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A Topological Approach of a Human Heart via Nano Pre-ideality

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Abstract A method for creating new neighbourhoods for graph vertices will be developed. The blood circulation of a human body may be characterised in this work by nano pre-I-open sets, which are suitably located between nano openness and nano preopenness independent of the nano topological ideal. Furthermore, we demonstrate that the class of nano pre-I-open sets is suitably situated between the classes of nano I-open and nano preopen sets. We decompose nano I-continuity by demonstrating that a function $f : (X, \tau, I) \longrightarrow (Y, \sigma)$ is nano I-continuous if and only if it is nano pre-I-continuous and nano *-I-continuous. Finally, we expand the graph of a human body's blood circulation with regard to ideals.

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1. Introduction

Topological models were used in biology [1–4], and medicine, [5]. Kuratowski [6] has been studying ideals in topology since 1930. The article by Vaidyanathaswamy [7] in 1945 established the topic's prominence. An ideal or dual filter on X is a nonempty collection of subsets of X satisfying hereditary and finite additivity criteria. A nonempty family $I \subseteq P(X)$ is termed an ideal iff (i) $A \in I$ yields $P(A) \subseteq I$ and (ii) $A, B \in I$ gives $A \cup B \in I$. Given a topological space (X, τ) and an ideal I on X, a set operator $()^* : P(X) \to P(X)$, is known as a local function [7] of A w.r.to τ and I is defined as follows: for $A \subseteq X$, $A^*(I, \tau) = \{x \in X : G \cap A \notin I, \forall\ G \in \tau(x)\}$, where, $\tau(x) = \{G \in \tau : x \in G\}$. A Kuratowski closure operator Cl^* for $\tau^*(I, \tau)$ (τ^*, for briefly) is called *-topology, finer than τ is defined by $Cl^*(A) = A \cup A^*(I, \tau)$ (A^*, for briefly) [7]. The space (X, τ, I) is called an ideal topology. The approximations are stated in terms of equivalence classes based on the overlaps with the approximated sets. Many ideas for generalizing and understanding rough sets have been offered, including those by [8–11]. Lellis Thivagar et al. [12] proposed the concept of a nano topology, which was described in terms of approximations and the boundary region of a subset of a universe using an equivalence

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relation on it, and they also defined nano closed sets, nano-interior, and nano-closure. Parimala et al. [13] developed the notion of nano ideal topological spaces and examined its features and characterizations. [14] addressed several weak kinds of continuity.

The links between certain weak forms of nano open sets are examined in this study, as well as near nano openness in nano ideal spaces. In addition, we note that the class of nano pre-I-open sets ($\mathcal{NPIO}$, for short) is suitably placed between the classes of nano I-open ($\mathcal{NIO}$, for short) and nano preopen sets. We decompose nano I-continuity by demonstrating that the function from (X, τ, I) into (Y, σ) is nano I-continuous iff it is nano pre-I-continuous and nano *-I-continuous. Finally, we use the findings to create an extension of a linked graph of a human blood circulation using $\mathcal{NPIO}$ sets.

2. Preliminaries

We recall the following definitions and results which are used in the sequel.

2.1. Nano Topological Spaces

Definition 2.1. [15] Let U represent a finite collection of objects known as the universe, and R represent an equivalence relation on U. After that, U is separated into disjoint equivalence classes. Elements from the same equivalence class are said to be indistinguishable from one another. The approximation space is a pair (U, R).

Definition 2.2. [15] Consider Figure 1. Let (U, R) be an approximation space and

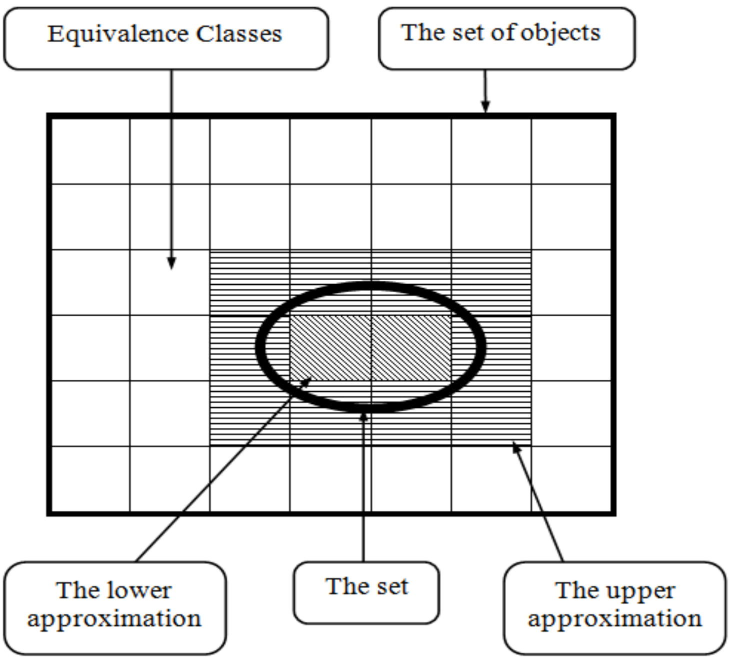

Figure 1. A rough set in a rough space.

$X \subseteq U$. Then:

(i) $L_R(X) = \bigcup_{x\in U} \{R(x) : R(x) \subseteq X\}$ is the lower approximation of X w.r.to R.

(ii) $H_R(X) = \bigcup_{x\in U} \{R(x) : R(x)\cap X \neq \phi\}$ is the upper approximation of X.

(iii) $B_R(X) = H_R(X) - L_R(X)$ is the boundary of X.

Using Pawlak's definition, X is roughif $H_R(X) \neq L_R(X)$.

Proposition 2.3. [15] *In (U, R), the following hold for $A, B \subseteq U$:*

(i) $L_R(A) \subseteq A \subseteq H_R(A)$ (Contraction and Extension).

(ii) $L_R(\phi) = H_R(\phi) = \phi$ (Normality) and $L_R(U) = H_R(U) = U$ (Co-normality).

(iii) $H_R(A \cup B) = H_R(A) \cup H_R(B)$ (Addition).

(iv) $H_R(A \cap B) \subseteq H_R(A) \cap H_R(B)$.

(v) $L_R(A \cup B) \supseteq L_R(A) \cup L_R(B)$.

(vi) $L_R(A \cap B) = L_R(A) \cap L_R(B)$ (Multiplication).

(vii) $L_R(A) \subseteq L_R(B)$ and $H_R(A) \subseteq H_R(B)$ whenever $A \subseteq B$ (Monotone).

(viii) $H_R(A^c) = [H_R(A)]^c$ and $L_R(A^c) = [H_R(A)]^c$ where A^c denotes the complement of A in U (Duality).

(ix) $H_R(H_R(A)) = L_R(H_R(A)) = H_R(A)$ (Idempotency).

(x) $L_R(L_R(A)) = H_R(L_R(A)) = L_R(A)$ (Idempotency).

Definition 2.4. [12] The class $\tau_R(X) = \{U, \phi, L_R(X), H_R(X), B_R(X)\}$, for $X \subseteq U$ is a nano topology on U and satisfies the three axioms of topology. $(U, \tau_R(X))$ is a nano space whose elements are named nano open sets ($\mathcal{NO}(U)$, for short). Its complement is nano closed ($\mathcal{NC}(U)$, for short).

Remark 2.5. [12] The basis for $\tau_R(X)$ is $\beta = \{U, L_R(X), B_R(X)\}$.

Definition 2.6. [12] The operators on nano topology $\tau_R(X)$ are

(i) Nano-interior of A (briefly, $n\mathfrak{int}(A)$) is $\bigcup\{B : B \in \mathcal{NO}(U, X), B \subseteq A\}$.

(ii) Nano-closure of A (briefly, $n\mathfrak{cl}(A)$) is $\bigcap\{F : F \in \mathcal{NC}(U, X), A \subseteq F\}$.

Definition 2.7. [12, 13, 16] $A \subseteq U$ is called:

(i) Nano regular if $A = n\mathfrak{int}(n\mathfrak{cl}(A))$,

(ii) Nano α-open if $A \subseteq n\mathfrak{int}(n\mathfrak{cl}(n\mathfrak{int}(A)))$,

(iii) Nano semi-open if $A \subseteq n\mathfrak{cl}(n\mathfrak{int}(A))$,

(iv) Nano preopen if $A \subseteq n\mathfrak{int}(n\mathfrak{cl}(A))$,

(v) Nano γ-open (or nano b-open) if $A \subseteq n\mathfrak{cl}(n\mathfrak{int}(A)) \cup n\mathfrak{int}(n\mathfrak{cl}(A))$,

(vi) Nano β-open if $A \subseteq n\mathfrak{cl}(n\mathfrak{int}(n\mathfrak{cl}(A)))$.

Definition 2.8. [17] A pair $G = (V, E)$ is graph. The elements of V are the vertices of G, and E the edges of G. The vertex set is named V_G and its edge set is named E_G. Directed (resp. undirected) graphs is a graph in which the edges have a direction (resp. no direction)

2.2. Nano Ideal Topological Spaces

In this section, the nano lacal function in a nano ideal space is investigated.

Definition 2.9. [13] Let $(U, \tau_R(X), I)$ be a nano ideal space. A set operator $()^*_n$ from $P(U)$ into itself is nano local function. For $A \subseteq U$, $A^*_n(I, \tau_R(X)) = \{x \in U : G_x \cap A \notin I, \forall\ G_x \in \tau_R(X)\}$ is named the nano local function of A w.r.to I and $\tau_R(X)$, A^*_n is used instead of $A^*_n(I, \tau_R(X))$, for short.

Example 2.10. Let $(U, \tau_R(X), I)$ be a nano ideal space. Then, $\forall\ A \subseteq U$:

(i) If $I = \{\phi\}$, then $A^*_n = n\mathfrak{cl}(A)$,
(ii) If $I = P(U)$, then $A^*_n = \phi$.

The following theorem provides numerous fundamental and relevant truths about the nano local function.

Theorem 2.11. [18] *Let $(U, \tau_R(X))$ be a nano space with ideals I and J on U. Then, the following are hold, for $A, B \subseteq U$:*

(i) If $A \subseteq B$, then $A^*_n \subseteq B^*_n$,
(ii) If $I \subseteq J$, then $A^*_n(J) \subseteq A^*_n(I)$,
(iii) $A^*_n = n\mathfrak{cl}(A^*_n) \subseteq n\mathfrak{cl}(A)$, i.e. $A^*_n \in \mathcal{NC}(n\mathfrak{cl}(A))$,
(iv) $(A^*_n)^*_n \subseteq A^*_n$,
(v) $(A \cup B)^*_n = A^*_n \cup B^*_n$,
(vi) $(A \cap B)^*_n \subseteq A^*_n \cap B^*_n$,
(vii) $A^*_n - B^*_n = (A - B)^*_n - B^*_n \subseteq (A - B)^*_n$,
(viii) If $V \in \mathcal{NO}(U, X)$, then $V \cap A^*_n = V \cap (V \cap A)^*_n \subseteq (V \cap A)^*_n$,
(ix) If $E \in I$, then $(A \cup E)^*_n = A^*_n = (A - E)^*_n$.

Theorem 2.12. *Let $(U, \tau_R(X), I)$ be a nono ideal space. Then:*

(i) $A^*_n \in \mathcal{NC}(U, X)$,
(ii) $\phi^*_n = \phi$,
(iii) $(U - E)^*_n = U^*_n$, if $E \in I$,
(iv) $[U - (A - E)]^*_n = [(U - A) \cup E]^*_n$, if $E \in I$ and $A \subseteq U$.

Definition 2.13. [13] Let $(U, \tau_R(X))$ be a nono space with I on U. The nano *-closure operator is defined by $n\mathfrak{cl}^*(A) = A \cup A^*_n$, for $A \subseteq X$.

Theorem 2.14. [13] $n\mathfrak{cl}^*$ *satisfies the conditions:*

(i) $A \subseteq n\mathfrak{cl}^*(A)$,
(ii) $n\mathfrak{cl}^*(\phi) = \phi$ and $n\mathfrak{cl}^*(U) = U$,
(iii) If $A \subseteq B$, then $n\mathfrak{cl}^*(A) \subseteq n\mathfrak{cl}^*(B)$,
(iv) $n\mathfrak{cl}^*(A) \cup n\mathfrak{cl}^*(B) = n\mathfrak{cl}^*(A \cup B)$,
(v) $n\mathfrak{cl}^*(n\mathfrak{cl}^*(A)) = n\mathfrak{cl}^*(A)$.

Definition 2.15. [12, 18] For the nono ideal space $(U, \tau_R(X), I)$, $A \subseteq U$ is called:

(i) Nano regular I-open (nano R-I-open, for short) if $A = n\mathfrak{int}(n\mathfrak{cl}^*(A))$,
(ii) Nano regular I-closed (nano R-I-closed, for short) if its complement is nano R-I-open,
(iii) Nano semi I-open if $A \subseteq n\mathfrak{cl}^*(n\mathfrak{int}(A))$,
(iv) Nano semi I-closed if its complement is nano semi I-open,
(v) Nano α-I-open if $A \subseteq n\mathfrak{int}(n\mathfrak{cl}^*(n\mathfrak{int}(A)))$,
(vi) Nano α-I-closed if its complement is nano α-I-open,
(vii) $\mathcal{NPIO}$ if $A \subseteq n\mathfrak{int}(n\mathfrak{cl}^*(A))$,
(viii) Nano pre-I-closed if its complement is $\mathcal{NPIO}$,
(ix) Nano β-I-open if $A \subseteq n\mathfrak{cl}(n\mathfrak{int}(n\mathfrak{cl}^*(A)))$,
(x) Nano β-I-closed if its complement is nano β-I-open,
(xi) Nano I-open if $A \subseteq n\mathfrak{int}(A^*_n)$.

Definition 2.16. [13] An ideal I in $(U, \tau_R(X), I)$ is named $\tau_R(X)$-codense ideal if $\tau_R(X) \cap I = \{\phi\}$.

Definition 2.17. [13] A subset A in $(U, \tau_R(X), I)$ is called:
(i) Nano $\star$-dense-in-itself if $A \subseteq A_n^*$,
(ii) Nano $\star$-closed if $A_n^* \subseteq A$,
(iii) Nano $\star$-perfect if $A = A_n^*$,
(iv) Nano I-dense if $A_n^* = U$.

3. Nano Pre-I-open Sets

$(U, \tau_R(X), I)$ is a nano ideal topological space in this section.

Definition 3.1. A subset A is $\mathcal{NPIO}$ if $A \subseteq n\mathfrak{int}(n\mathfrak{cl}^*(A))$.

$\mathcal{NPIO}(U, \tau_R(X), I)$ will denote to $\mathcal{NPIO}$ sets ($\mathcal{NPIO}(U)$, for short). The nano pre-I-closed sets are denoted by $\mathcal{NPIC}(U)$. Despite the fact that nano I-openness and nano openness are distinct concepts. The next two findings reveal that nano pre-I-openness is connected to both of them.

Proposition 3.2. *Every nano* I-*open set is* $\mathcal{NPIO}$.

Proof. Let $(U, \tau_R(X), I)$ be a nano ideal space and $A \subseteq U$ be nano I-open. Then, $A \subseteq n\mathfrak{int}(A_n^*) \subseteq n\mathfrak{int}(A_n^* \cup A) = n\mathfrak{int}(n\mathfrak{cl}^*(A))$. ∎

Proposition 3.3. *Every nano open set is* $\mathcal{NPIO}$.

Proof. Let $A \in \mathcal{NO}(U, X)$. Then, $A \subseteq n\mathfrak{int}(A) \subseteq n\mathfrak{int}(A_n^* \cup A) = n\mathfrak{int}(n\mathfrak{cl}^*(A))$. ∎

The converse of the above premises is not always true, as shown by the following case.

Example 3.4. Let $U = \{a, b, c, d\}$ with $U/R = \{\{b\}, \{d\}, \{a, c\}\}$ and $X = \{a, d\}$. One can deduce that $\tau_R(X) = \{U, \phi, \{d\}, \{a, c\}, \{a, c, d\}\}$. For $I = \{\phi, \{d\}\}$, if $A = \{a, d\}$, we get $A_n^* = \{a, b, c\}$ and $n\mathfrak{int}(A_n^*) = \{a, c\}$ and $n\mathfrak{cl}^*(A) = A \cup A_n^* = \{a, b, c, d\}$. So, $n\mathfrak{int}(n\mathfrak{cl}^*(A)) = U$. Hence, A is $\mathcal{NPIO}$ but nighter nano I-open nor nano open.

Together with Propositions 3.2 and 3.3, our following two findings indicate that the class of $\mathcal{NPIO}$ sets is correctly situated between the classes of nano I-open and nano preopen sets, as well as between the classes of nano open and nano preopen sets.

Proposition 3.5. *Every* $\mathcal{NPIO}$ *set is nano preopen.*

Proof. Let $A \in \mathcal{NPIO}(U)$. Then, $A \subseteq n\mathfrak{int}(n\mathfrak{cl}^*(A)) = n\mathfrak{int}(A_n^* \cup A) \subseteq n\mathfrak{int}(n\mathfrak{cl}(A) \cup A) = n\mathfrak{int}(n\mathfrak{cl}(A))$. ∎

Remark 3.6. Several types of weak nano open sets outlined above have the following consequences in $(U, \tau_R(X), I)$, as shown in Figure 2.

The converses of these implications of Figure 2, however, are not true in general, as shown by Examples 2.1, 2.2 [16], and the following cases.

Example 3.7. Let $U = \{a, b, c, d\}$ with $U/R = \{\{b\}, \{d\}, \{a, c\}\}$. One can deduce that $\tau_R(X) = \{U, \phi, \{d\}, \{a, c\}, \{a, c, d\}\}$.
(a) For $I = \{\phi, \{d\}\}$. Then:
(i) $A = \{a, d\} \in \mathcal{NPIO}(U)$, but nighter nano I-open nor nano α-I-open.
(ii) $B = \{a, b, c\}$ is nano semi-I-open, but not nano α-I-open.
(iii) B is nano β-I-open, but not $\mathcal{NPIO}$.
(iv) $C = \{b, d\}$ is nano semi-open, but not nano semi-I-open.

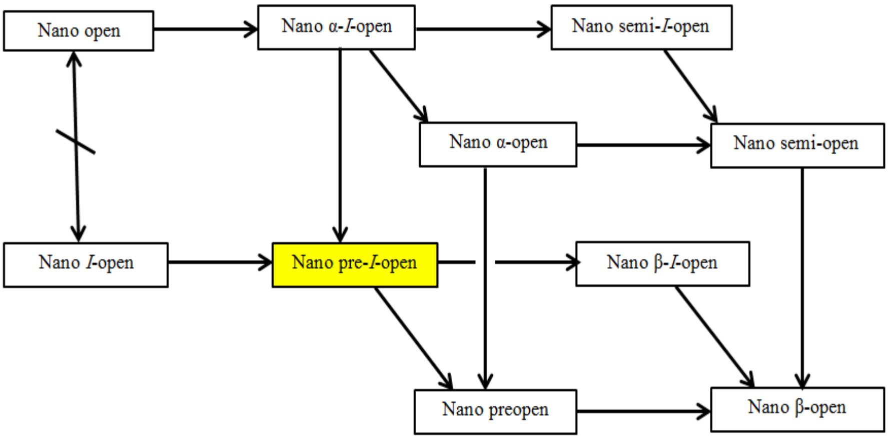

FIGURE 2. Comparison between weak nano open sets

(b) For $I = \{\phi, \{a\}\}$. Then, $A = \{a, d\}$ is nano preopen, but not $\mathcal{NPIO}$.
(c) For $I = \{\phi, \{a\}, \{d\}, \{a, d\}\}$. Then, $A = \{a, b, d\}$ is nano β-open, but not nano β-I-open.

Theorem 3.8. *For $(U, \tau_R(X), I)$. $\forall\ A \subseteq U$, we get:*

(i) If $I = \{\phi\}$, then $A \in \mathcal{NPIO}(U)$ iff A is nano preopen.
(ii) If $I = P(U)$, then $A \in \mathcal{NPIO}(U)$ iff $A \in \mathcal{NO}(U)$.
(iii) If $I \subseteq I_n$, then $A \in \mathcal{NPIO}(U)$ iff A is nano preopen, where I_n denote the ideal of nowhere dense sets.

Proof. (i) Using Proposition 3.5, $A \in \mathcal{NO}(U)$. For sufficiency, the minimal ideal $A^*_n = n\mathfrak{cl}(A)$.
(ii) If $A \in \mathcal{NPIO}(U)$, then $A \subseteq n\mathfrak{int}(n\mathfrak{cl}^*(A)) = n\mathfrak{int}(A^*_n \cup A) = n\mathfrak{int}(A \cup \phi) = n\mathfrak{int}(A)$. The other hand is given using Proposition 3.3.
(iii) By Proposition 3.5, we prove only the other hand. The nano local function of A w.r.to I_n and τ is given by $A^*_n(I_n) = n\mathfrak{cl}(n\mathfrak{int}(n\mathfrak{cl}(A)))$. Thus, A is nano pre-I-open iff $A \subseteq n\mathfrak{int}(A \cup n\mathfrak{cl}(n\mathfrak{int}(n\mathfrak{cl}(A))))$. Assume that A is nano preopen. Since $n\mathfrak{int}(n\mathfrak{cl}(A)) \subseteq A \cup\ n\mathfrak{cl}(n\mathfrak{int}(n\mathfrak{cl}(A)))$, then $A \subseteq n\mathfrak{int}(A \cup n\mathfrak{cl}(n\mathfrak{int}(n\mathfrak{cl}(A))))$ or equivalently A is $\mathcal{NPIO}$. ∎

Remark 3.9. The finite intersection of of elements in $\mathcal{NPIO}(U)$ need not be in $\mathcal{NPIO}(U)$.

Lemma 3.10. *In $(U, \tau_R(X), I)$, $G \in \mathcal{NO}(U)$ implies $G \cap A^*_n = G \cap (G \cap A)^*_n \subseteq (G \cap A)^*_n$, $\forall\ A \subseteq U$.*

Proof. Let $V \in \mathcal{NO}(U)$ and $x \in G \cap A^*_n$. Then, $x \in G$ and $x \in A^*_n$. Let $H \in \mathcal{NO}(U, x)$. Then, $H \cap G \in \mathcal{NO}(U)$ and $H \cap (G \cap A) = (H \cap G)\ \cap A \notin I$. Hence, $x \in (G \cap A)^*_n$ and so $G \cap A^*_n \subseteq (G \cap A)^*_n$. So, $G \cap A^*_n \subseteq G\ \cap (G \cap A)^*_n$. $(G \cap A)^*_n \subseteq A^*_n$ and $G \cap A^*_n \supseteq G\ \cap (G \cap A)^*_n$. Therefore, $G \cap A^*_n = G \cap (G \cap A)^*_n \subseteq (G \cap A)^*_n$. ∎

Proposition 3.11. *In $(U, \tau_R(X), I)$, we get*
(i) If $\{A_\alpha : \alpha \in \triangle\} \in \mathcal{NPIO}(U)$, then $\bigcup\{A_\alpha : \alpha \in \triangle\} \in \mathcal{NPIO}(U)$.

(ii) If $A \in \mathcal{NPIO}(U)$ and $G \in \mathcal{NO}(U)$, then $A \cap G \in \mathcal{NPIO}(U)$, where $\triangle$ an arbitrary index set.

Proof. (i) Since $\{A_\alpha : \alpha \in \triangle\} \in \mathcal{NPIO}(U)$, then $A_\alpha \subseteq n\mathfrak{int}(n\mathfrak{cl}^*(A_\alpha))\ \forall\ \alpha \in \triangle$. Thus, $\bigcup_{\alpha\in\triangle} A_\alpha \subseteq \bigcup_{\alpha\in\triangle} n\mathfrak{int}(n\mathfrak{cl}^*(A_\alpha)) \subseteq n\mathfrak{int}(\bigcup_{\alpha\in\triangle} n\mathfrak{cl}^*(A_\alpha)) = n\mathfrak{int}(\bigcup_{\alpha\in\triangle}(A^*_{n\alpha} \cup A_\alpha)) = n\mathfrak{int}((\bigcup_{\alpha\in\triangle} A^*_{n\alpha}) \cup (\bigcup_{\alpha\in\triangle} A_\alpha)) \subseteq n\mathfrak{int}((\bigcup_{\alpha\in\triangle} A_\alpha)^*_n \cup (\bigcup_{\alpha\in\triangle} A_\alpha)) = n\mathfrak{int}(n\mathfrak{cl}^*(\bigcup_{\alpha\in\triangle} A_\alpha))$.
(ii) From assumption, $A \subseteq n\mathfrak{int}(n\mathfrak{cl}^*(A))$ and $G \subseteq n\mathfrak{int}(G)$. Using Lemma 3.10, $A \cap G \subseteq n\mathfrak{int}(n\mathfrak{cl}^*(A)) \cap n\mathfrak{int}(G) \subseteq n\mathfrak{int}(n\mathfrak{cl}^*(A) \cap G) = n\mathfrak{int}((A^*_n \cup A) \cap G) = n\mathfrak{int}((A^*_n \cap G) \cup (A \cap G)) \subseteq n\mathfrak{int}((A \cap G)^*_n \cup (A \cap G)) = n\mathfrak{int}(n\mathfrak{cl}^*(A \cap G))$. ■

Corollary 3.12. *The arbitrary union of elements of $\mathcal{NPIC}(U)$ is in $\mathcal{NPIC}(U)$.*

4. Nano I-continuous Functions

In this part, we define and investigate the nano I-continuous functions, as well as some of their features.

Definition 4.1. A function $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$ is called nano I-continuous function if $\forall\ G \in \mathcal{NO}(V)$, $f^{-1}(G) \in \mathcal{NIO}(U)$, where $\mathcal{NIO}$ is nano-I-open sets on U.

Recall that $f : U \to V$ nano precontinuous if the inverse image of every nano open set in V is nano preopen in U.

Remark 4.2. It is possible to conclude from the above definitions that:

Nano I-continuity $\Rightarrow$ Nano precontinuity .

The reverse is not true in general, as shown by Example 4.3.

Example 4.3. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. $\tau_R(X) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{w\}, \{y, z\}\}$ and $Y = \{x, z\}$. $\tau_{R'}(Y) = \{V, \phi, \{x\}, \{y, z\}, \{x, y, z\}\}$. A function $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$ is defined by $f(a) = x$, $f(b) = y$, $f(c) = w$, $f(d) = z$. For $I = \{\phi, \{a\}, \{b\}, \{a, b\}\}$, f is nano precontinuous function, but not nano I-continuous.

Theorem 4.4. *For $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$, the following are equivalent:*

(i) f is nano I-continuous.
(ii) $\forall\ x \in U$ and $G \in \mathcal{NO}(Y, f(x))$, $\exists\ W \in \mathcal{NIO}(U, x)$ s.t. $f(W) \subseteq G$.
(iii) $\forall\ x \in U$ and $G \in \mathcal{NO}(Y, f(x))$, $(f^{-1}(G))^*_n$ is a nano neighborhood of x.

Proof. (i)$\Rightarrow$ (ii): Since $G \in \mathcal{NO}(Y, f(x))$, then by (i), $f^{-1}(G) \in \mathcal{NIO}(U)$. Putting $W = f^{-1}(G)$ which containing x. Therefore, $f(W) \subseteq G$.
(ii)$\Rightarrow$ (iii): Since $G \in \mathcal{NO}(Y, f(x))$, then by (ii), $\exists\ W \in \mathcal{NIO}(U)$ containing x s.t. $f(W) \subseteq G$. So, $x \in W \subseteq n\mathfrak{int}(W)^*_n \subseteq n\mathfrak{int}(f^{-1}(G))^*_n \subseteq (f^{-1}(G))^*_n$. Hence, $(f^{-1}(G))^*_n$ is a nano neighborhood of x.
(iii)$\Rightarrow$ (i): Obvious. ■

5. A Decomposition of Nano I-continuity

In this part, we look at how to combine nano I-continuous functions with other forms of nano continuity.

Definition 5.1. $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$ is called:

(i) Nano $*$-I-continuous if $f^{-1}(G)$ is nano $*$-dense-in-itself in $(U, \tau_R(X), I)$, $\forall\ H \in \mathcal{NO}(V)$.
(ii) Nano pre-I-continuous if $\forall\ H \in \mathcal{NO}(V)$, $f^{-1}(H) \in \mathcal{NPIO}(U)$.

Using Proposition 3.3, we have directly the following result.

Proposition 5.2. *Every nano continuity is nano pre-*I*-continuity.*

The contrary is not always true, as shown by the following case.

Example 5.3. Consider $U = \{a, b, c, d\}$ and $V = \{x, y, z, w\}$ are universes with $U/R = \{\{a\}, \{d\}, \{b, c\}\}$ and $V/R^{'} = \{\{x\}, \{z\}, \{y, w\}\}$, respectively. Nano topologies are $\tau_R(X) = \{U, \phi, \{a\}, \{d\}, \{a, d\}\}$ and $\tau_{R'}(Y) = \{V, \phi, \{x\}, \{y, w\}, \{x, y, w\}\}$. A function $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$ is defined by $f(a) = y = f(b)$, $f(c) = z$ and $f(d) = w$. For $I = \{\phi, \{d\}\}$. f is nano pre-I-continuous function, but not nano continuous.

We have the following outcome as a consequence of Proposition 3.2.

Proposition 5.4. *Every nano* I*-continuity is nano pre-*I*-continuity.*

As the following example demonstrates, the opposite is not true.

Example 5.5. (Continued for Example 5.3). f is nano pre-I-continuity, but not nano I-continuity.

Proposition 5.6. *Every nano pre-*I*-continuity is nano precontinuity.*

Proof. Follows directly by Proposition 3.5. ∎

Theorem 5.7. *For $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$, the following are equivalent:*
(i) f is nano pre-I-continuity.
(ii) $\forall\ x \in U$ and $G \in \mathcal{NO}(V, f(x))$, $\exists\ W \in \mathcal{NPIO}(U, x)$ s.t. $f(W) \subseteq G$.
(iii) $\forall\ x \in U$ and $G \in \mathcal{NO}(V, f(x))$, $\mathfrak{cl}^*_n(f^{-1}(G))$ is a nano neighbourhood of x.
(iv) $f^{-1}(F) \in \mathcal{NPIC}(U)$, $\forall\ F \in \mathcal{NC}(V)$.

Proof. (i) $\Rightarrow$ (ii): Let $x \in U$ and $G \in \mathcal{NO}(V, f(x))$. Put $W = f^{-1}(G)$. By (i), $W \in \mathcal{NPIO}(U, x)$ and $f(W) \subseteq G$.
(ii) $\Rightarrow$ (iii) : Since $G \in \mathcal{NO}(V, f(x))$, then by (ii), $\exists\ W \in \mathcal{NPIO}(U, x)$ s.t. $f(W) \subseteq G$. Thus, $x \in W \subseteq n\mathfrak{int}(n\mathfrak{cl}^*(W)) \subseteq n\mathfrak{int}(n\mathfrak{cl}^*(f^{-1}(G))) \subseteq n\mathfrak{cl}^*(f^{-1}(G))$. Hence, $n\mathfrak{cl}^*(f^{-1}(G))$ is nano neighbourhood of x.
(iii) $\Rightarrow$ (i) and (i) $\Leftrightarrow$ (iv) are obvious. ∎

Remark 5.8. The composition of two nano pre-I-continuous functions does not necessarily have to be nano pre-I-continuous. Because each nano pre-I-open is not a nano open, as seen in Figure 2.

Theorem 5.9. *Let $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y), J)$ and $g : (V, \tau_{R'}(Y), J) \longrightarrow (W, \tau_{R''}(Z))$ be two functions, where I and J are ideals on U and V, respectively. Then, the following satisfied:*
*(i) if f is a nano pre-I-continuous and g is nano continuous, then $g \circ f$ is nano pre-*I*-continuous.*
*(ii) if g is nano continuous and f is nano pre-*I*-continuous, then $g \circ f$ is nano pre-continuous.*

Proof. Obviously, from Figure 2. ∎

Theorem 5.10. *For $A \subseteq U$, the following are equivalent:*

(i) $A \in \mathcal{NIO}(U)$.

(ii) $A \in \mathcal{NPIO}(U)$ *and nano* $*$*-dense-in-itself.*

Proof. (i) $\Rightarrow$ (ii): Using Proposition 3.3, $\forall\ A \in \mathcal{NIO}(U)$, we get $A \in \mathcal{NPIO}(U)$. On the other hand, $A \subseteq n\mathfrak{int}(A_n^*) \subseteq A$ which means that A is nano $*$-dense-in-itself.
(ii) $\Rightarrow$ (i) By assumption $A \subseteq n\mathfrak{int}(n\mathfrak{cl}^*(A)) = n\mathfrak{int}(A_n^* \cup A) = n\mathfrak{int}(A_n^*)$ or equivalently $A \in \mathcal{NIO}(U)$. ∎

In what follows, we will attempt to dissect nano I-continuity using Theorem 5.10.

Theorem 5.11. *For $f : (U, \tau_R(X), I) \longrightarrow (V, \tau_{R'}(Y))$, the following are equivalent:*

(i) f is nano I*-continuity.*

*(ii) f is both nano pre-*I*-continuity and nano* $*$*-*I*-continuity.*

6. An Application on a Human Heart

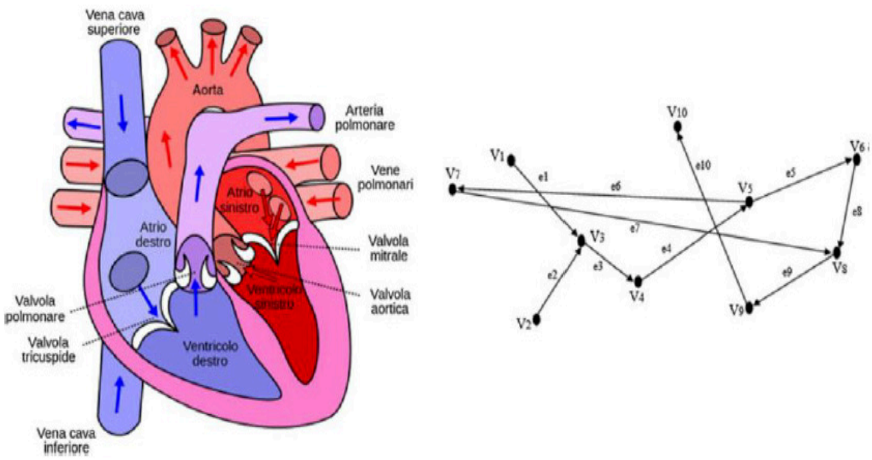

Figure 3. The connected graph of human body[wileyonlinelibrary.com]

In the following, we give a modification of Proposition 1.6 in [19] and Proposition 4.4.1 in [20]. We apply the ideal on topological graphs on a medical application. We conclude that the graph must be connected to modifying the medical state. In Figure 3, The connected graph $\mathcal{G}$ classifies the human heart into two sets of vertices and edges. Consider $V(\mathcal{G}) = \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7, \wp_8, \wp_9, \wp_{10}\}$ and the vertices of a subgraph $V(H) = \{\wp_1, \wp_3, \wp_5, \wp_{10}\}$. Using Definitions 2.2 and 3.1, we get $\mathcal{NO}(\mathcal{G})$ as follows: $L_R(V(H)) = \{\wp_1, \wp_{10}\}$, $H_R(V(H)) = \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_9, \wp_{10}\}$ and so $B_R(V(H)) = \{\wp_2, \wp_3, \wp_4, \wp_5, \wp_9\}$. Therefore, $\tau_R(V(H)) = \{V(H), \phi, \{\wp_1, \wp_{10}\}, \{\wp_2, \wp_3, \wp_4, \wp_5, \wp_9\}, \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_9, \wp_{10}\}\}$. We deduce that:

(i) Firstly, the $\mathcal{NO}(\mathcal{G})$ are $\{V(\mathcal{G}), \phi, \{\wp_1, \wp_{10}\}, \{\wp_2, \wp_3, \wp_4, \wp_5, \wp_9\}, \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_9, \wp_{10}\}, \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_9, \wp_{10}\}, \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_7, \wp_9, \wp_{10}\}, \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_8, \wp_9, \wp_{10}\}, \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7, \wp_9, \wp_{10}\}, \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_8, \wp_9, \wp_{10}\}\}$.

(ii) The basis of $\mathcal{NPO}(\mathcal{G})$ on $\mathcal{G}$ is $\beta_{\mathcal{NPO}(V(\mathcal{G}))} = \{V(\mathcal{G}),\ \phi,\ \{\wp_1,\ \wp_{10}\},\ \{\wp_2,\ \wp_4,\ \wp_5,\ \wp_9,\ \{\wp_2,\ \wp_3,\ \wp_4,\ \wp_5, \wp_9\ \},\ \{\wp_1,\ \wp_2,\ \wp_4, \wp_5,\ \wp_9, \wp_{10}\},\ \{\wp_1,\ \wp_2, \wp_3,\ \wp_4, \wp_5,\ \wp_9, \wp_{10}\},\ \{\wp_1,\ \wp_2,\ \wp_4,\ \wp_5,\ \wp_6,\ \wp_9,\ \wp_{10}\},\ \{\wp_1,\ \wp_2,\ \wp_4,\ \wp_5,\ \wp_7,\ \wp_9, \wp_{10}\},\ \{\wp_1,\ \wp_2,\ \wp_4,\ \wp_5,\ \wp_8,\ \wp_9, \wp_{10}\},\ \{\wp_1, \wp_2,\ \wp_3, \wp_4,\ \wp_5, \wp_6,\ \wp_9,\ \wp_{10}\},\ \{\wp_1,\ \wp_2,\ \wp_3, \wp_4,\ \wp_5, \wp_7,\ \wp_9, \wp_{10}\},\ \{\wp_1,\ \wp_2,\ \wp_3, \wp_4,\ \wp_5,\ \wp_8,\ \wp_9,\ \wp_{10}\},\ \{\wp_1,\ \wp_2,\ \wp_4, \wp_5,\ \wp_6,\ \wp_7,\ \wp_9, \wp_{10}\},\ \{\wp_1, \wp_2,\ \wp_4,\ \wp_5,\ \wp_6,\ \wp_8,\ \wp_9,\ \wp_{10}\},\ \{\wp_1, \wp_2,\ \wp_3,\ \wp_4,\ \wp_5,\ \wp_6,\ \wp_7,\ \wp_9,\ \wp_{10}\},\ \{\wp_1,\ \wp_2,\ \wp_3,\ \wp_4,\ \wp_5,\ \wp_6,\ \wp_8,\ \wp_9,\ \wp_{10}\},\ \{\wp_1,\ \wp_2,\ \wp_4,\ \wp_5,\ \wp_6, \wp_7,\ \wp_8,\ \wp_9,\ \wp_{10}\}\}$.

(iii) Finally, the ideality of $\mathcal{NPO}(\mathcal{G})$ on $\mathcal{G}$ through an ideal $I = \{\phi,\ \{\wp_3\}\}$ gives $\mathcal{NPIO}(V(\mathcal{G})) = \{V(\mathcal{G}),\ \phi,\ \{\wp_1,\ \wp_{10}\},\ \{\wp_1,\ \wp_2,\ \wp_{10}\},\ \{\wp_1,\ \wp_3,\ \wp_{10}\},\ \{\wp_1,\ \wp_4,\ \wp_{10}\},\ \{\wp_1,\ \wp_5,\ \wp_{10}\},\ \{\wp_1,\ \wp_6,\ \wp_{10}\},\ \{\wp_1,\ \wp_7,\ \wp_{10}\},\ \{\wp_1,\ \wp_8,\ \wp_{10}\},\ \{\wp_1,\ \wp_9,\ \wp_{10}\},\ \{\wp_1,\ \wp_2,\ \wp_3,\ \wp_{10}\},\ \{\wp_1,\ \wp_2,\ \wp_4,\ \wp_{10}\},\ \{\wp_1,\ \wp_2,\ \wp_5,\ \wp_{10}\},\ \{\wp_1,\ \wp_2,\ \wp_6,\ \wp_{10}\},\ \{\wp_1, \wp_2,\ \wp_7, \wp_{10}\},\ \{\wp_1,\ \wp_2,\ \wp_8,\ \wp_{10}\},\ \{\wp_1,\ \wp_2,\ \wp_9,\ \wp_{10}\},\ \{\wp_1, \wp_3,\ \wp_4,\ \wp_{10}\},\ \{\wp_1,\ \wp_3,\ \wp_5,\ \wp_{10}\},\ \{\wp_1,\ \wp_3,\ \wp_6,\ \wp_{10}\},\ \{\wp_1, \wp_3,\ \wp_7,\ \wp_{10}\},\ \{\wp_1,\ \wp_3,\ \wp_8,\ \wp_{10}\},\ \{\wp_1,\ \wp_3,\ \wp_9,\ \wp_{10}\},\ \{\wp_1,\ \wp_4,\ \wp_5, \wp_{10}\},\ 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\wp_{10}\},\ \{\wp_1,\ \wp_3,\ \wp_6,\ \wp_8,\ \wp_{10}\},\ \{\wp_1,\ \wp_3,\ \wp_6,\ \wp_9,\ \wp_{10}\},\ \{\wp_1,\ \wp_3,\ \wp_7,\ \wp_8,\ \wp_{10}\},\ \{\wp_1,\ \wp_3,\ \wp_7,\ \wp_9,\ \wp_{10}\},\ \{\wp_1,\ \wp_3,\ \wp_8,\ \wp_9,\ \wp_{10}\},\ \{\wp_1,\ \wp_4,\ \wp_5,\ \wp_6,\ \wp_{10}\},\ \{\wp_1,\ \wp_4,\ \wp_5,\ \wp_7,\ \wp_{10}\},\ \{\wp_1,\ \wp_4,\ \wp_5,\ \wp_8,\ \wp_{10}\},\ \{\wp_1,\ \wp_4,\ \wp_5,\ \wp_9,\ \wp_{10}\},\ \{\wp_1,\ \wp_4,\ \wp_6,\ \wp_7,\ \wp_{10}\},\ \{\wp_1,\ \wp_4,\ \wp_6,\ \wp_8,\ \wp_{10}\},\ \{\wp_1,\ \wp_4,\ \wp_6,\ \wp_9,\ \wp_{10}\},\ \{\wp_1,\ \wp_4,\ \wp_7,\ \wp_8,\ \wp_{10}\},\ \{\wp_1,\ \wp_4,\ \wp_7,\ \wp_9,\ \wp_{10}\},\ \{\wp_1,\ \wp_4,\ \wp_8,\ \wp_9,\ \wp_{10}\},\ \{\wp_1,\ \wp_5,\ \wp_6,\ \wp_7,\ \wp_{10}\},\ \{\wp_1,\ \wp_5,\ \wp_6,\ \wp_8,\ \wp_{10}\},\ \{\wp_1,\ \wp_5,\ \wp_6,\ \wp_9,\ \wp_{10}\},\ \{\wp_1,\ \wp_5,\ \wp_7,\ \wp_8,\ \wp_{10}\},\ \{\wp_1,\ \wp_5,\ \wp_7,\ \wp_9,\ \wp_{10}\},\ \{\wp_1,\ \wp_5,\ \wp_8,\ \wp_9,\ \wp_{10}\},\ 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$\wp_{3},\ \wp_{4},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{5},\ \wp_{6},\ \wp_{8},\ \wp_{10}\},\ \{\wp_{1},$
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$\wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{3},\ \wp_{4},\ \wp_{6},\ \wp_{7},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{3},\ \wp_{4},\ \wp_{6},\ \wp_{8},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},$
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$\{\wp_{1},\ \wp_{2},\ \wp_{3},\ \wp_{4},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{3},\ \wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{3},\ \wp_{5},\ \wp_{6},$
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$\{\wp_{1},\ \wp_{2},\ \wp_{3},\ \wp_{6},\ \wp_{7},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{3},\ \wp_{6},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{3},\ \wp_{7},\ \wp_{8},$
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$\wp_{4},\ \wp_{5},\ \wp_{6},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{4},\ \wp_{5},\ \wp_{7},\ \wp_{8},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{4},\ \wp_{5},\ \wp_{7},\ \wp_{9},\ \wp_{10}\},$
$\{\wp_{1},\ \wp_{2},\ \wp_{4},\ \wp_{5},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{4},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{4},\ \wp_{6},\ \wp_{7},$
$\wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{4},\ \wp_{6},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{4},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},$
$\wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{5},\ \wp_{6},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},$
$\{\wp_{1},\ \wp_{2},\ \wp_{5},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{4},\ \wp_{5},\ \wp_{6},$
$\wp_{7},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{4},\ \wp_{5},\ \wp_{6},\ \wp_{8},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{4},\ \wp_{5},\ \wp_{6},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},$
$\wp_{4},\ \wp_{5},\ \wp_{7},\ \wp_{8},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{4},\ \wp_{5},\ \wp_{7},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{4},\ \wp_{5},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},$
$\{\wp_{1},\ \wp_{3},\ \wp_{4},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{4},\ \wp_{6},\ \wp_{7},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{4},\ \wp_{6},\ \wp_{8},$
$\wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{4},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},$
$\wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{5},\ \wp_{6},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{5},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},$
$\{\wp_{1},\ \wp_{3},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{4},\ \wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{4},\ \wp_{5},\ \wp_{6},\ \wp_{7},$
$\wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{4},\ \wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{4},\ \wp_{5},\ \wp_{6},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{4},$
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$\{\wp_{1},\ \wp_{2},\ \wp_{3},\ \wp_{4},\ \wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{3},\ \wp_{4},\ \wp_{5},\ \wp_{6},\ \wp_{8},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{3},$
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$\wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{3},\ \wp_{4},\ \wp_{5},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{3},\ \wp_{4},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{10}\},\ \{\wp_{1},$
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$\wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{3},\ \wp_{5},\ \wp_{6},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{3},\ \wp_{5},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},$
$\wp_{2},\ \wp_{3},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{4},\ \wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{4},\ \wp_{5},$
$\wp_{6},\ \wp_{7},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{4},\ \wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{4},\ \wp_{5},\ \wp_{7},\ \wp_{8},\ \wp_{9},$
$\wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{4},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},$
$\wp_{3},\ \wp_{4},\ \wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{4},\ \wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{4},\ \wp_{5},$
$\wp_{6},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{4},\ \wp_{5},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{4},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{9},$
$\wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{4},\ \wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},$
$\wp_{2},\ \wp_{3},\ \wp_{4},\ \wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{3},\ \wp_{4},\ \wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},$
$\wp_{3},\ \wp_{4},\ \wp_{5},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{3},\ \wp_{4},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{3},$
$\wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{2},\ \wp_{4},\ \wp_{5},\ \wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{1},\ \wp_{3},\ \wp_{4},\ \wp_{5},$
$\wp_{6},\ \wp_{7},\ \wp_{8},\ \wp_{9},\ \wp_{10}\},\ \{\wp_{2},\ \wp_{3},\ \wp_{4},\ \wp_{5},\ \wp_{9}\}\}.$

Thus, $\mathfrak{int}(V(H)) = \{\wp_1, \wp_{10}\}$, $\mathfrak{cl}(V(H)) = V(\mathcal{G})$, and $B(V(H)) = \{\wp_2, \wp_3, \wp_4, \wp_5, \wp_6, \wp_7, \wp_8, \wp_9\}$. While, using Section 3, $np\mathfrak{int}^*(V(H)) = \{\wp_1, \wp_3, \wp_5, \wp_{10}\}$, $np\mathfrak{cl}^*(V(H)) = V(\mathcal{G})$ and $npB^*(V(H)) = \{\wp_2, \wp_4, \wp_6, \wp_7, \wp_8, \wp_9\}$, where $np\mathfrak{int}^*$ (resp $np\mathfrak{cl}^*$, npB^*) denote to the nano preinterior (nano preclosure, nano preboundary), respectively, w.r.to the ideal I.

We can also have the following:

(i) $np\mathfrak{cl}^*(V(H)) \supsetneq H_R(V(H))$. Because if we consider $V(H) = \{\wp_1, \wp_3, \wp_5, \wp_{10}\}$, then $H_R(V(H)) = \{\wp_1, \wp_2, \wp_3, \wp_4, \wp_5, \wp_9, \wp_{10}\}$. Take the ideal $I = \{\phi, \{\wp_3\}\}$, then $np\mathfrak{cl}^*(V(H)) = V(\mathcal{G})$.

(ii) $Neg_R(V(H)) \supsetneq npExt^*(V(H))$, where Neg_R denotes to the negative region w.r.to R and $npExt^*$ denotes to the nano preexterior w.r.to the ideal I. Because if $V(H) = \{\wp_1, \wp_3, \wp_5, \wp_{10}\}$, then $Neg_R(V(H)) = \{\wp_6, \wp_7, \wp_8\}$. Take $I = \{\phi, \{\wp_3\}\}$, we have $npExt^*(V(H)) = \phi$.

(iii) $np\mathfrak{int}^*(V(H)) \supsetneq L_R(V(H))$. Because if $V(H) = \{\wp_1, \wp_3, \wp_5, \wp_{10}\}$, then $L_R(V(H)) = \{\wp_1, \wp_{10}\}$. Take $I = \{\phi, \{\wp_3\}\}$, we have $np\mathfrak{int}^*(V(H)) = \{\wp_1, \wp_3, \wp_5, \wp_{10}\}$.

7. Conclusion

The use of topology makes difficult human heart research simpler. In applications, topology is very significant. The paper is just the start of a new structure. Many experts will be inspired to contribute to the development of nano ideal topology in the area of mathematical structures of nano approximations. We define a new pre-I-open set and study some of its features. We also compare the pre-I-open set to various other sorts of near nano openness. Some various sorts of nano I-continuity are studied, as are some of their features. We investigate the interaction of nano I-continuity with other forms of nano continuity. As a result, they are particularly important in decision making [21–24]. The provided ideas are quite valuable in practise since they pave the door for additional topological applications based on real-world situations. Finally, we explore the challenges associated with human blood circulation and provide a topological model for determining the validity of various biological solutions.

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