $G = \{H(v_2), H(v_6)\}$

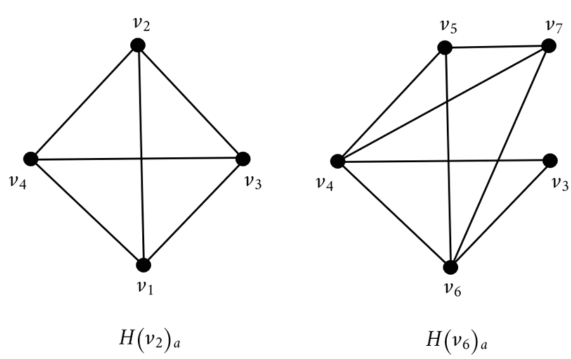


Figure 6. Adjacency Graph $G_a = \{H(v_2)_a, H(v_6)_a\}$

The adjacency graph G_a of this soft semigraph G is given by $G_a = \{H(v_2)_a, H(v_6)_a\}$ and is shown in Figure 6.

Theorem 4.3. *Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* given by $\{H(x) : x \in A\}$. Also let G^*_a and $G_a = \{H(x)_a : x \in A\}$ be the adjacency graphs of G^* and G respectively. Then, the p-part adjacency graph $H(x)_a$ is a subgraph of G^*_a, for all $x \in A$.*

Proof. If $G^* = (V, X)$, then V is the vertex set of both G^* and G^*_a. Consider any p-part $H(x)$ of G. Then, the vertex set of the p-part adjacency graph $H(x)_a$ is $Q(x) \subseteq V$. Also an f-edge present in $H(x)$ is an edge in G^* or a partial edge of an edge in G^*. Therefore, if u and v are adjacent in $H(x)$, they are also adjacent in G^*. So any edge present in $H(x)_a$ is also an edge in G^*_a. That is, the vertex set and edge set of $H(x)_a$ are subsets of the vertex set and edge set of G^*_a respectively. So $H(x)_a$ is a subgraph of G^*_a, for all $x \in A$. ■

4.2. Consecutive Adjacency Graph of a Soft Semigraph

Definition 4.4. Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* given by $\{H(x) : x \in A\}$. Then, the *consecutive adjacency graph* G_{ca} of the soft semigraph G is given by $G_{ca} = \{H(x)_{ca} : x \in A\}$ where $H(x)_{ca}$ is a graph having vertex set $Q(x)$ and two vertices in $H(x)_{ca}$ are adjacent if they are consecutively adjacent in the p-part $H(x)$. $H(x)_{ca}$ is called *p-part consecutive adjacency graph* of $H(x)$.

Example 4.5. Consider the semigraph $G^* = (V, X)$ and its soft semigraph $G = \{H(v_2), H(v_6)\}$ given in Figure 4 and in Figure 5 respectively. Then, the consecutive adjacency graph G_{ca} of G is given by $G_{ca} = \{H(v_2)_{ca}, H(v_6)_{ca}\}$ and is shown in Figure 7.

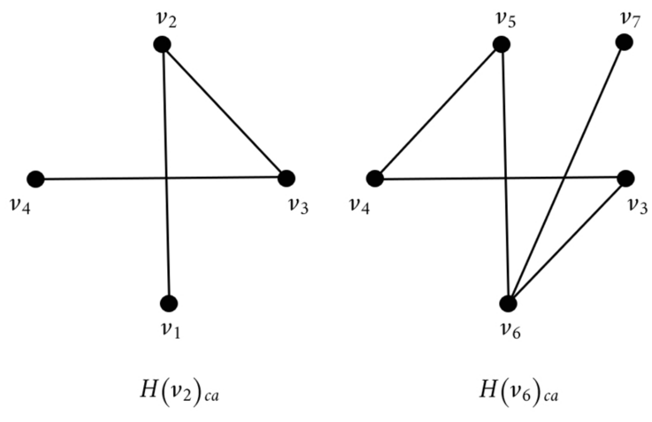


Figure 7. Consecutive Adjacency Graph $G_{ca} = \{H(v_2)_{ca}, H(v_6)_{ca}\}$

Theorem 4.6. *Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* given by $\{H(x) : x \in A\}$. Then,*
(1) $H(x)_{ca}$ is a spanning subgraph of $H(x)_a$, for all x in A,
(2) $H(x)_{ca}$ is a subgraph of G^*_a, for all x in A and
(3) $H(x)_{ca}$ is a subgraph of G^*_{ca}, for all x in A.

Proof. *(1)* Consider the p-part $H(x)$ of G for any $x \in A$. The vertex set of both $H(x)_{ca}$ and $H(x)_a$ is $Q(x)$. Also if two vertices u and v are consecutively adjacent then they are definitely adjacent. Therefore, any edge present in $H(x)_{ca}$ will also be in $H(x)_a$. That is, vertex sets of both $H(x)_{ca}$ and $H(x)_a$ are the same and the edge set of $H(x)_{ca}$ is a subset of the edge set of $H(x)_a$. So $H(x)_{ca}$ is a spanning subgraph of $H(x)_a$, for all x in A.
(2) By applying Theorem 4.3 and part *(1)* of this theorem, we can say $H(x)_{ca}$ is a subgraph of G^*_a, for all x in A.
(3) The vertex set of the p-part consecutive adjacent graph $H(x)_{ca}$ is $Q(x) \subseteq V$, where V is the vertex set of G^*_{ca}. Also if two vertices u and v are consecutively adjacent in $H(x)$, then they are definitely consecutively adjacent in G^*. Therefore, any edge present in $H(x)_{ca}$ is also an edge in G^*_{ca}. That is, the vertex set and edge set of $H(x)_{ca}$ are subsets of the vertex set and edge set of G^*_{ca} respectively. Therefore, $H(x)_{ca}$ is a subgraph of G^*_{ca}, for all x in A. ∎

4.3. One End Vertex Graph of a Soft Semigraph

Definition 4.7. Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* given by $\{H(x) : x \in A\}$. Then, the *one end vertex graph* G_{1e} of the soft semigraph G is given by $G_{1e} = \{H(x)_{1e} : x \in A\}$ where $H(x)_{1e}$ is a graph having vertex set $Q(x)$ and two vertices u and v in $H(x)_{1e}$ are adjacent if one of them is an end vertex or a partial end vertex of an f-edge containing these vertices in the p-part $H(x)$. $H(x)_{1e}$ is called *p-part one end vertex graph* of $H(x)$.

Example 4.8. Consider the semigraph $G^* = (V, X)$ and its soft semigraph $G = \{H(v_2), H(v_6)\}$ given in Figure 4 and in Figure 5 respectively. Then, the one end vertex graph G_{1e} of G is given by $G_{1e} = \{H(v_2)_{1e}, H(v_6)_{1e}\}$ and is shown in Figure 8.

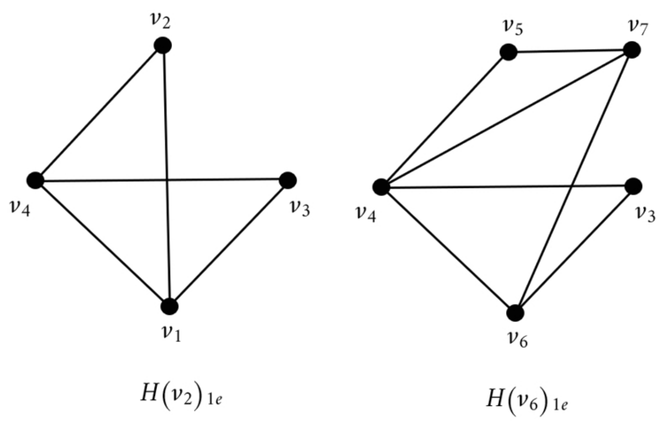


Figure 8. One End Vertex Graph $G_{1e} = \{H(v_2)_{1e}, H(v_6)_{1e}\}$

Theorem 4.9. *Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* given by $\{H(x) : x \in A\}$. Then,*

(1) $H(x)_{1e}$ is a spanning subgraph of $H(x)_a$, for all $x \in A$ and
(2) $H(x)_{1e}$ is a subgraph of G_a^*, for all $x \in A$.

Proof. *(1)* Consider the p-part $H(x)$ of G for any $x \in A$. The vertex set of both $H(x)_{1e}$ and $H(x)_a$ is $Q(x)$. In $H(x)_{1e}$, two vertices u and v are adjacent if one of them is an end vertex or a partial end vertex of an f-edge containing these two vertices in $H(x)$. Then definitely u and v are adjacent in $H(x)$. Therefore, any edge present in $H(x)_{1e}$ is also an edge in $H(x)_a$. That is, vertex sets of both $H(x)_{1e}$ and $H(x)_a$ are the same and the edge set of $H(x)_{1e}$ is a subset of the edge set of $H(x)_a$. So $H(x)_{1e}$ is a spanning subgraph of $H(x)_a$, for all x in A.
(2) By applying Theorem 4.3 and part *(1)* of this theorem we can say $H(x)_{1e}$ is a subgraph of G_a^*, for all x in A. ∎

4.4. End Vertex Graph of a Soft Semigraph

Definition 4.10. Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* given by $\{H(x) : x \in A\}$. Then, the *end vertex graph* G_e of the soft semigraph G is given by $G_e = \{H(x)_e : x \in A\}$ where $H(x)_e$ is a graph having vertex set $Q(x)$ and two vertices u and v in $H(x)_e$ are adjacent if they are the end vertices or a partial end vertices of an f-edge containing these vertices in the p-part $H(x)$. $H(x)_e$ is called *p-part end vertex graph* of $H(x)$.

Example 4.11. Consider the semigraph $G^* = (V, X)$ and its soft semigraph $G = \{H(v_2), H(v_6)\}$ given in Figure 4 and in Figure 5 respectively. Then, the end vertex graph G_e of G is given by $G_e = \{H(v_2)_e, H(v_6)_e\}$ and is shown in Figure 9.

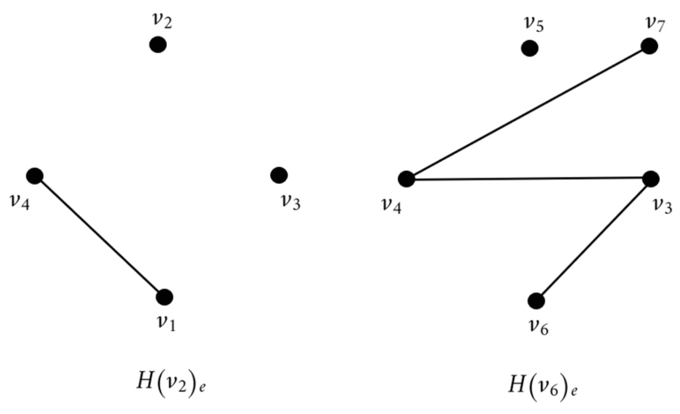


Figure 9. End Vertex Graph $G_e = \{H(v_2)_e, H(v_6)_e\}$

Theorem 4.12. *Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* given by $\{H(x) : x \in A\}$. Then,*
(1) $H(x)_e$ is a spanning subgraph of $H(x)_a$, for all x in A,
(2) $H(x)_e$ is a subgraph of G_a^*, for all x in A and
(3) $H(x)_e$ is a spanning subgraph of $H(x)_{1e}$, for all x in A.

Proof. (1) Consider the p-part $H(x)$ of G for any $x \in A$. The vertex set of both $H(x)_e$ and $H(x)_a$ is $Q(x)$.In $H(x)_e$, two vertices u and v are adjacent if they are end vertices or partial end vertices of an f-edge containing these two vertices in $H(x)$. Then definitely u and v are adjacent in $H(x)$. Therefore, any edge present in $H(x)_e$ is also an edge in $H(x)_a$. That is, vertex sets of both $H(x)_e$ and $H(x)_a$ are the same and the edge set of $H(x)_e$ is a subset of the edge set of $H(x)_a$. So $H(x)_e$ is a spanning subgraph of $H(x)_a$, for all x in A.
(2) By applying Theorem 4.3 and part *(1)* of this theorem we can say $H(x)_e$ is a subgraph of G^*_a, for all x in A.
(3) Consider the p-part $H(x)$ of G for any $x \in A$. The vertex set of both $H(x)_e$ and $H(x)_{1e}$ is $Q(x)$.In $H(x)_e$, two vertices u and v are adjacent if they are end vertices or partial end vertices of an f-edge containing these two vertices in $H(x)$. In $H(x)_{1e}$, two vertices u and v are adjacent if one of them is an end vertex or a partial end vertex of an f-edge containing these two vertices in $H(x)$. Definitely if u and v are adjacent in $H(x)_e$, then they are adjacent in $H(x)_{1e}$. Therefore, any edge present in $H(x)_e$ is also an edge in $H(x)_{1e}$. That is, vertex sets of both $H(x)_e$ and $H(x)_{1e}$ are the same and the edge set of $H(x)_e$ is a subset of the edge set of $H(x)_{1e}$. So $H(x)_e$ is a spanning subgraph of $H(x)_{1e}$, for all x in A. ∎

5. Degrees Associated with a Soft Semigraph

For a vertex v in a soft semigraph $G = (G^*, Q, W, A)$, we define various types of degrees as follows.

5.1. Degree of a Vertex in a Soft Semigraph

Definition 5.1. Let $H(x)$ be any p-part of the soft semigraph G and let v be any vertex in $H(x)$. Then, the *p-part degree* of v in $H(x)$ denoted by $deg\ v[H(x)]$ is defined as the number of f-edges having v as an end vertex in $H(x)$. We define $deg\ v[H(x)]$, for all $v \in Q(x)$.

Definition 5.2. *Degree* of a vertex v in soft semigraph G, denoted by $deg\ v$ is defined as $deg\ v = max\{deg\ v[H(x)] : x \in A\}$, where $deg\ v[H(x)]$ denotes the p-part degree of v in $H(x)$. We define $deg\ v$, for all $v \in \bigcup_{x\in A} Q(x)$.

Example 5.3. Consider the semigraph $G^* = (V, X)$ given in Figure 2 and its soft semigraph $G = \{H(C), H(M)\}$ given in Figure 3. Here $\bigcup_{x\in A} Q(x) = \{A, B, C, D, E, F, L, M, N\}$. We have $deg\ A[H(C)] = 1,\ deg\ B[H(C)] = 1,\ deg\ C[H(C)] = 1,\ deg\ D[H(C)] = 0,\ deg\ E[H(C)] = 1,\ deg\ L[H(C)] = 0$. Also $deg\ D[H(M)] = 1, deg\ F[H(M)] = 1, deg\ M[H(M)] = 1, deg\ N[H(M)] = 1$. Then $deg\ A = max\{deg\ A[H(C)]\} = max\{1\} = 1$. Similarly $deg\ B = 1, deg\ C = 1, deg\ D = 1, deg\ E = 1, deg\ F = 1, deg\ L = 0, deg\ M = 1$ and $deg\ N = 1$.

5.2. End Degree of a Vertex in a Soft Semigraph

Definition 5.4. Let $H(x)$ be any p-part of the soft semigraph G and let v be any vertex in $H(x)$. Then, the *p-part end degree* of v in $H(x)$ denoted by $deg_{ep}v[H(x)]$ is defined as the number of f-edges having v as an end vertex or partial end vertex in $H(x)$. We define $deg_{ep}v[H(x)]$, for all $v \in Q(x)$.

Definition 5.5. *End degree* of a vertex v in soft semigraph G, denoted by $deg_{ep}v$ is defined as $deg_{ep}v = max\{deg_{ep}v[H(x)] : x \in A\}$, where $deg_{ep}v[H(x)]$ denotes the p-part end degree of v in $H(x)$. We define $deg_{ep}v$, for all $v \in \bigcup_{x\in A} Q(x)$.

Example 5.6. In Example 5.3, all f-edges of G have end verices on both ends and have no partial end vertices. So the end degree of a vertex of G coincides with the degree of that vertex. That is, $deg_{ep}B = 1, deg_{ep}C = 1, deg_{ep}D = 1, deg_{ep}E = 1, deg_{ep}F = 1, deg_{ep}L = 0, deg_{ep}M = 1$ and $deg_{ep}N = 1$.

5.3. Edge Degree of a Vertex in a Soft Semigraph

Definition 5.7. Let $H(x)$ be any p-part of the soft semigraph G and let v be any vertex in $H(x)$. Then, the *p-part edge degree* of v in $H(x)$ denoted by $deg_e v[H(x)]$ is defined as the number of f-edges containing v in $H(x)$. We define $deg_e v[H(x)]$, for all $v \in Q(x)$.

Definition 5.8. *Edge degree* of a vertex v in a soft semigraph G, denoted by $deg_e v$ is defined as $deg_e v = max\{deg_e v[H(x)] : x \in A\}$, where $deg_e v[H(x)]$ denotes the p-part edge degree of v in $H(x)$. We define $deg_e v$, for all $v \in \bigcup_{x\in A} Q(x)$.

Example 5.9. Consider the semigraph $G^* = (V, X)$ given in Figure 4 and its soft semigraph $G = \{H(v_2), H(v_6)\}$ given in Figure 5. Here $\bigcup_{x\in A} Q(x) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$. We have $deg_e v_1[H(v_2)] = 1, deg_e v_2[H(v_2)] = 1, deg_e v_3[H(v_2)] = 1, deg_e v_4[H(v_2)] = 1$. Also we have $deg_e v_3[H(v_6)] = 2, deg_e v_4[H(v_6)] = 2, deg_e v_5[H(v_6)] = 1, deg_e v_6[H(v_6)] = 2, deg_e v_7[H(v_6)] = 1$. Then $deg_e v_1 = max\{deg_e v_1[H(v_2)]\} = max\{1\} = 1$. Similarly $deg_e v_2 = max\{1\} = 1, deg_e v_3 = max\{1, 2\} = 2, deg_e v_4 = max\{1, 2\} = 2, deg_e v_5 = max\{1\} = 1, deg_e v_6 = max\{2\} = 2, deg_e v_7 = max\{1\} = 1$.

5.4. Adjacent Degree of a Vertex in a Soft Semigraph

Definition 5.10. Let $H(x)$ be any p-part of the soft semigraph G and let v be any vertex in $H(x)$. Then, the *p-part adjacent degree* of v in $H(x)$ denoted by $deg_a v[H(x)]$ is defined as the number of vertices adjacent to v in $H(x)$. We define $deg_a v[H(x)]$, for all $v \in Q(x)$.

Definition 5.11. *Adjacent degree* of a vertex v in soft semigraph G, denoted by $deg_a v$ is defined as $deg_a v = max\{deg_a v[H(x)] : x \in A\}$, where $deg_a v[H(x)]$ denotes the p-part adjacent degree of v in $H(x)$. We define $deg_a v$, for all $v \in \bigcup_{x\in A} Q(x)$.

Example 5.12. Consider the semigraph $G^* = (V, X)$ given in Figure 4 and its soft semigraph $G = \{H(v_2), H(v_6)\}$ given in Figure 5. Here $\bigcup_{x\in A} Q(x) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$. We have $deg_a v_1[H(v_2)] = 3, deg_a v_2[H(v_2)] = 3, deg_a v_3[H(v_2)] = 3, deg_a v_4[H(v_2)] = 3$. Also we have $deg_a v_3[H(v_6)] = 2, deg_a v_4[H(v_6)] = 4, deg_a v_5[H(v_6)] = 3, deg_a v_6[H(v_6)] = 4, deg_a v_7[H(v_6)] = 3$. Then $deg_a v_1 = max\{deg_a v_1[H(v_2)]\} = max\{3\} = 3$. Similarly $deg_a v_2 = max\{3\} = 3, deg_a v_3 = max\{3, 2\} = 3, deg_a v_4 = max\{3, 4\} = 4, deg_a v_5 = max\{3\} = 3, deg_a v_6 = max\{4\} = 4, deg_a v_7 = max\{3\} = 3$.

5.5. Consecutive Adjacent Degree of a Vertex in a Soft Semigraph

Definition 5.13. Let $H(x)$ be any p-part of the soft semigraph G and let v be any vertex in $H(x)$. Then, the *p-part consecutive adjacent degree* of v in $H(x)$ denoted by $deg_{ca}v[H(x)]$ is defined as the number of vertices consecutively adjacent to v in $H(x)$. We define $deg_{ca}v[H(x)]$, for all $v \in Q(x)$.

Definition 5.14. *Consecutive adjacent degree* of a vertex v in soft semigraph G, denoted by $deg_{ca}v$ is defined as $deg_{ca}v = max\{deg_{ca}v[H(x)] : x \in A\}$, where $deg_{ca}v[H(x)]$ denotes the p-part consecutive adjacent degree of v in $H(x)$. We define $deg_{ca}v$, for all $v \in \bigcup_{x\in A} Q(x)$.

Example 5.15. Consider the semigraph $G^* = (V, X)$ given in Figure 4 and its soft semigraph $G = \{H(v_2), H(v_6)\}$ given in Figure 5. Here $\bigcup_{x\in A} Q(x) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$. We have $deg_{ca}v_1[H(v_2)] = 1, deg_{ca}v_2[H(v_2)] = 2, deg_{ca}v_3[H(v_2)] = 2, deg_{ca}v_4[H(v_2)] = 1$. Also we have $deg_{ca}v_3[H(v_6)] = 2, deg_{ca}v_4[H(v_6)] = 2, deg_{ca}v_5[H(v_6)] = 2, deg_{ca}v_6[H(v_6)] = 3, deg_{ca}v_7[H(v_6)] = 1$. Then $deg_{ca}v_1 = max\{deg_{ca}v_1[H(v_2)]\} = max\{1\} = 1$. Similarly $deg_{ca}v_2 = max\{2\} = 2, deg_{ca}v_3 = max\{2, 2\} = 2, deg_{ca}v_4 = max\{1, 2\} = 2, deg_{ca}v_5 = max\{2\} = 2, deg_{ca}v_6 = max\{3\} = 3, deg_{ca}v_7 = max\{1\} = 1$.

Theorem 5.16. *For any vertex v in the soft semigraph G, $deg\ v \leq deg_{ep}v \leq deg_e v \leq deg_{ca}v \leq deg_a v$.*

Proof. Consider a soft semigraph $G = (G^*, Q, W, A)$ which is represented by $\{H(x) : x \in A\}$. Consider any p-part $H(x)$ of G. Let v be any vertex in that p-part. If $deg\ v[H(x)] \neq 0$, v is the end vertex of some f-edge present in $H(x)$. This vertex v may be a partial end vertex of some other f-edges also. Therefore, $deg\ v[H(x)] \leq deg_{ep}v[H(x)]$. Also v may be a middle vertex of some other f-edges in $H(x)$. So $deg\ v[H(x)] \leq deg_{ep}v[H(x)] \leq deg_e v[H(x)]$. If v is an end vertex, partial end vertex or middle vertex of an f-edge in the p-part $H(x)$, it is consecutively adjacent to one or more vertices. Hence $deg\ v[H(x)] \leq deg_{ep}v[H(x)] \leq deg_e v[H(x)] \leq deg_{ca}v[H(x)]$. If v is consecutively adjacent to, say, n vertices in $H(x)$, it is definitely adjacent to these n vertices and there may be more vertices in $H(x)$ which are adjacent to v, but are not consecutively adjacent to v. So $deg\ v[H(x)] \leq deg_{ep}v[H(x)] \leq deg_e v[H(x)] \leq deg_{ca}v[H(x)] \leq deg_a v[H(x)]$. Clearly $max\{deg\ v[H(x)] : x \in A\} \leq max\{deg_{ep}v[H(x)] : x \in A\} \leq max\{deg_e v[H(x)] : x \in A\} \leq max\{deg_{ca}v[H(x)] : x \in A\} \leq max\{deg_a v[H(x)] : x \in A\}$. That is, $deg\ v \leq deg_{ep}v \leq deg_e v \leq deg_{ca}v \leq deg_a v$, for any vertex v in the soft semigraph G. ∎

Definition 5.17. Consider the collection of all vertices in the soft semigraph G which is a subset of V given by $\bigcup_{x\in A} Q(x)$. A vertex v is said to be a *pure end vertex* of G if it is an end vertex of an f-edge in $H(x)$ for some $x \in A$ and is not a partial end vertex or middle vertex of an f-edge in any p-part $H(x)$ of G, where $x \in A$.

Definition 5.18. Consider the collection of all vertices in the soft semigraph G which is a subset of V given by $\bigcup_{x\in A} Q(x)$. A vertex v is said to be a *pure middle vertex* of G if it is a middle vertex of an f-edge in $H(x)$ for some $x \in A$ and is not an end vertex or a partial end vertex of an f-edge in any p-part $H(x)$ of G, where $x \in A$.

Theorem 5.19. *Let V_e be the collection of pure end vertices of the soft semigraph G. Then $deg\ v = deg_e v = deg_{ca}v$, for all $v \in V_e$.*

Proof. Consider a soft semigraph G represented by $\{H(x) : x \in A\}$. Consider any p-part $H(x)$ of G. Since $v \in V_e$, the number of edges whose end vertex is v is the same as the number of edges containing the vertex v in $H(x)$. Also if v is an end vertex of p edges, say, $E_1, E_2, \ldots, E_p$, then there is exactly one vertex v_i in each E_i which is consecutively adjacent to the vertex v. Hence $deg\ v[H(x)] = deg_e v[H(x)] = deg_{ca}v[H(x)]$. So $max\{deg\ v[H(x)] : x \in A\} = max\{deg_e v[H(x)] : x \in A\} = max\{deg_{ca}v[H(x)] : x \in A\}$, for all $v \in V_e$. That is, $deg\ v = deg_e v = deg_{ca}v$, for all $v \in V_e$. ∎

Theorem 5.20. *Let V_m be the collection of pure middle vertices of the soft semigraph G. Then, for all v in V_m,*
(1) $deg\ v = 0$,
(2) $deg_{ep}v = 0$,
(3) $deg_e v \geq 1$.

Proof. *(1)* Since $v \in V_m$, no f-edge E exists in G such that v is an end vertex of E. Hence $deg\ v[H(x)] = 0$, for all x in A. Therefore, $max\{deg\ v[H(x)] : x \in A\} = 0$. That is, $deg\ v = 0$, for all $v \in V_m$.
(2) Since $v \in V_m$, no f-edge E exists in G such that v is an end vertex or partial end vertex of E. Hence $deg_{ep}v[H(x)] = 0$, for all x in A. Therefore, $max\{deg_{ep}v[H(x)] : x \in A\} = 0$. That is, $deg_{ep}v = 0$, for all $v \in V_m$.
(3) Since $v \in V_m$, v belongs to at least one f-edge in the p-part $H(x)$, for some $x \in A$. Therefore, $deg_e v[H(x)] \geq 1$, for some $x \in A$. So $max\{deg_e v[H(x)] : x \in A\} \geq 1$. That is, $deg_e v \geq 1$, for all $v \in V_m$. ∎

6. Matrices Associated with a Soft Semigraph

We define various types of matrices associated with a soft semigraph as follows.

6.1. Consecutive Adjacency Matrix of a Soft Semigraph

Definition 6.1. Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* which is represented by $\{H(x) : x \in A\}$. Let $H(x) = (Q(x), W(x))$ be any p-part of G. If $Q(x)$ contains m vertices $v_1, v_2, \ldots, v_m$, then then the *p-part consecutive adjacency matrix* $M_{ca}[H(x)]$ is an $m \times m$ matrix $[c_{ij}]$, where

$$c_{ij} = \begin{cases} 1, \text{ if } v_i \text{ and } v_j \text{ are consecutively adjacent in } H(x) \\ 0, \text{ if not.} \end{cases}$$

Then the *consecutive adjacency matrix* $M_{ca}(G)$ of the soft semigraph G is given by $M_{ca}(G) = \{M_{ca}[H(x)] : x \in A\}$.

Example 6.2. Consider the semigraph $G^* = (V, X)$ given in Figure 4 and its soft semigraph $G = \{H(v_2), H(v_6)\}$ given in Figure 5. For this soft semigraph G, the consecutive adjacency matrix is given by $M_{ca}(G) = \{M_{ca}[H(v_2)], M_{ca}[H(v_6)]\}$, where the p-part consecutive adjacency matrices $M_{ca}[H(v_2)]$ and $M_{ca}[H(v_6)]$ are as given below:

$$M_{ca}[H(v_2)] = \begin{array}{c c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} & \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} & \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \end{array}, \quad M_{ca}[H(v_6)] = \begin{array}{c c} \begin{array}{ccccc} v_3 & v_4 & v_5 & v_6 & v_7 \end{array} & \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} & \begin{array}{c} v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \end{array}$$

Remark 6.3. The p-part consecutive adjacency matrix $M_{ca}[H(x)]$ has the following properties:
(1) $M_{ca}[H(x)]$ is a symmetric matrix of order m, for all x in A, where m is the total number of vertices in $Q(x)$.
(2) $M_{ca}[H(x)]$ contains only 0 and 1 as its entries, for all x in A.

(3) The diagonal entries of $M_{ca}[H(x)]$ are zeros, for all x in A.
(4) In each $M_{ca}[H(x)]$, sum of a row or column gives the p-part consecutive adjacent degree of the corresponding vertex in that p-part $H(x)$.
(5) Maximum of row sums of the rows or column sums of the columns, corresponding to a vertex v among all p-part adjacency matrices $M_{ca}[H(x)]$, gives the consecutive adjacent degree of that vertex in the soft semigraph G.
(6) The p-part consecutive adjacency matrix $M_{ca}[H(x)]$ is same as the adjacency matrix of the p-part consecutive adjacency graph $H(x)_{ca}$, for all x in A.

6.2. Adjacency Matrix of a Soft Semigraph

Definition 6.4. Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* which is represented by $\{H(x) : x \in A\}$. Let $H(x) = (Q(x), W(x))$ be any p-part of G. If $Q(x)$ contains m vertices $v_1, v_2, \ldots, v_m$, then then the *p-part adjacency matrix* $M_{ad}[H(x)]$ is an $m \times m$ matrix $[a_{ij}]$, where

$$a_{ij} = \begin{cases} 1, \text{ if } v_i \text{ and } v_j \text{ are adjacent in } H(x) \\ 0, \text{ if not.} \end{cases}$$

Then, the *adjacency matrix* $M_{ad}(G)$ of the soft semigraph G is given by $M_{ad}(G) = \{M_{ad}[H(x)] : x \in A\}$.

Example 6.5. Consider the semigraph $G^* = (V, X)$ given in Figure 4 and its soft semigraph $G = \{H(v_2), H(v_6)\}$ given in Figure 5. For this soft semigraph G, the adjacency matrix is given by $M_{ad}(G) = \{M_{ad}[H(v_2)], M_{ad}[H(v_6)]\}$, where the p-part adjacency matrices $M_{ad}[H(v_2)]$ and $M_{ad}[H(v_6)]$ are as given below:

$$M_{ad}[H(v_2)] = \begin{array}{cc} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} & \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} & \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \end{array}, M_{ad}[H(v_6)] = \begin{array}{cc} \begin{array}{ccccc} v_3 & v_4 & v_5 & v_6 & v_7 \end{array} & \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} & \begin{array}{c} v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \end{array}$$

Remark 6.6. The p-part adjacency matrix $M_{ad}[H(x)]$ has the following properties:
(1) $M_{ad}[H(x)]$ is a symmetric matrix of order m, for all x in A, where m is the total number of vertices in $Q(x)$.
(2) $M_{ad}[H(x)]$ contains only 0 and 1 as its entries, for all x in A.
(3) The diagonal entries of $M_{ad}[H(x)]$ are zeros, for all x in A.
(4) In each $M_{ad}[H(x)]$, sum of a row or column gives the p-part adjacent degree of the corresponding vertex in that p-part $H(x)$.
(5) Maximum of row sums of the rows or column sums of the columns, corresponding to a vertex v among all p-part adjacency matrices $M_{ad}[H(x)]$, gives the adjacent degree of that vertex in the soft semigraph G.
(6) The p-part adjacency matrix $M_{ad}[H(x)]$ is same as the adjacency matrix of the p-part adjacency graph $H(x)_a$, for all x in A.

Remark 6.7. The adjacency matrix $M_{ad}(G)$ does not represent the soft semigraph G uniquely. To see this, consider the following example. Here, we give two different soft semigraphs with identical adjacency matrices.

Example 6.8. Let $G_1^* = (V_1, X_1)$ be a semigraph given in Figure 10 having vertex set $V_1 = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$ and the edge set $X_1 = \{(v_3, v_4, v_2, v_5), (v_0, v_2, v_1), (v_4, v_6), (v_3, v_8, v_9), (v_7, v_8)\}$.

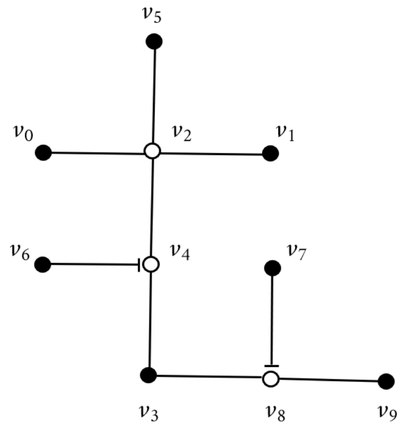


FIGURE 10. Semigraph $G_1^* = (V_1, X_1)$

Let the parameter set be $A_1 = \{v_2, v_8\} \subseteq V$. Define $Q_1 : A_1 \to \mathcal{P}(V_1)$ by $Q_1(x) = \{y \in V_1 | xRy \Leftrightarrow x = y$ or x and y are adjacent$\}$, for all x in A_1 and $W_1 : A_1 \to \mathcal{P}(X_{1p})$ by $W_1(x) = \{mp\ edges < Q_1(x) >\}$, for all x in A_1. That is, $Q_1(v_2) = \{v_0, v_1, v_2, v_3, v_4, v_5\}$ and $Q_1(v_8) = \{v_3, v_7, v_8, v_9\}$. Also $W_1(v_2) = \{(v_3, v_4, v_2, v_5), (v_0, v_2, v_1)\}$ and $W_1(v_8) = \{(v_7, v_8), (v_3, v_8, v_9)\}$. Then $H_1(v_2) = (Q_1(v_2), W_1(v_2))$ and $H_1(v_8) = (Q_1(v_8), W_1(v_8))$ are partial semigraphs of G_1^* as shown below in Figure 11. Hence $G_1 = \{H_1(v_2), H_1(v_8)\}$ is a soft semigraph of G_1^*.

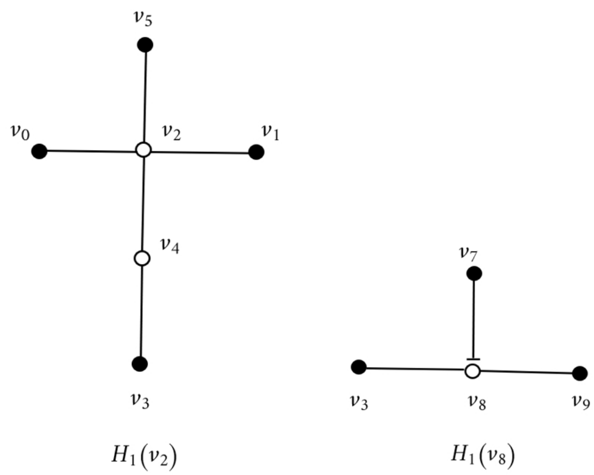


FIGURE 11. Soft Semigraph $G_1 = \{H_1(v_2), H_1(v_8)\}$

The adjacency matrix of G_1 is given by $M_{ad}(G_1) = \{M_{ad}[H_1(v_2)], M_{ad}[H_1(v_8)]\}$, where $M_{ad}[H_1(v_2)]$ and $M_{ad}[H_1(v_8)]$ are as given below:

$$M_{ad}[H_1(v_2)] = \begin{array}{c} \begin{array}{cccccc} v_0 & v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}, M_{ad}[H_1(v_8)] = \begin{array}{c} \begin{array}{cccc} v_3 & v_7 & v_8 & v_9 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} v_3 \\ v_7 \\ v_8 \\ v_9 \end{array}$$

Let $G_2^* = (V_2, X_2)$ be a semigraph given in Figure 12 having vertex set $V_2 = \{v_0, v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$ and edge set $X_2 = \{(v_2, v_3, v_4, v_5), (v_0, v_1, v_2), (v_0, v_6, v_7), (v_7, v_8), (v_3, v_9, v_8)\}$.

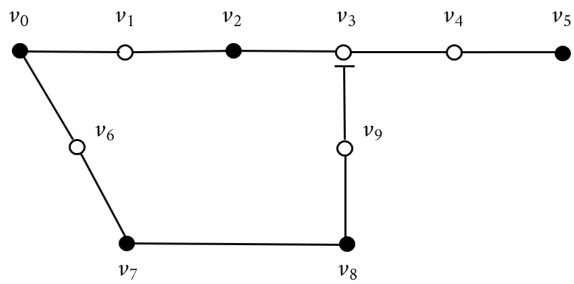


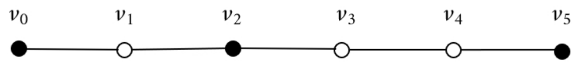
FIGURE 12. Semigraph $G_2^* = (V_2, X_2)$

Let the parameter set be $A_2 = \{v_2, v_8\} \subseteq V$. Define $Q_2 : A_2 \to \mathcal{P}(V_2)$ by $Q_2(x) = \{y \in V_2 | xRy \Leftrightarrow x = y$ or x and y are adjacent$\}$, for all x in A_2 and $W_2 : A_2 \to \mathcal{P}(X_{2p})$ by $W_2(x) = \{mp\ edges < Q_2(x) >\}$, for all x in A_2. That is, $Q_2(v_2) = \{v_0, v_1, v_2, v_3, v_4, v_5\}$ and $Q_2(v_8) = \{v_3, v_7, v_8, v_9\}$. Also $W_2(v_2) = \{(v_2, v_3, v_4, v_5), (v_0, v_1, v_2)\}$ and $W_2(v_8) = \{(v_7, v_8), (v_3, v_9, v_8)\}$. Then $H_2(v_2) = (Q_2(v_2), W_2(v_2))$ and $H_2(v_8) = (Q_2(v_8), W_2(v_8))$ are partial semigraphs of G_2^* as shown below in Figure 13. Hence $G_2 = \{H_2(v_2), H_2(v_8)\}$ is a soft semigraph of G_2^*.

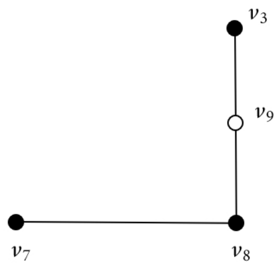
The adjacency matrix of G_2 is given by $M_{ad}(G_2) = \{M_{ad}[H_2(v_2)], M_{ad}[H_2(v_8)]\}$, where $M_{ad}[H_2(v_2)]$ and $M_{ad}[H_2(v_8)]$ are as given below:

$$M_{ad}[H_2(v_2)] = \begin{array}{c} \begin{array}{cccccc} v_0 & v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}, M_{ad}[H_2(v_8)] = \begin{array}{c} \begin{array}{cccc} v_3 & v_7 & v_8 & v_9 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} v_3 \\ v_7 \\ v_8 \\ v_9 \end{array}$$

That is, the adjacency matrices $M_{ad}(G_1)$ and $M_{ad}(G_2)$ are exactly same, even though the semigraphs G_1 and G_2 are entirely different.



$H_2(v_2)$



$H_2(v_8)$

FIGURE 13. Soft Semigraph $G_2 = \{H_2(v_2), H_2(v_8)\}$

6.3. Unique Adjacency Matrix of a Soft Semigraph

We can extend the unique representation of the adjacency matrix of a semigraph given in [10] to a soft semigraph as given below.

Definition 6.9. Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* given by $\{H(x) : x \in A\}$. Then, the *unique adjacency matrix* $M_{ud}(G)$ of the soft semigraph G is given by $M_{ud}(G) = \{M_{ud}[H(x)] : x \in A\}$, where $M_{ud}[H(x)]$ denotes the unique adjacency matrix of the p-part $H(x)$ which is defined as given below.
Let $H(x) = (Q(x), W(x))$ be a p-part of G, where $Q(x) = \{v_1, v_2, \ldots, v_m\}$ and $W(x) = \{E_1, E_2, \ldots, E_n\}$. Then, the unique adjacency matrix, $M_{ud}[H(x)]$ of $H(x)$ is an $m \times m$ matrix $[u_{ij}]$, which is defined as follows:

(1) For every f-edge $E_i = (v_{i_1}, v_{i_2}, \ldots, v_{i_q})$ of the p-part $H(x)$,
    (a) $u_{i_1 i_p} = p - 1, \forall v_{i_p} \in E_i; p = 1, 2, \ldots, q.$
    (b) $u_{i_q i_p} = q - p, \forall v_{i_p} \in E_i; p = 1, 2, \ldots, q.$
(2) All other entries of $M_{ud}[H(x)]$ are zeros.

We can see that if vertices v_i and v_j are not adjacent in $H(x)$, $u_{ij} = 0$ in $M_{ud}[H(x)]$. If the vertices v_i and v_j are adjacent in $H(x)$, then there is precisely one f-edge in $H(x)$ containing these vertices since, the p-part $H(x)$ is a partial semigraph of G^* and in a semigraph two edges have at most one vertex in common. Therefore, the entry u_{ij} is defined precisely by one f-edge.

Example 6.10. The unique adjacency matrix of the soft semigraph G_1 given in Figure 11 is given by $M_{ud}(G_1) = \{M_{ud}[H_1(v_2)], M_{ud}[H_1(v_8)]\}$, where $M_{ud}[H_1(v_2)]$ and $M_{ud}[H_1(v_8)]$ are as given below:

$$M_{ud}[H_1(v_2)] = \begin{array}{c} \begin{array}{cccccc} v_0 & v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 2 & 1 & 0 & 0 & 0 \\ 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 & 2 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}, \quad M_{ud}[H_1(v_8)] = \begin{array}{c} \begin{array}{cccc} v_3 & v_7 & v_8 & v_9 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ v_3 \\ v_7 \\ v_8 \\ v_9 \end{array}$$

The unique adjacency matrix of the soft semigraph G_2 given in Figure 13 is given by $M_{ud}(G_2) = \{M_{ud}[H_2(v_2)], M_{ud}[H_2(v_8)]\}$, where $M_{ud}[H_2(v_2)]$ and $M_{ud}[H_2(v_8)]$ are as given below:

$$M_{ud}[H_2(v_2)] = \begin{array}{c} \begin{array}{cccccc} v_0 & v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 2 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array}, \quad M_{ud}[H_2(v_8)] = \begin{array}{c} \begin{array}{cccc} v_3 & v_7 & v_8 & v_9 \end{array} \\ \begin{bmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ v_3 \\ v_7 \\ v_8 \\ v_9 \end{array}$$

So,we can observe that the unique adjacency matrices $M_{ud}(G_1)$ and $M_{ud}(G_2)$ are different even though the adjacency matrices $M_{ad}(G_1)$ and $M_{ad}(G_2)$ are exactly same.

Remark 6.11. The unique adjacency matrix $M_{ud}(G) = \{M_{ud}[H(x)] : x \in A\}$ of the soft semigraph G has the following properties:
(1) Entries in the unique adjacency matrix due to one edge are not altered by any other edge. Therefore, while writing entries in $M_{ud}[H(x)]$ for some x in A, the order of considering the f-edges in $H(x)$ is not important.
(2) If vertices v_i and v_j are not adjacent in $H(x)$ for some x in A, $u_{ij} = 0$ in $M_{ud}[H(x)]$.
(3) If the vertices v_i and v_j are adjacent in a p-part $H(x)$ for some x in A, then there is precisely one f-edge in $H(x)$ containing these vertices since, $H(x)$ is a partial semigraph of G^*. Therefore, the entry u_{ij} is defined precisely by one f-edge in $H(x)$.
(4) If $Q(x)$ contains m vertices for some x in A, then $M_{ud}[H(x)]$ will be an $m \times m$ matrix.
(5) $M_{ud}[H(x)]$ is not a symmetric matrix, in general. $M_{ud}[H(x)]$ will be symmetric for some x in A, if $W(x)$ contains only f-edges having two vertices.
(6) If $Q(x)$ contains m vertices for some x in A then, the entries in $M_{ud}[H(x)]$ are any numbers between 0 and $m-1$.
(7) A vertex v_i in $Q(x)$ is an isolated vertex in $H(x)$ for some x in A if and only if all the entries in the i^{th} row and i^{th} column of $M_{ud}[H(x)]$ are zeros.
(8) The i^{th} row and i^{th} column of $M_{ud}[H(x)]$ contain non zero entries for some x in A if and only if the corresponding vertex v_i is the end vertex or partial end vertex of some f-edge in $H(x)$.
(9) v_i is a pure middle vertex of the p-part $H(x)$ for some x in A, that is, v_i is not an end vertex or a partial end vertex of an f-edge in $H(x)$ if and only if i^{th} row of $M_{ud}[H(x)]$ contains only zero entries and i^{th} column contains non-zero entries.
(10) If i^{th} row of $M_{ud}[H(x)]$ has only zero entries and the i^{th} column contains $2r$ non-zero entries for some x in A, then v_i is a pure middle vertex and exactly r f-edges contain v_i

as a middle vertex in that p-part $H(x)$ of the soft semigraph G.
(11) Number of $1's$ (that is, number of entries equal to 1) in the i^{th} row of $M_{ud}[H(x)]$ gives the number of f-edges in $H(x)$ which contain v_i as an end vertex or a partial end vertex.
(12) Number of f-edges containing v_i as a middle vertex in $H(x)$ for some x in A, is the half of the difference between the number of non zero entries in the i^{th} column and the number of $1's$ in the i^{th} row in $M_{ud}[H(x)]$.
(13) Total number of f-edges in a p-part $H(x)$ for some x in A is the half of the total number of entries equal to 1 in $M_{ud}[H(x)]$.
(14) Total number of f-edges in the soft semigraph G is the total number of entries equal to 1 in $M_{ud}(G)$, that is, total number of $1's$ in $M_{ud}[H(x)]$ for all x in A.

6.4. Incidence Matrix of a Soft Semigraph

Definition 6.12. Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* which is represented by $\{H(x) : x \in A\}$. Let $H(x) = (Q(x), W(x))$ be any p-part of G. If $H(x)$ contains m vertices $v_1, v_2, \ldots, v_m$, and n f-edges $e_1, e_2, \ldots, e_n$ then then the *p-part incidence matrix* $M_{in}[H(x)]$ is an $m \times n$ matrix $[b_{ij}]$, where

$$b_{ij} = \begin{cases} 1, \text{ if the vertex } v_i \text{ belongs to the } f\text{-edge } \ e_j \text{ in } H(x) \\ 0, \text{ if not.} \end{cases}$$

Then, the *incidence matrix* $M_{in}(G)$ of the soft semigraph G is given by $M_{in}(G) = \{M_{in}[H(x)] : x \in A\}$.

Example 6.13. Consider the semigraph $G^* = (V, X)$ given in Figure 4 and its soft semigraph $G = \{H(v_2), H(v_6)\}$ given in Figure 5. The p-part $H(v_2)$ contains 4 vertices v_1, v_2, v_3, v_4 and an f-edge $e_1 = (v_1, v_2, v_3, v_4)$. The p-part $H(v_6)$ contains 5 vertices v_3, v_4, v_5, v_6, v_7 and 3 f-edges $e_2 = (v_3, v_4)$, $e_3 = (v_4, v_5, v_6, v_7)$ and $e_4 = (v_3, v_6)$. For this soft semigraph G, the incidence matrix is given by $M_{in}(G) = \{M_{in}[H(v_2)], M_{in}[H(v_6)]\}$, where the p-part incidence matrices $M_{in}[H(v_2)]$ and $M_{in}[H(v_6)]$ are as given below:

$$M_{in}[H(v_2)] = \begin{array}{c} e_1 \\ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \end{array} \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}, M_{in}[H(v_6)] = \begin{array}{c} \begin{array}{ccc} e_2 & e_3 & e_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array}$$

Remark 6.14. The p-part incidence matrix $M_{in}[H(x)]$ has the following properties:
(1) $M_{in}[H(x)]$ is an $m \times n$ matrix, if $H(x)$ contains m vertices and n f-edges.
(2) $M_{in}[H(x)]$ contains only 0 and 1 as its entries, for all x in A.
(3) In each $Min[H(x)]$, sum of entries in each column gives the number of vertices in the corresponding f-edge in $H(x)$.
(4) Sum of entries in each column of $M_{in}[H(x)]$ will be at least two.

Remark 6.15. The incidence matrix $M_{in}(G)$ does not represent the soft semigraph G uniquely. To see this, consider the following example. Here, we give two different soft semigraphs with identical incidence matrices.

Example 6.16. Let $G_1^* = (V_1, X_1)$ be a semigraph given in Figure 14 having vertex set $V_1 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and the edge set $X_1 = \{(v_2, v_1, v_3), (v_2, v_4, v_5), (v_3, v_6), (v_5, v_7, v_6)\}$.

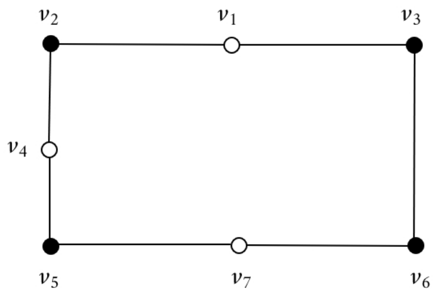


FIGURE 14. Semigraph $G_1^* = (V_1, X_1)$

Let the parameter set be $A_1 = \{v_1, v_6\} \subseteq V$. Define $Q_1 : A_1 \to \mathcal{P}(V_1)$ by $Q_1(x) = \{y \in V_1 | xRy \Leftrightarrow x = y$ or x and y are adjacent$\}$, for all x in A_1 and $W_1 : A_1 \to \mathcal{P}(X_{1p})$ by $W_1(x) = \{mp\ edges < Q_1(x) >\}$, for all x in A_1. That is, $Q_1(v_1) = \{v_1, v_2, v_3\}$ and $Q_1(v_6) = \{v_3, v_5, v_6, v_7\}$. Also $W_1(v_1) = \{(v_2, v_1, v_3)\}$ and $W_1(v_6) = \{(v_3, v_6), (v_5, v_7, v_6)\}$. Then $H_1(v_1) = (Q_1(v_1), W_1(v_1))$ and $H_1(v_6) = (Q_1(v_6), W_1(v_6))$ are partial semigraphs of G_1^* as shown below in Figure 15. Hence $G_1 = \{H_1(v_1), H_1(v_6)\}$ is a soft semigraph of G_1^*.

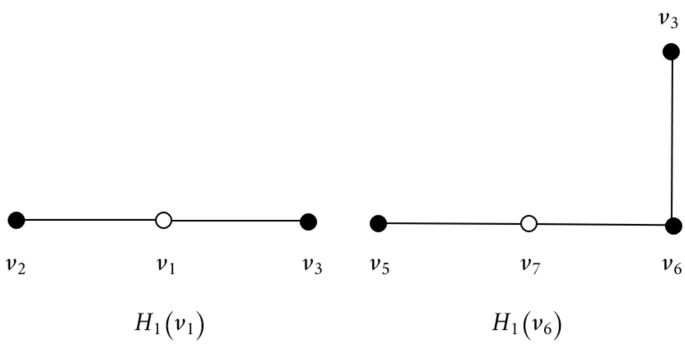


FIGURE 15. Soft Semigraph $G_1 = \{H_1(v_1), H_1(v_6)\}$

Suppose that the f-edge (v_2, v_1, v_3) in $H_1(v_1)$ is e_1 and the f-edges (v_3, v_6) and (v_5, v_7, v_6) in $H_1(v_6)$ are respectively e_2 and e_3. Then, the incidence matrix of G_1 is given by $M_{in}(G_1) = \{M_{in}[H_1(v_1)], M_{in}[H_1(v_6)]\}$, where $M_{in}[H_1(v_1)]$ and $M_{in}[H_1(v_6)]$ are as given below:

$$M_{in}[H_1(v_1)] = \begin{array}{c c} e_1 & \\ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} & \begin{array}{c} v_1 \\ v_2 \\ v_3 \end{array} \end{array}, \quad M_{in}[H_1(v_6)] = \begin{array}{c c} \begin{array}{cc} e_2 & e_3 \end{array} & \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} & \begin{array}{c} v_3 \\ v_5 \\ v_6 \\ v_7 \end{array} \end{array}$$

Let $G_2^* = (V_2, X_2)$ be a semigraph given in Figure 16 having vertex set $V_2 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ and the edge set $X_2 = \{(v_4, v_3, v_2, v_1), (v_3, v_6), (v_8, v_5, v_6, v_7)\}$.

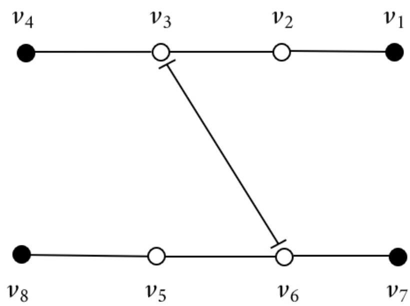


FIGURE 16. Semigraph $G_2^* = (V_2, X_2)$

Let the parameter set be $A_2 = \{v_2, v_6\} \subseteq V$. Define $Q_2 : A_2 \to \mathcal{P}(V_2)$ by $Q_2(x) = \{y \in V_2 | xRy \Leftrightarrow x = y$ or x and y are consecutively adjacent$\}$, for all x in A_2 and $W_2 : A_2 \to \mathcal{P}(X_{2p})$ by $W_2(x) = \{mp\ edges < Q_2(x) >\}$, for all x in A_2. That is, $Q_2(v_2) = \{v_1, v_2, v_3\}$ and $Q_2(v_6) = \{v_3, v_5, v_6, v_7\}$. Also $W_2(v_2) = \{(v_3, v_2, v_1)\}$ and $W_2(v_6) = \{(v_3, v_6), (v_5, v_6, v_7)\}$. Then $H_2(v_2) = (Q_2(v_2), W_2(v_2))$ and $H_2(v_6) = (Q_2(v_6), W_2(v_6))$ are partial semigraphs of G_2^* as shown below in Figure 17. Hence $G_2 = \{H_2(v_2), H_2(v_6)\}$ is a soft semigraph of G_2^*.

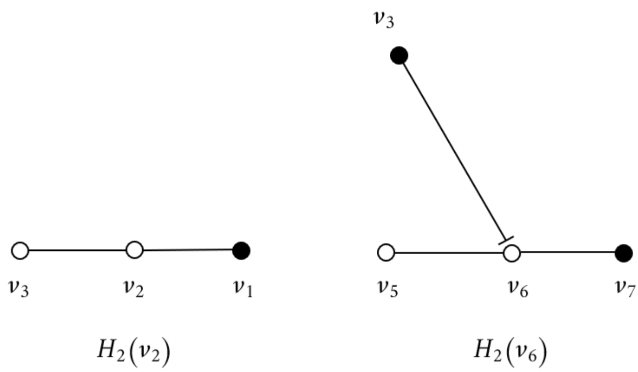


FIGURE 17. Soft Semigraph $G_2 = \{H_2(v_2), H_2(v_6)\}$

Suppose that the f-edge (v_3, v_2, v_1) in $H_2(v_2)$ is e_1 and the f-edges (v_3, v_6) and (v_5, v_6, v_7) in $H_2(v_6)$ are respectively e_2 and e_3. Then, the incidence matrix of G_2 is given by $M_{in}(G_2) = \{M_{in}[H_2(v_2)], M_{in}[H_2(v_6)]\}$, where $M_{in}[H_2(v_2)]$ and $M_{in}[H_2(v_6)]$ are as given below:

$$M_{in}[H_2(v_2)] = \begin{array}{cc} e_1 & \\ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} & \begin{array}{c} v_1 \\ v_2 \\ v_3 \end{array} \end{array}, \quad M_{in}[H_2(v_6)] = \begin{array}{cc} \begin{array}{cc} e_2 & e_3 \end{array} & \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} & \begin{array}{c} v_3 \\ v_5 \\ v_6 \\ v_7 \end{array} \end{array}$$

That is, the incidence matrices $M_{in}(G_1)$ and $M_{in}(G_2)$ are exactly same, even though the semigraphs G_1 and G_2 are entirely different.

6.5. Unique Incidence Matrix of a Soft Semigraph

We can extend the unique representation of the incidence matrix of a semigraph given in [10] to a soft semigraph as given below.

Definition 6.17. Let $G^* = (V, X)$ be a semigraph and $G = (G^*, Q, W, A)$ be a soft semigraph of G^* given by $\{H(x) : x \in A\}$. Then, the *unique incidence matrix* $M_{un}(G)$ of the soft semigraph G is given by $M_{un}(G) = \{M_{un}[H(x)] : x \in A\}$, where $M_{un}[H(x)]$ denotes the unique incidence matrix of the p-part $H(x)$ which is defined as given below. Let $H(x) = (Q(x), W(x))$ be a p-part of G, where $Q(x) = \{v_1, v_2, \ldots, v_m\}$ and $W(x) = \{E_1, E_2, \ldots, E_n\}$ where $E_j = (v_{i_1}, v_{i_2}, \ldots, v_{i_{q_j}})$. For each f-edge $E_j, j = 1, 2, \ldots, n$, define

$$h_{ij} = \begin{cases} 0, \text{ if } v_i \notin \{v_{i1}, v_{i2}, \ldots v_{iq_j}\} \\ 1, \text{ if } v_i = v_{i1}, v_{i2}; \text{ and } q_j = 2 \\ s, \text{ if } v_i = v_{i_s}, \text{ for } s = 1, 2, 3, \ldots, q_j \text{ and } q_j \geq 3. \end{cases}$$

The $m \times n$ matrix given by $M_{un}[H(x)] = [h_{ij}]$ is the unique incidence matrix associated with the p-part $H(x)$ of the soft semigraph G.

Example 6.18. The unique incidence matrix of the soft semigraph G_1 given in Figure 15 is given by $M_{un}(G_1) = \{M_{un}[H_1(v_1)], M_{un}[H_1(v_6)]\}$, where $M_{un}[H_1(v_1)]$ and $M_{un}[H_1(v_6)]$ are as given below:

$$M_{un}[H_1(v_1)] = \begin{array}{cc} e_1 & \\ \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} & \begin{array}{c} v_1 \\ v_2 \\ v_3 \end{array} \end{array}, \quad M_{un}[H_1(v_6)] = \begin{array}{cc} \begin{array}{cc} e_2 & e_3 \end{array} & \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 3 \\ 0 & 2 \end{bmatrix} & \begin{array}{c} v_3 \\ v_5 \\ v_6 \\ v_7 \end{array} \end{array}$$

The unique incidence matrix of the soft semigraph G_2 given in Figure 17 is given by $M_{un}(G_2) = \{M_{un}[H_2(v_2)], M_{un}[H_2(v_6)]\}$, where $M_{un}[H_2(v_2)]$ and $M_{un}[H_2(v_6)]$ are as given below:

$$M_{un}[H_2(v_2)] = \begin{array}{cc} e_1 & \\ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} & \begin{array}{c} v_1 \\ v_2 \\ v_3 \end{array} \end{array}, \quad M_{un}[H_2(v_6)] = \begin{array}{cc} \begin{array}{cc} e_2 & e_3 \end{array} & \\ \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \\ 0 & 3 \end{bmatrix} & \begin{array}{c} v_3 \\ v_5 \\ v_6 \\ v_7 \end{array} \end{array}$$

So,we can observe that the unique incidence matrices $M_{un}(G_1)$ and $M_{un}(G_2)$ are different even though the incidence matrices $M_{in}(G_1)$ and $M_{in}(G_2)$ are exactly same.

Remark 6.19. The unique incidence matrix $M_{un}(G) = \{M_{un}[H(x)] : x \in A\}$ of the soft semigraph G has the following properties:
(1) If the p-part $H(x) = (Q(x), W(x))$ of G for some x in A contains m vertices and n f-edges then, $M_{un}[H(x)]$ will be an $m \times n$ matrix.
(2) If $Q(x)$ contains m vertices for some x in A, then, the entries in $M_{un}[H(x)]$ are any number between 0 and m.

(3) If any column of $M_{un}[H(x)]$ contains two $1's$ (that is, two entries equal to 1) for some x in A, then, all other entries in that column are zeros.
(4) If any column of $M_{un}[H(x)]$ contains only one entry equal to 1 for some x in A, then, the number of non zero entries of that column is greater than 2 and the non zero entries are $1, 2, \dots, q_j$ where q_j is the largest entry in that column.
(5) The number of non zero entries in the j^{th} column of $M_{un}[H(x)]$ for some x in A gives the number of vertices present in the f-edge E_j in $W(x)$.
(6) If all entries in the i^{th} row of $M_{un}[H(x)]$ are zeros for some x in A, then, the corresponding vertex v_i in $H(x)$ is an isolated vertex.
(7) Interchanging of any two columns or rows in $M_{un}[H(x)]$ for some x in A corresponds to the re-labeling of f-edges or vertices of $H(x)$.
(8) Number of non-zero entries in the i^{th} row of $M_{un}[H(x)]$ for some x in A gives the edge degree of v_i in $H(x)$, that is, $deg_e v_i[H(x)]$.

7. Conclusion

Soft semigraph was introduced by applying the concept of soft set in semigraph. By means of parameterization, a soft semigraph produces a series of descriptions of a relationship described using a semigraph. Definitely, the theory of soft semigraphs will become an important part of semigraph theory due to its capability to deal with the parameterization tool.

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