Thai Journal of Mathematics
Volume 20 Number 4 (2022)
Pages 1734–1744

http://thaijmath.in.cmu.ac.th

ISSN 1686-0209

Fixed Point Theorems for $F-$Contractive Type Fuzzy Mapping in $\mathbb{G}-$Metric Spaces

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Abstract In this article, inspired by the concepts of $\mathbb{G}-$metric spaces, we introduce the notion of $F-$contractive type fuzzy mappings in $\mathbb{G}-$metric spaces. Using this new idea, some fixed point theorems are proved. Examples are also provided to support the hypotheses of our obtained results. The established ideas herein improve some related work in the existing literature.

MSC: 46S40; 47H10; 54H25
Keywords: fixed point; fuzzy set; fuzzy mapping; $F-$contraction; $\mathbb{G}-$metric space

Submission date: 30.12.2019 / Acceptance date: 15.02.2022

1. Introduction

Throughout this article, denoted by $\mathbb{R}$, $\mathbb{R}^+$ and $\mathbb{N}$ are the set of all real numbers, positive real numbers and natural numbers, respectively. Also, $(\mathcal{F}, d)$, ($\mathcal{F}$ for short), represents a metric space with the metric d.

Fixed point theory is a renowned and huge field of research in mathematical sciences. This field is known as the combination of analysis which includes topology, geometry and algebra. The first most well known result in fixed point theory with metric space structure is the Banach fixed point theorem [1](which is also called the contraction mapping principle). In the literature, there are several extensions of the Banach contraction principle [1], which states that every self mapping $\mathcal{S}$ defined on a complete metric space $(\mathcal{F}, d)$ satisfying for all $\tau, \sigma \in \mathcal{F}$, $d(\mathcal{S}\tau, \mathcal{S}\sigma) \leq \kappa d(\tau, \sigma)$, where $\kappa \in (0, 1)$, has a unique fixed point. Some improvements of the Banach fixed point theorem concern the contractive inequality while others deal with generalizing the space. A particular extension of metric space is the so-called $\mathbb{G}-$metric space initiated by Mustafa and Sims [2] in 2006. In the first paper on $\mathbb{G}-$metric spaces, Sims and Mustafa [2] introduced some properties

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Published by The Mathematical Association of Thailand.

of $\mathbb{G}-$metric spaces and also discussed its topology, compactness, completeness, product and the criteria regarding the convergence and continuity of sequences in $\mathbb{G}-$metric space. Some theorems concerning these properties were also proved. Another famous generalization of the contraction mapping principle due to Banach was presented by Wardowski [3], the concept of which is called $F-$contraction. The idea of F-contractions has been extended both for single-valued (see, e.g. [4]) and set-valued mappings (see, e.g. [5]) For some comprehensive surveys in this direction, we refer the interested reader to the work of Taskovic [6] or Rhoades [7].

As a natural extension of crisp sets, fuzzy sets was introduced initially by Zadeh [8]. After the introduction of this concept, several researches were conducted on various applications and improvements of fuzzy sets in different directions. Along this trend, Heilpern [9] introduced the concept of fuzzy mapping and proved a fixed point theorem for fuzzy contraction mappings which is a generalization of the fixed point theorem for multi-valued mappings of Nadler [10]. Thereafter, other authors [11–13] have studied the existence of fixed point of fuzzy mappings.

The aim of this paper is to establish fixed point theorems, common fixed point theorems for $F-$contraction type fuzzy mappings in $\mathbb{G}-$metric spaces. Our results generalize and extend a few known results in the comparable literature.

2. Preliminaries

In this section, we recall some basic concepts that are necessary in the establishment of our main results. Most of these preliminaries are recorded from [2, 14–16].

Definition 2.1. Let $\mathcal{F} \neq \emptyset$ and $\mathbb{G} : \mathcal{F} \times \mathcal{F} \times \mathcal{F} \longrightarrow \mathbb{R}^+$ be a function such that the following conditions are satisfied:

($\mathbb{G}1$) $\mathbb{G}(\tau, \sigma, \upsilon) = 0$ if $\tau = \sigma = \upsilon$,
($\mathbb{G}2$) $\mathbb{G}(\tau, \tau, \sigma) > 0$ for all $\tau, \sigma \in \mathcal{F}$ with $\tau \neq \sigma$,
($\mathbb{G}3$) $\mathbb{G}(\tau, \tau, \sigma) \leq \mathbb{G}(\tau, \sigma, \upsilon)$ for all $\tau, \sigma, \upsilon \in \mathcal{F}$ with $\upsilon \neq \sigma$,
($\mathbb{G}4$) $\mathbb{G}(\tau, \sigma, \upsilon) = \mathbb{G}(\tau, \upsilon, \sigma) = \mathbb{G}(\sigma, \upsilon, \tau) = ...$(symmetric with respect to τ, σ, υ),
($\mathbb{G}5$) $\mathbb{G}(\tau, \sigma, \upsilon) \leq \mathbb{G}(\tau, a, a) + \mathbb{G}(a, \sigma, \upsilon)$ for all $\tau, \sigma, \upsilon, a \in \mathcal{F}$ (rectangular property).

Then $\mathbb{G}$ is called a $\mathbb{G}-$M function and $(\mathcal{F}, \mathbb{G})$ is said to be a $\mathbb{G}-$metric space.

Example 2.2. Consider $\mathcal{F} = \mathbb{R}$, then a $\mathbb{G}-$metric space on $\mathbb{R}$ is defined as:

$$\mathbb{G}(\tau, \sigma, \upsilon) = |\tau - \sigma| + |\sigma - \upsilon| + |\tau - \upsilon| \text{ for all } \tau, \sigma, \upsilon \in \mathcal{F}.$$

Definition 2.3. Let $(\mathcal{F}, \mathbb{G})$ be a $\mathbb{G}-$metric space. A sequence $\{\tau_e\}$ in $\mathcal{F}$ is $\mathbb{G}-$Convergent sequence if, for any $\delta > 0$, there exists $\tau \in \mathcal{F}$, $O(\delta) \in \mathbb{N}$ such that $\mathbb{G}(\tau, \tau_e, \tau_\rho) < \delta$, for all $e, \rho \geq O(\delta)$. We call τ the limit of the sequence and write $\tau_e \to \tau$ or $\lim_{e\to\infty} \tau_e = \tau$.

Definition 2.4. Let $(\mathcal{F}, \mathbb{G})$ be a $\mathbb{G}-$metric space. A sequence $\{\tau_e\}$ in $\mathcal{F}$ is called $\mathbb{G}-$Cauchy sequence if, for any $\delta > 0$, there exists $O(\delta) \in \mathbb{N}$ such that $\mathbb{G}(\tau_\varsigma, \tau_e, \tau_\rho) < \delta$, for each $e, \rho, \varsigma \geq O(\delta)$, that is, $\mathbb{G}(\tau_\varsigma, \tau_e, \tau_\rho) \to 0$ as $e, \rho, \varsigma \to \infty$.

Definition 2.5. Let $(\mathcal{F}, \mathbb{G})$ be a $\mathbb{G}-$metric space. A sequence $\{\tau_e\}$ in $\mathcal{F}$ is called $\mathbb{G}-$Complete if every $\mathbb{G}-$Cauchy sequence in $(\mathcal{F}, \mathbb{G})$ is convergent in $\mathcal{F}$.

Lemma 2.6. [2]. *Let $(\mathcal{F}, \mathbb{G})$ be a $\mathbb{G}-$metric space and $\{\tau_e\}$ be a sequence in $\mathcal{F}$. Then the following statements are equivalent:*

(i) $\{\tau_e\}$ *is* $\mathbb{G}-$*convergent to* τ.
(ii) $\mathbb{G}(\tau_e, \tau_e, \tau) \to 0$ *as* e *approaches infinity.*
(iii) $\mathbb{G}(\tau_e, \tau, \tau) \to 0$ *as* e *approaches infinity.*
(iv) $\mathbb{G}(\tau_e, \tau_\rho, \tau) \to 0$ *as* e, ρ *approaches infinity.*

Definition 2.7. Kaewcharoen and Kaewkhao [15] introduced the concept of Hausdorff $\mathbb{G}-$distance as follows: Let $\mathcal{F}$ be a $\mathbb{G}-$metric space and $\mathcal{CB}(\mathcal{F})$ be the family of all non empty closed and bounded subsets of $\mathcal{F}$. Then, the Hausdorff $\mathbb{G}-$ distance function is defined as follows:

$$\mathcal{H}_{\mathbb{G}}(\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3) = \max\left\{ \sup_{\tau\in\mathcal{Z}_1} \mathbb{G}(\tau, \mathcal{Z}_2, \mathcal{Z}_3), \sup_{\tau\in\mathcal{Z}_2} \mathbb{G}(\tau, \mathcal{Z}_1, \mathcal{Z}_3), \sup_{\tau\in\mathcal{Z}_3} \mathbb{G}(\tau, \mathcal{Z}_1, \mathcal{Z}_2)\right\},$$

where

$$\begin{aligned}\mathbb{G}(\tau, \mathcal{Z}_2, \mathcal{Z}_3) =&\mu_{\mathbb{G}}(\tau, \mathcal{Z}_2) + \mu_{\mathbb{G}}(\mathcal{Z}_2, \mathcal{Z}_3) + \mu_{\mathbb{G}}(\tau, \mathcal{Z}_3),\\ \mu_{\mathbb{G}}(\tau, \mathcal{Z}_2) =& \inf_{\sigma\in\mathcal{Z}_2} \mu_{\mathbb{G}}(\tau, \sigma),\\ \mu_{\mathbb{G}}(\mathcal{Z}_1, \mathcal{Z}_2) =& \inf_{\tau\in\mathcal{Z}_1,\sigma\in\mathcal{Z}_2} \mu_{\mathbb{G}}(\tau, \sigma).\end{aligned}$$

Remark 2.8. [15]. Let $(\mathcal{F}, \mathbb{G})$ be a $\mathbb{G}-$metric space, $\tau \in \mathcal{F}$ and $\mathcal{Z} \subseteq \mathcal{F}$. For each $\sigma \in \mathcal{Z}$ we have:

$$\begin{aligned}\mathbb{G}(\tau, \mathcal{Z}, \mathcal{Z}) &= \mu_{\mathbb{G}}(\tau, \mathcal{Z}) + \mu_{\mathbb{G}}(\mathcal{Z}, \mathcal{Z}) + \mu_{\mathbb{G}}(\tau, \mathcal{Z})\\ &\leq 2\mu_{\mathbb{G}}(\tau, \sigma)\\ &= 2[\mu_{\mathbb{G}}(\tau, \tau, \sigma) + \mu_{\mathbb{G}}(\tau, \sigma, \sigma)]\\ &\leq 2[\mu_{\mathbb{G}}(\tau, \sigma, \sigma) + \mu_{\mathbb{G}}(\tau, \sigma, \sigma) + \mu_{\mathbb{G}}(\tau, \sigma, \sigma)]\\ &= 6\mu_{\mathbb{G}}(\tau, \sigma, \sigma).\end{aligned}$$

Let $(\mathcal{F}, d_{\mathbb{G}})$ be a metric space. A fuzzy set in $\mathcal{F}$ is a function with domain $\mathcal{F}$ and values in $\mathcal{I} = [0, 1]$. If $\mathcal{Z}$ is a fuzzy set and $\tau \in \mathcal{F}$ then the function value $\eta_{\mathcal{Z}}(\tau)$ is called the degree of membership of τ in $\mathcal{Z}$.
The $\alpha-$level set of $\mathcal{Z}$, denoted by $[\mathcal{Z}]_\alpha$ is defined as

$$\begin{aligned}&[\mathcal{Z}]_\alpha = \{\tau : \ \eta_{\mathcal{Z}}(\tau) \geq \alpha,\ \alpha \in (0, 1]\}\\ &[\mathcal{Z}]_0 = \overline{\{\tau : \eta_{\mathcal{Z}}(\tau) > 0\}},\end{aligned}$$

where $\overline{A}$ is the closure of the non-fuzzy set A.
Let $\mathcal{C}(\mathcal{F})$ be the family of all nonempty compact subsets of $\mathcal{F}$. Denote by $\mathfrak{Z}(\mathcal{F})$ the totality of fuzzy sets which satisfy that for each $\alpha \in \mathcal{I}$, $[\mathcal{Z}]_\alpha \in \mathcal{C}(\mathcal{F})$. Let $\mathcal{Z}_1, \mathcal{Z}_2 \in \mathfrak{Z}(\mathcal{F})$, then $\mathcal{Z}_1$ is said to be more accurate than $\mathcal{Z}_2$, denoted by $\mathcal{Z}_1 \subset \mathcal{Z}_2$ iff $\eta_{\mathcal{Z}_1}(\tau) \leq \eta_{\mathcal{Z}_2}(\tau)$ for each $\tau \in \mathcal{F}$. $\mathcal{Z}_1 = \mathcal{Z}_2$ if and only if $\mathcal{Z}_1 \subset \mathcal{Z}_2$ and $\mathcal{Z}_2 \subset \mathcal{Z}_1$.
Let $\mathcal{Z}_1, \mathcal{Z}_2 \in \mathfrak{Z}(\mathcal{F})$, then define

$$\begin{aligned}\mathcal{D}_\infty(\mathcal{Z}_1, \mathcal{Z}_2) &= \sup_{0\leq\alpha\leq1} \mathcal{H}([\mathcal{Z}_1]_\alpha, [\mathcal{Z}_2]_\alpha)\\ &= \sup_{0\leq\alpha\leq1} \max\left\{ \sup_{\tau\in[\mathcal{Z}_1]_\alpha} \mu_{\mathbb{G}}(\tau, [\mathcal{Z}_2]_\alpha), \sup_{\sigma\in[\mathcal{Z}_2]_\alpha} \mu_{\mathbb{G}}(\sigma, [\mathcal{Z}_1]_\alpha)\right\}.\end{aligned} \tag{2.1}$$

Definition 2.9. [14]. Let $(\mathcal{F}, d)$ be a metric space and $\mathcal{Z}_1, \mathcal{Z}_2 \in \mathfrak{Z}(\mathcal{F})$ such that $[\mathcal{Z}_1]_\alpha$ and $[\mathcal{Z}_2]_\alpha$ are compact subsets of $\mathcal{F}$, the following identities are defined as,

$$\begin{aligned} P_\alpha(\mathcal{Z}_1, \mathcal{Z}_2) &= \inf_{\tau \in [\mathcal{Z}_1]_\alpha, \sigma \in [\mathcal{Z}_2]_\alpha} \mu(\tau, \sigma), \\ P(\mathcal{Z}_1, \mathcal{Z}_2) &= \sup_\alpha P_\alpha(\mathcal{Z}_1, \mathcal{Z}_2), \\ \mathcal{D}_\alpha(\mathcal{Z}_1, \mathcal{Z}_2) &= \mu_{\mathcal{H}}([\mathcal{Z}_1]_\alpha, [\mathcal{Z}_2]_\alpha). \end{aligned}$$

Definition 2.10. [14]. Let $(\mathcal{F}, d)$ be a metric space. The distance function $\mathcal{D}_\infty : \mathfrak{Z}(\mathcal{F}) \times \mathfrak{Z}(\mathcal{F}) \to \mathbb{R}$ is defined as:

$$\mathcal{D}_\infty(\mathcal{Z}_1, \mathcal{Z}_2) = \sup_\alpha \mathcal{D}_\alpha(\mathcal{Z}_1, \mathcal{Z}_2).$$

Definition 2.11. [10]. Let $(\mathcal{F}, d)$ be a metric space, $\mathcal{T} : \mathcal{F} \longrightarrow I^{\mathcal{F}}$ and $\mathcal{Q} : \mathcal{F} \longrightarrow I^{\mathcal{F}}$ be two fuzzy mappings. A point $\nu \in \mathcal{F}$ is called

(i) fuzzy fixed point of $\mathcal{T}$ if $\nu \in [\mathcal{T}_\nu]_\alpha$ for some $\alpha \in [0, 1]$.
(ii) common fuzzy fixed point if $\nu \in [\mathcal{T}_\nu]_\alpha \cap [\mathcal{Q}_\nu]_\alpha$.

Definition 2.12. [3]. Let $F : \mathbb{R}^+ \to \mathbb{R}$ be a mapping satisfying:

(F1) F is strictly increasing, i.e. for all $\beta, \gamma \in \mathbb{R}_+$ such that $\beta < \gamma, F(\beta) < F(\gamma)$.
(F2) For each sequence $\{\beta_n\}_{n \in \mathbb{N}}$ of positive numbers $\lim_{n\to\infty} \beta_n = 0$ if and only if $\lim_{n\to\infty} F(\beta_n) = -\infty$.
(F3) There exists $k \in (0, 1)$ such that $\lim_{\beta \to 0^+} \beta^k F(\beta) = 0$.

Subsequently, Altun et al. [4] modified the above definition by adding comprehensive condition (F4) which is stated as:

(F4) $F(\inf A) = \inf F(A)$ for all $A \subset (0, \infty)$ with $\inf A > 0$.

We denote the set of all functions satisfying properties $(F1) - (F4)$ by $\mathcal{X}$.

Definition 2.13. [17] Let $(\mathcal{F}, \mathbb{G})$ be a $\mathbb{G}-$metric space. A mapping $\mathcal{T} : \mathcal{F} \to \mathcal{F}$ is said to be an $F-$contraction if there exists $\omega > 0$ such that for all $\tau, \sigma, \upsilon \in \mathcal{F}$,

$$\mathbb{G}(\mathcal{T}\tau, \mathcal{T}\sigma, \mathcal{T}\upsilon) > 0 \;\Rightarrow\; \omega + F(\mathbb{G}(\mathcal{T}\tau, \mathcal{T}\sigma, \mathcal{T}\upsilon)) \leq F(\mathbb{G}(\tau, \sigma, \upsilon)). \tag{2.2}$$

Example 2.14. [17] Let $F : \mathbb{R}_+ \to \mathbb{R}$ be given by $F(\beta) = \ln(\beta)$. It is clear that F satisfies $F(1) - F(3)$, (F(3) for any $k \in (0, 1)$). Each mapping $\mathcal{T} : \mathcal{F} \to \mathcal{F}$ satisfying (2.2) is an $F-$contraction such that

$$\mathbb{G}(\mathcal{T}\tau, \mathcal{T}\sigma, \mathcal{T}\upsilon) \leq e^{-\omega}(\mathbb{G}(\tau, \sigma, \upsilon)), \text{ for all } \tau, \sigma, \upsilon \in \mathcal{F}, \mathcal{T}\tau \neq \mathcal{T}\sigma \neq \mathcal{T}\upsilon. \tag{2.3}$$

It is clear that for $\tau, \sigma, \upsilon \in \mathcal{F}$ such that $\mathcal{T}\tau = \mathcal{T}\sigma = \mathcal{T}\upsilon$ the inequality (2.3) also hold.

Example 2.15. [17] Let $F : \mathbb{R}_+ \to \mathbb{R}$ be given by $F(\beta) = \ln(\beta) + \beta,\ \beta > 0$. Then F satisfies $F(1) - F(3)$, (F(3) for any $k \in (0, 1)$). Each mapping $\mathcal{T} : \mathcal{F} \to \mathcal{F}$ satisfying (2.2) is an $F-$contraction such that

$$\frac{\mathbb{G}(\mathcal{T}\tau, \mathcal{T}\sigma, \mathcal{T}\upsilon)}{\mathbb{G}(\tau, \sigma, \upsilon)} e^{[\mathbb{G}(\mathcal{T}\tau, \mathcal{T}\sigma, \mathcal{T}\upsilon) - \mathbb{G}(\tau, \sigma, \upsilon)]} \leq e^{-\omega}, \tag{2.4}$$

for all $\tau, \sigma, \upsilon \in \mathcal{F}, \mathcal{T}\tau \neq \mathcal{T}\sigma \neq \mathcal{T}\upsilon$. It is clear that for $\tau, \sigma, \upsilon \in \mathcal{F}$ such that $\mathcal{T}\tau = \mathcal{T}\sigma = \mathcal{T}\upsilon$ the inequality (2.3) is also true.

Remark 2.16. From $(F1)$ and (2.2) it is easy to conclude that every $F-$contraction $\mathcal{T}$ is contractive mapping, i.e.

$$\mathbb{G}(\mathcal{T}\tau, \mathcal{T}\sigma, \mathcal{T}\upsilon) \leq \mathbb{G}(\tau, \sigma, \upsilon), \tag{2.5}$$

for all $\tau, \sigma, \upsilon \in \mathcal{F}, \mathcal{T}\tau \neq \mathcal{T}\sigma \neq \mathcal{T}\upsilon$. Then every $F-$contraction is a continuous mapping.

Lemma 2.17. [16] *Let $(\mathcal{F}, \mathbb{G})$ be a $\mathbb{G}-$metric space and $\mathcal{Z}_1, \mathcal{Z}_2 \in \mathcal{CB}(\mathcal{F})$, then for each $\tau \in \mathcal{Z}_1$, we have*

$$\mathbb{G}(\tau, \mathcal{Z}_2, \mathcal{Z}_2) \leq \mathcal{H}_{\mathbb{G}}(\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_2).$$

Lemma 2.18. [16] *Let $(\mathcal{F}, \mathbb{G})$ be a $\mathbb{G}-$metric space. If $\mathcal{Z}_1, \mathcal{Z}_2 \in \mathcal{CB}(\mathcal{F})$ and $\tau \in \mathcal{Z}_1$, then for each $\epsilon > 0$ there exists $\sigma \in \mathcal{Z}_2$ s.t.*

$$\mathbb{G}(\tau, \sigma, \sigma) \leq \mathcal{H}_{\mathbb{G}}(\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_2) + \epsilon.$$

Lemma 2.19. [5] *Let $\mathcal{V}$ be a metric linear space, $\mathcal{T} : \mathcal{F} \to \mathcal{W}(\mathcal{V})$ and $\tau_0 \in \mathcal{V}$. Then there exists $\tau_1 \in \mathcal{V}$ such that $\{\tau_1\} \subset \mathcal{T}(\tau_0)$.*

3. Main Result

We begin this section with some auxiliary concepts as follows.

Definition 3.1. Let $(\mathcal{F}, \mathbb{G})$ be a $\mathbb{G}-$metric space, $F \in \mathcal{X}$ and $\mathcal{T} : \mathcal{F} \to I^{\mathcal{F}}$ be a fuzzy mapping. Then $\mathcal{T}$ is said to be an $F-$contractive type fuzzy mapping if there exists $\omega > 0$ such that

$$\omega + \mathcal{H}_{\mathbb{G}}([\mathcal{T}\tau]_\lambda, [\mathcal{T}\sigma]_\lambda, [\mathcal{T}\upsilon]_\lambda) \leq F(\mathbb{G}(\tau, \sigma, \upsilon)), \tag{3.1}$$

for all $\tau, \sigma, \upsilon \in \mathcal{F}$ with $\mathcal{H}_{\mathbb{G}}([\mathcal{T}\tau]_\lambda, [\mathcal{T}\sigma]_\lambda, [\mathcal{T}\upsilon]_\lambda) > 0$ and $\lambda \in [0, 1]$.

Example 3.2. Let $\mathcal{F} = [0, 1]$ and define $\mathbb{G} : \mathcal{F} \times \mathcal{F} \times \mathcal{F} \to \mathbb{R}^+$ as follows

$$\mathbb{G}(\tau, \sigma, \upsilon) = |\tau - \sigma| + |\sigma - \upsilon| + |\upsilon - \tau|.$$

Define $\mathcal{T} : \mathcal{F} \to I^{\mathcal{F}}$, for $\lambda \in [0, 1]$ as follows:
For $\tau \in \mathcal{F}$, we have

$$\mathcal{T}(\tau)(t) = \begin{cases} \lambda \text{ if } t \in [0, \frac{\tau}{2}], \\ \frac{\lambda}{2} \text{ if } t \in (\frac{\tau}{2}, 1] \end{cases}$$

such that

$$[\mathcal{T}\tau]_\lambda = \left[0, \frac{\tau}{2}\right].$$

Let $F(\beta) = -\frac{1}{\sqrt{\beta}}$, $\beta > 0$. Then F satisfies $F(1) - F(3)$, ($F(3)$ for any$k \in (\frac{1}{2}, 1)$). In this case each $F-$contraction $\mathcal{T}$ satisfies,

$$\mathbb{G}([\mathcal{T}\tau]_\lambda, [\mathcal{T}\sigma]_\lambda, [\mathcal{T}\upsilon]_\lambda) \leq \frac{1}{(1 + \omega\sqrt{\mathbb{G}(\tau, \sigma, \upsilon)})^2}\mathbb{G}(\tau, \sigma, \upsilon),$$

for all $\tau, \sigma, \upsilon \in \mathcal{F}, \mathcal{T}\tau \neq \mathcal{T}\sigma \neq \mathcal{T}\upsilon$.

Our main theorem runs as follows.

Theorem 3.3. *Let $(\mathcal{F}, \mathbb{G})$ be a $\mathbb{G}-$complete metric space and let $\mathcal{S}, \mathcal{T} : \mathcal{F} \to I^{\mathcal{F}}$ be fuzzy mapping such that for each $\tau, \sigma \in \mathcal{F}$, there exist $\alpha \in (0, 1]$ with $[\mathcal{S}\tau]_\lambda, [\mathcal{T}\tau]_\lambda \in \mathcal{C}(\mathcal{F})$. Assume there exist some $F \in \mathcal{X}$ and $\tau > 0$ such that*

$$\omega + F(\mathcal{H}_{\mathbb{G}}([\mathcal{S}\tau]_\lambda, [\mathcal{T}\sigma]_\lambda, [\mathcal{T}\upsilon]_\lambda)) \leq F(\mathbb{G}(\tau, \sigma, \upsilon)) \tag{3.2}$$

for all $\tau, \sigma, \upsilon \in \mathcal{F}$ with $\mathcal{H}_{\mathbb{G}}([\mathcal{S}\tau]_\lambda, [\mathcal{T}\sigma]_\lambda, [\mathcal{T}\upsilon]_\lambda)) > 0$. Then $\mathcal{S}$ and $\mathcal{T}$ have a common fixed point.

Proof. Let $\tau_0 \in \mathcal{F}$ be an arbitrary point of $\mathcal{F}$. Then by assumption there exists $\lambda \in (0,1]$ such that $[\mathcal{S}\tau_0]_\lambda \in \mathcal{C}(\mathcal{F})$. Let $\tau_1 \in [\mathcal{S}\tau_0]_\lambda$. For this τ_1, there exists $\lambda \in (0,1]$ such that $[\mathcal{T}\tau_1]_\lambda \in \mathcal{C}(\mathcal{F})$. By Lemma 2.17, $F(1)$ and (3.2), we have

$$\begin{aligned}\omega + F(\mathbb{G}(\tau_1, [\mathcal{T}\tau_1]_\lambda, [\mathcal{T}\tau_1]_\lambda)) &\leq \omega + F(\mathcal{H}_{\mathbb{G}}([\mathcal{S}\tau_0]_\lambda, [\mathcal{T}\tau_1]_\lambda, [\mathcal{T}\tau_1]_\lambda)) \\ &\leq F(\mathbb{G}(\tau_0, \tau_1, \tau_1)).\end{aligned}$$

From $(F4)$, we know that

$$F(\mathbb{G}(\tau_1, [\mathcal{T}\tau_1]_\lambda, [\mathcal{T}\tau_1]_\lambda)) = \inf_{\sigma \in [\mathcal{T}\tau_1]_\lambda} F(\mathbb{G}(\tau_1, \sigma, \sigma)).$$

Thus,

$$\omega + \inf_{\sigma \in [\mathcal{T}\tau_1]_\lambda} F(\mathbb{G}(\tau_1, \sigma, \sigma)) \leq F(\mathbb{G}(\tau_0, \tau_1, \tau_1)). \tag{3.3}$$

Then, from equation (3.3), there exists $\tau_2 \in [\mathcal{T}\tau_1]_\lambda$ such that

$$\omega + F(\mathbb{G}(\tau_1, \tau_2, \tau_2)) \leq F(\mathbb{G}(\tau_0, \tau_1, \tau_1)).$$

For this τ_2, there exists $\lambda \in (0,1]$ such that $[\mathcal{S}\tau_2]_\lambda \in \mathcal{C}(\mathcal{F})$. By Lemma 2.17, $F(1)$ and (3.2), we have

$$\begin{aligned}\omega + \mathbb{G}(\tau_2, [\mathcal{S}\tau_2]_\lambda, [\mathcal{S}\tau_2]_\lambda) &\leq \omega + \mathcal{H}_{\mathbb{G}}([\mathcal{T}\tau_1]_\lambda, [\mathcal{S}\tau_2]_\lambda, [\mathcal{S}\tau_2]_\lambda) \\ &\leq \omega + \mathcal{H}_{\mathbb{G}}([\mathcal{S}\tau_2]_\lambda, [\mathcal{S}\tau_2]_\lambda, [\mathcal{T}\tau_1]_\lambda) \\ &\leq F(\mathbb{G}(\tau_2, \tau_2, \tau_1)) \\ &\leq F(\mathbb{G}(\tau_1, \tau_2, \tau_2)).\end{aligned}$$

From $(F4)$ we know that

$$F(\mathbb{G}(\tau_2, [\mathcal{S}\tau_2]_\lambda, [\mathcal{S}\tau_2]_\lambda)) = \inf_{\sigma_1 \in [\mathcal{T}\tau_2]_\lambda} F(\mathbb{G}(\tau_2, \sigma_1, \sigma_1)).$$

Hence,

$$\omega + \inf_{\sigma_1 \in [\mathcal{S}\tau_2]_\lambda} F(\mathbb{G}(\tau_2, \sigma_1, \sigma_1)) \leq F(\mathbb{G}(\tau_1, \tau_2, \tau_2)).$$

Then, from equation (3.3), there exists $\tau_3 \in [\mathcal{S}\tau_2]_\lambda$ such that

$$\omega + F(\mathbb{G}(\tau_2, \tau_3, \tau_3)) \leq F(\mathbb{G}(\tau_1, \tau_2, \tau_2)).$$

Continuing in this way, we get a sequence $\{\tau_n\} \in \mathcal{F}$ such that $\tau_{2n+1} \in [\mathcal{S}\tau_{2n}]_\lambda$ and $\tau_{2n+2} \in [\mathcal{T}\tau_{2n+1}]_\lambda$ and

$$\omega + F(\mathbb{G}(\tau_{2n+1}, \tau_{2n+2}, \tau_{2n+2})) \leq F(\mathbb{G}(\tau_{2n}, \tau_{2n+1}, \tau_{2n+1})) \tag{3.4}$$

and

$$\omega + F(\mathbb{G}(\tau_{2n+2}, \tau_{2n+3}, \tau_{2n+3})) \leq F(\mathbb{G}(\tau_{2n+1}, \tau_{2n+2}, \tau_{2n+2})) \tag{3.5}$$

for all $n \in \mathbb{N}$. By (3.4) and (3.5), we have

$$\omega + F(\mathbb{G}(\tau_n, \tau_{n+1}, \tau_{n+1})) \leq F(\mathbb{G}(\tau_{n-1}, \tau_n, \tau_n)).$$

Therefore,

$$\begin{aligned}F(\mathbb{G}(\tau_n, \tau_{n+1}, \tau_{n+1})) &\leq F(\mathbb{G}(\tau_{n-1}, \tau_n, \tau_n)) - \omega \\ &\leq F(\mathbb{G}(\tau_{n-2}, \tau_{n-1}, \tau_{n-1})) - 2\omega \\ &\leq ... \leq F(\mathbb{G}(\tau_0, \tau_1, \tau_1)) - n\omega.\end{aligned} \tag{3.6}$$

Taking limit as $n \to \infty$ in (3.6), we get

$$\lim_{n\to\infty} F(\mathbb{G}(\tau_n, \tau_{n+1}\tau_{n+1})) = -\infty.$$

Then by $F(2)$, we have

$$\lim_{n\to\infty} \mathbb{G}(\tau_n, \tau_{n+1}, \tau_{n+1}) = 0.$$

Now, by $F(3)$, there exists $k \in (0,1)$ such that

$$\lim_{n\to\infty} [\mathbb{G}(\tau_n, \tau_{n+1}, \tau_{n+1})]^k F(\mathbb{G}(\tau_n, \tau_{n+1}, \tau_{n+1})) = 0.$$

From (3.6), we have

$$\begin{aligned}
[\mathbb{G}(\tau_n, \tau_{n+1}, \tau_{n+1})]^k F(\mathbb{G}(\tau_n, \tau_{n+1}, \tau_{n+1})) - \mathbb{G}(\tau_n, \tau_{n+1}, \tau_{n+1})]^k F(\mathbb{G}(\tau_0, \tau_1, \tau_1)) \\
\leq -n\omega[\mathbb{G}(\tau_n, \tau_{n+1}, \tau_{n+1})]^k \\
\leq 0.
\end{aligned} \tag{3.7}$$

Letting $n \to \infty$ in (3.7), we get

$$\lim_{n\to\infty} n[\mathbb{G}(\tau_n, \tau_{n+1}, \tau_{n+1})]^k = 0.$$

Thus,

$$\lim_{n\to\infty} n^{\frac{1}{k}}[\mathbb{G}(\tau_n, \tau_{n+1}, \tau_{n+1})] = 0$$

and there exists $n_1 \in \mathbb{N}$ such that

$$n^{\frac{1}{k}}\mathbb{G}(\tau_n, \tau_{n+1}, \tau_{n+1}) \leq 1 \text{ for all } n \geq n_1.$$

So we have

$$\mathbb{G}(\tau_n, \tau_{n+1}, \tau_{n+1}) \leq \frac{1}{n^{\frac{1}{k}}} \text{ for all } n \geq n_1.$$

Now, consider $m, n \in \mathbb{N}$ such that $m > n \geq n_1$, we have

$$\begin{aligned}
\mathbb{G}(\tau_n, \tau_m, \tau_m) &\leq \mathbb{G}(\tau_n, \tau_{n+1}, \tau_{n+1}) + \mathbb{G}(\tau_{n+1}, \tau_{n+2}, \tau_{n+2}) + ... + \mathbb{G}(\tau_{m-1}, \tau_m, \tau_m) \\
&\leq \sum_{i=n}^{\infty} \frac{1}{i^{\frac{1}{k}}}.
\end{aligned}$$

Since $\sum_{i=n}^{\infty} \frac{1}{i^{\frac{1}{k}}}$ is convergent, we have $\mathbb{G}(\tau_n, \tau_m, \tau_m) \to 0$ as $n, m \to \infty$. Hence $\{\tau_n\}$ is a Cauchy sequence in $\mathcal{F}$. Since $(\mathcal{F}, \mathbb{G})$ is complete, there exists $u \in \mathcal{F}$ such that, $\lim_{n\to\infty} \tau_n \to u$. Now, we show that $u \in [\mathcal{T}u]_\lambda$. Assume on contrary that $u \notin [\mathcal{T}u]_\lambda$, so there exist $n_0 \in \mathbb{N}$ and a subsequence $\{\tau_{n_k}\}$ of $\{\tau_n\}$ such that $\mathbb{G}(\tau_{2n_k+1}, [\mathcal{T}u]_\lambda, [\mathcal{T}u]_\lambda) > 0$ for all $n_k \geq n_0$. Since $\mathbb{G}(\tau_{2n_k+1}, [\mathcal{T}u]_\lambda, [\mathcal{T}u]_\lambda) > 0$ for all $n_k \geq n_0$, so by Lemma 2.17, $F(1)$ and (3.2), we get

$$\begin{aligned}
\omega + F(\mathbb{G}(\tau_{n_k+1}, [\mathcal{T}u]_\lambda, [\mathcal{T}u]_\lambda)) &\leq \omega + F[\mathcal{H}_{\mathbb{G}}([\mathcal{S}\tau_{2n_k}]_\lambda, [\mathcal{T}u]_\lambda, [\mathcal{T}u]_\lambda))] \\
&\leq F(\mathbb{G}(\tau_{2n_k}, u, u)).
\end{aligned}$$

This implies that

$$\begin{aligned}
F(\mathbb{G}(\tau_{2n_k+1}, [\mathcal{T}u]_\lambda, [\mathcal{T}u]_\lambda)) &\leq F(\mathbb{G}(\tau_{2n_k}, u, u)) - \omega \\
&< F(\mathbb{G}(\tau_{2n_k}, u, u)).
\end{aligned}$$

By $F(1)$, we get

$$\mathbb{G}(\tau_{2n_k+1}, [\mathcal{T}u]_\lambda, [\mathcal{T}u]_\lambda) < \mathbb{G}(\tau_{2n_k}, u, u) \tag{3.8}$$

Taking $n \to \infty$ in (3.8), we obtain

$$\mathbb{G}(u, [\mathcal{T}u]_\lambda, [\mathcal{T}u]_\lambda) \leq 0.$$

Therefore, $u \in [\mathcal{T}u]_\lambda$. Similarly, we can show that in $u \in [\mathcal{S}u]_\lambda$. Thus $u \in [\mathcal{S}u]_\lambda \cap [\mathcal{T}u]_\lambda$. Hence, u is a common fixed point of the mappings $\mathcal{S}$ and $\mathcal{T}$. ∎

Corollary 3.4. *Let $(\mathcal{F}, \mathbb{G})$ be a $\mathbb{G}-$complete metric space and let $\mathcal{S} : \mathcal{F} \to I^{\mathcal{F}}$ for each $\tau, \sigma \in \mathcal{F}$, there exist $\lambda \in (0, 1]$ such that $[\mathcal{S}\tau]_\lambda \in \mathcal{C}(\mathcal{F})$. Assume there exist some $F \in \mathcal{X}$ and $\tau > 0$ such that*

$$\omega + F(\mathcal{H}_\mathbb{G}([\mathcal{S}\tau]_\lambda, [\mathcal{S}\sigma]_\lambda, [\mathcal{S}\upsilon]_\lambda)) \leq F(\mathbb{G}(\tau, \sigma, \upsilon))$$

for all $\tau, \sigma, \upsilon \in \mathcal{F}$ with $\mathcal{H}_\mathbb{G}([\mathcal{S}\tau]_\lambda, [\mathcal{S}\sigma]_\lambda, [\mathcal{S}\upsilon]_\lambda)) > 0$. Then $\mathcal{S}$ has a fixed point.

Proof. Put $\mathcal{S} = \mathcal{T}$ in the proof of Theorem 3.3, we get the required result. ∎

Corollary 3.5. *Let $(\mathcal{F}, \mathbb{G})$ be a $\mathbb{G}-$complete metric space and let $\mathcal{P}_1, \mathcal{P}_2 : \mathcal{F} \to \mathcal{C}(\mathcal{F})$. Suppose that there exist some $F \in \mathcal{X}$ and $\tau > 0$ such that*

$$F(\mathcal{H}_\mathbb{G}(\mathcal{P}_1\tau, \mathcal{P}_2\sigma, \mathcal{P}_2\upsilon)) \leq F(\mathbb{G}(\tau, \sigma, \upsilon))$$

for all $\tau, \sigma, \upsilon \in \mathcal{F}$ with $\mathcal{H}_\mathbb{G}(\mathcal{P}_1\tau, \mathcal{P}_2\sigma, \mathcal{P}_2\upsilon) > 0$. Then $\mathcal{P}_1$ and $\mathcal{P}_2$ have a common fixed point.

Proof. Consider $\lambda : \mathcal{F} \to (0, 1]$ and $\mathcal{S}, \mathcal{T} : \mathcal{F} \to I^{\mathcal{F}}$ defined by

$$\mathcal{S}(\tau)(t) = \begin{cases} \lambda \text{ if } t \in \mathcal{P}_1\tau, \\ 0 \text{ if } t \notin \mathcal{P}_1\tau. \end{cases}$$

and

$$\mathcal{T}(\tau)(t) = \begin{cases} \lambda \text{ if } t \in \mathcal{P}_2\tau, \\ 0 \text{ if } t \notin \mathcal{P}_2\tau. \end{cases}$$

Then

$$[\mathcal{S}\tau]_\lambda = \{t : \mathcal{S}(\tau)(t) \geq \lambda\} = \mathcal{P}_1\tau$$

and

$$[\mathcal{T}\tau]_\lambda = \{t : \mathcal{T}(\tau)(t) \geq \lambda\} = \mathcal{P}_2\tau$$

Thus, by Theorem 3.3, we get $u \in \mathcal{F}$ such that

$$u \in [\mathcal{S}u]_\lambda \cap [\mathcal{T}u]_\lambda = \mathcal{P}_1 u \cap \mathcal{P}_2 u.$$

Hence, the required result. ∎

Corollary 3.6. *Let $(\mathcal{F}, \mathbb{G})$ be a $\mathbb{G}-$complete metric space and let $\mathcal{P} : \mathcal{F} \to \mathcal{C}(\mathcal{F})$. Suppose that there exist some $F \in \mathcal{X}$ and $\tau > 0$ such that*

$$F(\mathcal{H}_\mathbb{G}(\mathcal{P}\tau, \mathcal{P}\sigma, \mathcal{P}\upsilon)) \leq F(\mathbb{G}(\tau, \sigma, \upsilon))$$

for all $\tau, \sigma, \upsilon \in \mathcal{F}$ with $\mathcal{H}_\mathbb{G}(\mathcal{P}\tau, \mathcal{P}\sigma, \mathcal{P}\upsilon) > 0$. Then $\mathcal{P}$ has a common fixed point.

Proof. Put $\mathcal{P}_1 = \mathcal{P}_2$ in the proof of Corollary 3.5, we get the required result. ∎

Corollary 3.7. *Let $(\mathcal{F},\mathbb{G})$ be a $\mathbb{G}-$complete metric linear space and $\mathcal{S},\mathcal{T}:\mathcal{F}\to\mathcal{W}(\mathcal{F})$. Suppose that there exist some $F\in\mathcal{X}$ and $\tau>0$ such that*

$$\omega+F(\mathbb{G}_\infty([\mathcal{S}\tau]_\lambda,[\mathcal{T}\sigma]_\lambda,[\mathcal{T}\upsilon]_\lambda))\le F(\mathbb{G}(\tau,\sigma,\upsilon))$$

for all $\tau,\sigma,\upsilon\in\mathcal{F}$ with $\mathbb{G}_\infty([\mathcal{S}\tau]_\lambda,[\mathcal{T}\sigma]_\lambda,[\mathcal{T}\upsilon]_\lambda))>0$. There exists $u\in\mathcal{F}$ such that $\{u\}\subset[\mathcal{S}u]_\lambda$ and $\{u\}\subset[\mathcal{T}u]_\lambda$.

Proof. Let $\tau\in\mathcal{F}$, then by Lemma 2.17, there exists $\sigma\in\mathcal{F}$ such that $\sigma\in[\mathcal{S}\tau]_1$. Similarly we can find $\upsilon\in\mathcal{F}$ such that $\upsilon\in[\mathcal{S}\tau]_1$. It follows that for each $\tau\in\mathcal{F}$, $[\mathcal{S}\tau]_\lambda,[\mathcal{S}\tau]_\lambda\in\mathcal{C}(\mathcal{F})$. As $\lambda=1$, by definition of $\mathbb{G}_\infty-$metric for fuzzy sets, we have

$$\mathcal{H}_{\mathbb{G}}([\mathcal{S}\tau]_\lambda,[\mathcal{T}\sigma]_\lambda,[\mathcal{T}\sigma]_\lambda)\le\mathbb{G}_\infty(\mathcal{S}\tau,\mathcal{T}\sigma,\mathcal{T}\sigma)$$

for all $\tau,\sigma\in\mathcal{F}$. From $(F1)$, we have

$$\begin{aligned}\omega+F(\mathcal{H}_{\mathbb{G}}([\mathcal{S}\tau]_\lambda,[\mathcal{T}\sigma]_\lambda,[\mathcal{T}\sigma]_\lambda))&\le\omega+\mathbb{G}_\infty(\mathcal{S}\tau,\mathcal{T}\sigma,\mathcal{T}\sigma)\\&\le F(\mathbb{G}(\tau,\sigma,\sigma))\end{aligned}$$

for all $\tau,\sigma\in\mathcal{F}$. Since $[\mathcal{S}\tau]_1\subseteq[\mathcal{S}\tau]_\lambda$ for each $\lambda\in(0,1)$. Therefore, $\mathbb{G}(\tau,[\mathcal{S}\tau]_\lambda,[\mathcal{S}\tau]_\lambda))\le\mathbb{G}(\tau,[\mathcal{S}\tau]_1,[\mathcal{S}\tau]_1))$ for each $\lambda\in(0,1)$. It implies that $\mathbb{G}(\tau,\mathcal{S}\tau,\mathcal{S}\tau))\le\mathbb{G}(\tau,[\mathcal{S}\tau]_1,[\mathcal{S}\tau]_1))$. Similarly, $\mathbb{G}(\tau,\mathcal{T}\tau,\mathcal{T}\tau))\le\mathbb{G}(\tau,[\mathcal{T}\tau]_1,[\mathcal{T}\tau]_1))$. This further implies that for all $\tau,\sigma\in\mathcal{F}$,

$$\omega+F(\mathcal{H}_{\mathbb{G}}([\mathcal{S}\tau]_1,[\mathcal{T}\sigma]_1,[\mathcal{T}\sigma]_1))\le F(\mathbb{G}(\tau,\sigma,\sigma)).$$

By Theorem 3.3, we get $u\in\mathcal{F}$ such that $u\in[\mathcal{S}u]_\lambda\cap[\mathcal{T}u]_\lambda$, i.e.,

$$\{u\}\subset[\mathcal{S}u]_\lambda\text{ and }\{u\}\subset[\mathcal{T}u]_\lambda.$$

∎

Example 3.8. Let $\mathcal{F}=[0,1]$ and define $\mathbb{G}:\mathcal{F}\times\mathcal{F}\times\mathcal{F}\to\mathbb{R}^+$ as follows

$$\mathbb{G}(\tau,\sigma,\upsilon)=|\tau-\sigma|+|\sigma-\upsilon|+|\upsilon-\tau|.$$

Define $\mathcal{S},\mathcal{T}:\mathcal{F}\to I^{\mathcal{F}}$, for $\lambda\in[0,1]$ as follows:
For $\tau\in\mathcal{F}$, we have

$$\mathcal{S}(\tau)(t)=\begin{cases}\lambda\text{ if }t\in[0,\frac{\tau}{30}],\\ \frac{\lambda}{2}\text{ if }t\in(\frac{\tau}{30},\frac{\tau}{20}]\\ \frac{\lambda}{5}\text{ if }t\in(\frac{\tau}{20},1]\end{cases}\quad\text{and }\mathcal{T}(\tau)(t)=\begin{cases}\alpha\text{ if }t\in[0,\frac{\tau}{20}],\\ \frac{\lambda}{3}\text{ if }t\in(\frac{\tau}{20},\frac{\tau}{10}]\\ \frac{\lambda}{7}\text{ if }t\in(\frac{\tau}{10},1]\end{cases}$$

such that

$$[\mathcal{S}\tau]_\lambda=\left[0,\frac{\tau}{30}\right]\text{ and }[\mathcal{T}\tau]_\lambda=\left[0,\frac{\tau}{20}\right].$$

Let $F(t)=\ln(t)$, for $t>0$. Then there exist $\omega>0$ with $\tau\ne\sigma\ne\upsilon$ such that

$$\omega+F(\mathcal{H}_{\mathbb{G}}([\mathcal{S}\tau]_\lambda,[\mathcal{T}\sigma]_\lambda,[\mathcal{T}\sigma]_\lambda))\le F(\mathbb{G}(\tau,\sigma,\sigma))$$

for all $\tau,\sigma,\in\mathcal{F}$ with $\mathcal{H}_{\mathbb{G}}([\mathcal{S}\tau]_\lambda,[\mathcal{T}\sigma]_\lambda,[\mathcal{T}\sigma]_\lambda))>0$ is satisfied. Then, clearly $0\in[\mathcal{S}0]_\lambda\cap[\mathcal{T}0]_\lambda$. Hence, 0 is a common fixed point of the mappings $\mathcal{S}$ and $\mathcal{T}$.

Conclusion

In this paper, we studied fixed point theorems and common fixed point results for $F-$contraction type fuzzy mappings in $\mathbb{G}-$metric space. Starting from the notion of $\mathbb{G}-$metric space, our results complement several significant fixed point theorems of $\mathbb{G}-$metric space in the frame of fuzzy mappings. We hope that the presented idea herein will be a source of motivation for interested researchers to extend and improve these results suitable for areas of applications such as in the investigation of existence of solutions of differential and integral equations of different types and related problems.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

The authors are thankful to the editors and the anonymous reviewers for their valuable suggestions and comments on the manuscript.

References

[1] S. Banach, Sur les oprations dans les ensembles abstraits et leur application aux quations intgrales. Fund. math. 3 (1) (1922) 133–181.

[2] Z. Mustafa, B. Sims, A new approach to generalized metric spaces, J. Nonl. Convex Anal. 7 (2) (2006) 289–297.

[3] D. Wardowski, Fixed points of a new type of contractive mappings in complete metric spaces, Fixed Point Theory Appl. 2012 (1) (2012) 94–100.

[4] I. Altun, G. Minak, H. Dag, Multivalued F-contractions on complete metric space, J. Nonlinear Convex Anal. 16 (4) (2015) 659–666.

[5] H. Aydi, T. Rashid, Q.H. Khan, Z. Mustafa, M.M. Jaradat, Fixed point results using Ft-Contractions in ordered metric spaces having t-property, Symmetry 11 (3) (2019) 313.

[6] M.R. Taskovic, A generalization of Banachs contraction principle, Publ. Inst. Math, 37 (1978) 179–191.

[7] B.E. Rhoades, A comparison of various definitions of contractive mappings, Trans. Amer. Math. Soc. 226 (1977) 257–290.

[8] L.A. Zadeh, Fuzzy sets, Info. Cont. 8 (3) (1965) 338–353.

[9] S. Heilpern, Fuzzy mappings and fixed point theorem, Journal of Mathematical Analysis and Applications 83 (2) (1981) 566–569.

[10] S.B. Nadler, Multi-valued contraction mappings, Pacific Journal of Mathematics 30 (2) (1969) 475–488.

[11] B. Samet, C. Vetro, P. Vetro, Fixed point theorems for $\alpha-\psi-$contractive type mappings, Nonl. Anal.; Theory, Methods Appl. 75 (4) (2012) 2154–2165.

[12] I.Y. Alber, S.D. Guerre, Principle of weakly contractive maps in Hilbert spaces, In New Results in Oper. Theory Appl. 3 (5) (1997) 7–22.

[13] V. Bernide, On the approximation of fixed points of weak contractive mappings, Carpathian Journal of Mathematics 4 (1) (2003) 7–22.

[14] A. Azam, Fuzzy fixed points of fuzzy mappings via a rational inequality, Hacettepe J. Math. Stat. 40 (3) (2011) 421–431.

[15] A. Kaewcharoen, A. Kaewkhao, Common fixed points for single-valued and multi-valued mappings in G-metric spaces. Int. J. Math. Anal. 5 (36) (2011) 1775–1790.

[16] Z. Mustafa, M. Arshad, S.U. Khan, J. Ahmad, M. Jaradat, Common fixed points for multivalued mappings in G-metric spaces with applications. J. Nonlinear Sci. Appl. 10 (2017) 2550–2564

[17] A. Gupta, P.S. Kaurav, Fixed points of a new type of contractive mappings in G-metric spaces, Int. J. Math. And Appl. 7 (4) (2019) 95–100.