Thai Journal of Mathematics
Volume 20 Number 3 (2022)
Pages 1089–1097

http://thaijmath.in.cmu.ac.th

ISSN 1686-0209

On ps-ro Fuzzy Strongly α-Continuous Function

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Abstract Here we introduce a new class of function between two fuzzy topological spaces termed as *ps-ro* fuzzy strongly α-continuous. It is seen that this class of function is independent of the known notion of fuzzy strongly α-continuity and this motivates to study the concept and use it as a tool to investigate fuzzy topological spaces. Also it is observed that this function is stronger than the existing *ps-ro* fuzzy irresolute, *ps-ro* fuzzy α-irresolute and *ps-ro* fuzzy semicontinuous functions. Along with the characterizations of this function, their several properties for the existence are also established.

MSC: 54A40; 54C08

Keywords: Fuzzy strongly α-continuous; *ps-ro* fuzzy topology; *ps-ro* fuzzy strongly α-continuous

Submission date: 12.08.2020 / Acceptance date: 19.03.2022

1. Introduction and Preliminaries

L.A Zadeh generalized the classical set known as fuzzy set [1], which was further used by C.L Chang to initiate and explore the notion of fuzzy topological space (in short, fts) [2]. Later, various concepts of fuzzy topology have been studied by different researchers. The introduction of *ps-ro* fuzzy topology [3] opened a new direction and tool to study fts and their interrelations. In *ps-ro* fuzzy topology, different type of functions between two fuzzy topological spaces such as *ps-ro* fuzzy continuous [4], [5] *ps-ro* fuzzy semicontinuous [6], *ps-ro* fuzzy irresolute [7], *ps-ro* fuzzy strongly α-irresolute function [8] and *ps-ro* fuzzy α-irresolute [9], *ps-ro* fuzzy semi α-irresolute [10], *ps-ro* fuzzy semi-homeomorphism [11] etc. were introduced and explored. Fuzzy strongly α-continuouity was introduced and studied by R. K. Saraf, S. Mishra and Govindappa Navalagi [12].

The main motive of this paper is to initiate and explore the notion *ps-ro* fuzzy strongly α-continuous function between two fts and study their various properties. Independence of this function with the familiar idea of fuzzy strongly α-continuous is established. It is found that this new function is stronger than *ps-ro* fuzzy semicontinuous functions, *ps-ro* fuzzy irresolute and *ps-ro* fuzzy α-irresolute. Also, along with the characterizations of this function, several properties for the existence of this function are also obtained.

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Published by The Mathematical Association of Thailand.

On a nonempty set P, a fuzzy set U is a function from P into $[0,1] = I$. If g is a mapping between two sets P and Q and U, V are fuzzy sets on P and Q respectively, then $1-U$, $g^{-1}(V)$ and $g(U)$ are fuzzy sets respectively on P, P and Q and are given by $(1-U)(p) = 1-U(p)\ \forall\ p \in P$, $g^{-1}(V)(a) = V(g(a))\ \forall\ a \in P$ and $g(U)(t) = \begin{cases} \sup_{r\in g^{-1}(t)} U(r), when\ g^{-1}(t) \neq \emptyset \\ 0,\ otherwise \end{cases}$. Any fuzzy sets U, V on A, U is subset of V if $U(t) \leq V(t)\ \forall\ t \in A$ and is written as $U \leq V$. A fuzzy set x_r is called fuzzy point whose value is $r \in (0,1] = I_1$ at x, otherwise the value is 0, also it is q-coincident to a fuzzy set U for $r + U(x) > 1$ and is denoted by $x_r q U$ [1].

A fts (U,σ) is a pair where σ is a collection of some fuzzy sets on U with $0,1 \in \sigma$ and finite intersection and arbitrary union of members of σ belongs to σ [2]. A set B is called regular open if $int(clB) = B$, where B is a subset of a topological space [13]. A fuzzy set P defined on fts (U,σ) is fuzzy regular open for $P = int(clP)$ [14]. For a fts (U,σ), the collection $i_\alpha(\sigma) = \{P^\alpha : P \in \sigma$ and $\alpha \in I_1$, where $P^\alpha = \{s \in U; P(s) > \alpha\}$, is a topology on U named as strong α-level topology. P^α for being regular open in $(U, i_\alpha(\sigma))$, $\forall\ \alpha \in I_1$, the fuzzy open set P on fts (U,σ) is called pseudo regular open fuzzy set, the collection of which generates a fuzzy topology on U, named as *ps-ro* fuzzy topology on U, the elements of which are termed as *ps-ro* open fuzzy sets and as usual their complements as *ps-ro* closed fuzzy sets [3], [4], [5]. Fuzzy *ps*-interior of a fuzzy set P in the fts (U,σ), written as *ps-int*(P) is the biggest *ps-ro* open fuzzy set on U that is subset of P also its fuzzy *ps*-closure written as *ps-cl*(P) is the tiniest *ps-ro* closed fuzzy set that contains P [4], [5]. A fuzzy set P on a fts (U,σ) is known as fuzzy α-open [16](resp. fuzzy α-closed [16], *ps-ro* semiopen [6], *ps-ro* semiclosed [6], *ps-ro* α-open [15], *ps-ro* α-closed [15]) on U if $P \leq int(cl(int(P)))$ (resp. $P \geq cl(int(cl(P)))$, $P \leq$ *ps-cl*(*ps-int*(P)), *ps-int*(*ps-cl*(P)) $\leq P$, $P \leq$ *ps-int*(*ps-cl*(*ps-int*(P))), $P \geq$ *ps-cl*(*ps-int*$(cl(P))$)). Let us use the symbols *ps*-(r, U_σ), *ps*-(o, U_σ), *ps*-(s, U_σ) and *ps*-(α, U_σ) respectively to denote the set of all pseudo regular open, *ps-ro* open, *ps-ro* semiopen and *ps-ro* α-open fuzzy sets on the fts (U,σ) and also by *ps*-(r^c, U_σ), *ps*-(o^c, U_σ), *ps*-(s^c, U_σ) and *ps*-(α^c, U_σ) respectively for the set of all pseudo regular closed, *ps-ro* closed, *ps-ro* semiclosed and *ps-ro* α-closed fuzzy sets on the fts (U,σ). Also, P is *ps-ro* fuzzy dense, nowhere *ps-ro* fuzzy dense respectively for *ps-cl*$(P) = 1$ and *ps-int*(*ps-cl*(P)) $= 0$ [9] and P is called *ps-ro* fuzzy semi-nbd of x_t, if we get Q, a *ps-ro* semiopen fuzzy set satisfying $x_t \in Q \leq P$ [6]. Similarly, *ps-ro* fuzzy α-nbd is defined [15]. In the line of *ps-int* and *ps-cl*, similar concepts of *ps*-semi closure(*ps-scl*), *ps*-semi interior(*ps-sint*) and *ps*-α closure(*ps*-α*cl*), *ps*-α interior(*ps*-α*int*) operators are defined [6], [15].

A function f between two fts (U,σ_1) and (V,σ_2) is called fuzzy strongly α-continuous [12] (resp. *ps-ro* fuzzy continuous [5], *ps-ro* fuzzy semicontinuous [6], *ps-ro* fuzzy irresolute [7], *ps-ro* fuzzy α-irresolute [9], *ps-ro* fuzzy semi α-irresolute [10], *ps-ro* fuzzy strongly α-irresolute [8]) if $f^{-1}(Q)$ is fuzzy α-open (resp. member of *ps*-(o, U_{σ_1}), *ps*-(s, U_{σ_1}), *ps*-(s, U_{σ_1}), *ps*-(α, U_{σ_1}), *ps*-(s, U_{σ_1}), *ps*-(o, U_{σ_1})) on U for any fuzzy semiopen set Q (resp. member of *ps*-(o, V_{σ_2}), *ps*-(o, V_{σ_2}), *ps*-(s, V_{σ_2}), *ps*-(α, V_{σ_2}), *ps*-(α, V_{σ_2}), *ps*-(α, V_{σ_2}) on V .

2. *ps-ro* FUZZY STRONGLY α-CONTINUOUS FUNCTION

Definition 2.1. *A mapping g between two fts (U,σ_1) and (V,σ_2) is called ps-ro fuzzy strongly α-continuous if for every $Q \in$ ps-(s, V_{σ_2}), $g^{-1}(Q) \in$ ps-(α, U_{σ_1}).*

We shall now establish relationship of *ps-ro* fuzzy strongly α-continuous function with existing well-known allied concepts.

Example 2.2. *Consider $P = \{e,b,s,d\}$, $Q = \{w,i,m,z\}$ and let us take K, L, M and N the fuzzy sets on P given as $K(r) = 0.1\ \forall\ r \in P$; $L(e) = 0.4, L(b) = 0.4, L(s) = 0.5, L(d) = 0.5$; $M(r) = 0.5\ \forall\ r \in P$ and $N(e) = 0.1, N(b) = 0.1, N(s) = 0.2, N(d) = 0.2$. Let U,V,W and J be the fuzzy sets on Q given as $U(r) = 0.5\ \forall\ r \in Q$; $V(w) = 0.7, V(i) = 0.7, V(m) = 0.8, V(z) = 0.8$; $W(r) = 0.3\ \forall\ r \in Q$ and $J(w) = 0.6, J(i) = 0.6, J(m) = 0.7, J(z) = 0.7$. Then $\sigma_1 = \{0,1,K,L,M,N\}$ and $\sigma_2 = \{0,1,U,V,W,J\}$ are fuzzy topologies in P and Q respectively.*
The open sets in the corresponding strong α-level topological space $(P, i_\alpha(\sigma_1))$ are ϕ, P, K^α, L^α, M^α and N^α, $\forall\ \alpha \in I_1$, where

$$K^\alpha = \begin{cases} P, & \text{if } \alpha < 0.1 \\ \phi, & \text{if } \alpha \geq 0.1 \end{cases},\ L^\alpha = \begin{cases} P, & \text{if } \alpha < 0.4 \\ \{s,d\}, & \text{if } 0.4 \leq \alpha < 0.5 \\ \phi, & \text{if } \alpha \geq 0.5 \end{cases},$$

$$M^\alpha = \begin{cases} P, & \text{if } \alpha < 0.5 \\ \phi, & \text{if } \alpha \geq 0.5 \end{cases} \text{ and } N^\alpha = \begin{cases} P, & \text{if } \alpha < 0.1 \\ \{s,d\}, & \text{if } 0.1 \leq \alpha < 0.2 \\ \phi, & \text{if } \alpha \geq 0.2 \end{cases},$$

L^α and N^α are not regular open on $(P, i_\alpha(\sigma_1))$, as for $\alpha \in [.4,.5)$, $int(cl(K^\alpha)) = P$ and for $\alpha \in [.1,.2)$, $int(cl(N^\alpha)) = P$. Hence, $L, N \notin ps\text{-}(r, P_{\sigma_1})$. As, $int(cl(K^\alpha)) = K^\alpha$ and $int(cl(M^\alpha)) = M^\alpha$, $\forall\ \alpha \in I_1$, K^α and M^α are regular open on $(P, i_\alpha(\sigma_1))$, $\forall\ \alpha \in I_1$. Hence, $ps\text{-}(r, P_{\sigma_1}) = \{0, K, M, 1\}$ which implies that $\{0, 1, K, M\}$ is ps-ro fuzzy topology on P. Again, the open sets in the corresponding strong α-level topological space $(Q, i_\alpha(\sigma_2))$, $\forall\ \alpha \in I_1$ are $\phi, Q, U^\alpha, V^\alpha, W^\alpha$ and J^α, where

$$U^\alpha = \begin{cases} Q, & \text{if } \alpha < 0.5 \\ \phi, & \text{if } \alpha \geq 0.5 \end{cases},\ V^\alpha = \begin{cases} Q, & \text{if } \alpha < 0.7 \\ \{m,z\}, & \text{if } 0.7 \leq \alpha < 0.8 \\ \phi, & \text{if } \alpha \geq 0.8 \end{cases},$$

$$W^\alpha = \begin{cases} Q, & \text{if } \alpha < 0.3 \\ \phi, & \text{if } \alpha \geq 0.3 \end{cases} \text{ and } J^\alpha = \begin{cases} Q, & \text{if } \alpha < 0.6 \\ \{m,z\}, & \text{if } 0.6 \leq \alpha < 0.7 \\ \phi, & \text{if } \alpha \geq 0.7 \end{cases},$$

V^α and J^α are not regular open in $(Q, i_\alpha(\sigma_2))$ for $\alpha \in [.7,.8)$ and $\alpha \in [.6,.7)$ respectively. So, $V, J \notin ps\text{-}(r, Q_{\sigma_2})$. Now, $\forall\ \alpha \in I_1$, U^α and W^α are regular open on $(Q, i_\alpha(\sigma_2))$. Hence, $ps\text{-}(r, Q_{\sigma_2}) = \{0, U, W, 1\}$ which implies that ps-ro fuzzy topology on Q is $\{0, U, W, 1\}$.
Let us take a function h from P to Q by $h(e) = w$, $h(b) = i$, $h(s) = m$ and $h(d) = z$. As, $U, W \in ps\text{-}(o, Q_{\sigma_2})$ we have $U, W \in ps\text{-}(s, Q_{\sigma_2})$. Here, $h^{-1}(U) = M$ and $ps\text{-}int(ps\text{-}cl(ps\text{-}int(h^{-1}(U)))) = M$. Thus, $h^{-1}(U) \leq ps\text{-}int(ps\text{-}cl(ps\text{-}int(h^{-1}(U))))$. So, $h^{-1}(U) \in ps\text{-}(\alpha, P_{\sigma_1})$. Similarly, $h^{-1}(W) \in ps\text{-}(\alpha, P_{\sigma_1})$. Any fuzzy set Z on Q satisfying $S \leq Z \leq ps\text{-}cl(S)$, $Z \in ps\text{-}(s, Q_{\sigma_2})$ where $S \in ps\text{-}(o, Q_{\sigma_2})$. Also, $h^{-1}(Z) \in ps\text{-}(\alpha, P_{\sigma_1})$. Hence, h is ps-ro fuzzy strongly α-continuous from P to Q.
Here, $J \in ps\text{-}(s, Q_{\sigma_2})$ and $int(cl(int(h^{-1}(J)))) = M$, $h^{-1}(J) > int(cl(int(h^{-1}(J))))$. Hence, $h^{-1}(J)$ is not fuzzy α-open on P, showing h is not fuzzy strongly α-continuous from P to Q.

Example 2.3. *Consider $P = \{e,b,c,d\}$, $Q = \{w,i,m,z\}$. Let us take K, L, M and N be the fuzzy sets on P given as $K(e) = 0.5, K(b) = 0.5, K(c) = 0.6, K(d) = 0.6$; $L(r) = 0.2$*

$\forall\ r \in P$; $M(r) = 0.4\ \forall\ r \in P$ and $N(e) = 0.4, N(L) = 0.4, N(c) = 0.5, N(d) = 0.5$. Let U, V, W and J be the fuzzy sets on Q given as $U(r) = 0.5\ \forall\ r \in Q$; $V(w) = 0.3, V(i) = 0.3, V(m) = 0.4, V(z) = 0.4$; $W(r) = 0.3\ \forall\ r \in Q$ and $J(w) = 0.2, J(i) = 0.2, J(m) = 0.3, J(z) = 0.3$. Then, $\sigma_1 = \{0, K, L, M, N, 1\}$ and $\sigma_2 = \{0, 1, U, V, W, J\}$ are fuzzy topologies in P and Q respectively.
In $(P, i_\alpha(\sigma_1))$, $\forall\ \alpha \in I_1$, the open sets are $\phi, P, K^\alpha, L^\alpha, M^\alpha$ and N^α. For $\alpha \in [.5, .6)$, $int(cl(K^\alpha) = P$ and for $\alpha \in [.1, .2)$, $int(cl(N^\alpha)) = P$, proving that K^α and N^α are not regular open on $(P, i_\alpha(\sigma_1))$. Hence, $K, N \notin$ ps-(r, P_{σ_1}). As, $int(cl(L^\alpha)) = L^\alpha$ and $int(cl(M^\alpha)) = M^\alpha$, we have L^α and M^α are regular open on $(P, i_\alpha(\sigma_1))$, $\forall\ \alpha \in I_1$. Hence, ps-$(r, P_{\sigma_1}) = \{0, L, M, 1\}$ and ps-ro fuzzy topology on P is $\{1, L, M, 0\}$. In the similar manner, $V, J \notin$ ps-(r, Q_{σ_2}). Thus, ps-$(r, Q_{\sigma_2}) = \{0, V, W, 1\}$ and ps-ro fuzzy topology on Q is $\{0, U, W, 1\}$.
Let us define a mapping h from P to Q by $h(e) = i$, $h(b) = i$, $h(c) = m$ and $h(d) = z$. Now, $h^{-1}(B)$ is fuzzy α-open on P for any fuzzy semiopen set B on Q. Therefore, h is fuzzy strongly α-continuous from P to Q. But, $U \in$ ps-(s, Q_{σ_2}) and $h^{-1}(U) \notin$ ps-(α, P_{σ_1}), proving that h fails to be ps-ro fuzzy strongly α-continuous from P to Q.

Remark 2.4. *From Example (2.2) and Example (2.3) we can conclude that fuzzy strongly α-continuity and ps-ro fuzzy strongly α-continuity are independent of each other.*

Remark 2.5. *From the definition it is clear a ps-ro fuzzy strongly α-continuous is ps-ro fuzzy α-irresolute but not conversely is given below:*

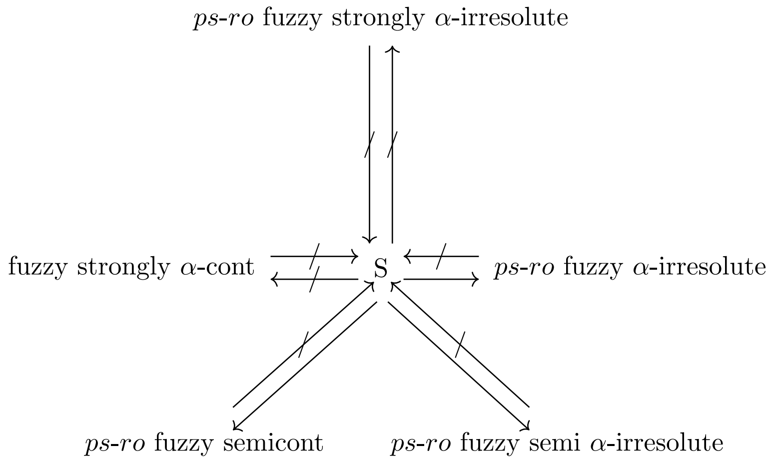
Example 2.6. *Consider $P = \{e, b, s, d\}$, $Q = \{w, i, m, z\}$ and let K, L, M and N be the fuzzy sets on P given as $K(e) = 0.7, K(b) = 0.7, K(s) = 0.6, K(d) = 0.6$; $L(r) = 0.2\ \forall\ r \in P$; $M(r) = 0.5\ \forall\ r \in P$ and $N(e) = 0.3, N(b) = 0.3, N(s) = 0.2, N(d) = 0.2$. Let U, V, W and J be the fuzzy sets on Q given as $U(r) = 0.3\ \forall\ r \in P$; $V(w) = 0.1, V(i) = 0.1, V(m) = 0.2, V(z) = 0.2$; $W(r) = 0.4\ \forall\ r \in P$ and $J(w) = 0.3, J(i) = 0.3, J(m) = 0.4, J(z) = 0.4$. Then, the fuzzy topologies on P and Q are $\sigma_1 = \{0, 1, K, L, M, N\}$ and $\sigma_2 = \{0, 1, U, V, W, J\}$ respectively.*
In $(P, i_\alpha(\sigma_1))$, $\forall\ \alpha \in I_1$, the open sets are $\phi, P, K^\alpha, L^\alpha, M^\alpha$ and N^α. Here, $K, N \notin$ ps-(r, P_{σ_1}) for $\alpha \in [.6, .7)$ and $\alpha \in [.2, .3)$ respectively. So, ps-ro fuzzy topology on P is $\{M, N, U, 0, 1\}$.
Similarly, $V, J \notin$ ps-(r, Q_{σ_2}) for $\alpha \in [.1, .2)$ and $\alpha \in [.3, .4)$ respectively. Thus, ps-ro fuzzy topology on Q is $\{1, U, W, 0\}$.
Now, we define a function h from P to Q by $h(e) = w$, $h(b) = i$, $h(s) = m$, $h(d) = z$. Here, it can be verified that $h^{-1}(B) \in$ ps-(α, P_{σ_1}) for each $B \in$ inps-(α, Q_{σ_2}). Therefore, h is ps-ro fuzzy α-irresolute P to Q. But $h^{-1}(U) \notin$ inps-(α, P_{σ_1}) where $U \in$ inps-(s, Q_{σ_2}) given as $U(r) = 0.6\ \forall\ r \in Q$. Thus, h is not ps-ro fuzzy strongly α-continuous.

Remark 2.7. *As every ps-ro fuzzy α-irresolute is both ps-ro fuzzy semi α-irresolute and ps-ro fuzzy semicontinuous [10], [6], by remark (2.5) we have ps-ro fuzzy strongly α-continuous implies ps-ro fuzzy semi α-irresolute and ps-ro fuzzy semicontinuous functions respectively but the converses are not true.*

Remark 2.8. *From Example (2.2), $W \in$ ps-(α, Q_{σ_2}) but $h^{-1}(W) \notin$ ps-(o, P_{σ_1}), showing h failes to be ps-ro fuzzy strongly α-irresolute, which shows that ps-ro fuzzy strongly α-continuity does not imply ps-ro fuzzy strongly α-irresoluteness.*
Also, every ps-ro fuzzy strongly α-irresolute is ps-ro fuzzy α-irresolute [8], but from the

Remark (2.5), ps-ro fuzzy strongly α-irresolute does not imply ps-ro fuzzy strongly α-continuous. Hence, ps-ro fuzzy strongly α-continuous and ps-ro fuzzy strongly α-irresolute are two independent notions.

Schematic diagram of above proven relations is given below:



Here, S denots *ps-ro* fuzzy strongly α-continuous

Theorem 2.9. *Let (U, σ_1) and (V, σ_2) be two fts and h be a function between them, then the following equivalent conditions hold.*
(a) h is ps-ro fuzzy strongly α-continuous.
(b) For any x_t on U and each $Q \in ps\text{-}(s, V_{\sigma_2})$ and $h(x_t) \in Q$, there exist $P \in ps\text{-}(\alpha, U_{\sigma_1})$ such that $x_t \in P$ and $h(P) \leq Q$.
(c) The preimage of $Q \in ps\text{-}(s^c, V_{\sigma_2})$ belongs to $ps\text{-}(\alpha^c, U_{\sigma_1})$.
(d) $ps\text{-}cl(ps\text{-}int(ps\text{-}cl(h^{-1}(Q)))) \leq h^{-1}(ps\text{-}scl(Q))$, for any fuzzy set Q on V.
(e) $h(ps\text{-}cl(ps\text{-}int(ps\text{-}cl(P)))) \leq ps\text{-}scl(h(P))$, $\forall$ fuzzy set P on U.
(f) $h(ps\text{-}\alpha cl(P)) \leq ps\text{-}scl(h(P))$, for any fuzzy set P on U.
(g) $ps\text{-}\alpha cl(h^{-1}(Q)) \leq h^{-1}(ps\text{-}scl(Q))$, for any fuzzy set Q on V.
(h) $h^{-1}(ps\text{-}sint(Q)) \leq ps\text{-}\alpha int(h^{-1}(Q))$, for every fuzzy set Q on V.

Proof: $(a) \Rightarrow (b)$ For any x_t on U and $Q \in ps\text{-}(s, V_{\sigma_2})$ such that $h(x_t) \in Q$, there exist $h^{-1}(Q) \in ps\text{-}(\alpha, U_{\sigma_1})$ which contains x_t. The result follows by taking $h^{-1}(Q) = P$.
$(b) \Rightarrow (a)$ $Q \in ps\text{-}(s, V_{\sigma_2})$. If $h^{-1}(Q) = 0$, result follows.
Let $h^{-1}(Q) \neq 0$, then $\exists\, x_t$ on U such that $x_t \in h^{-1}(Q)$. So, $\exists\, Z_{x_t} \in ps\text{-}(\alpha, U_{\sigma_1})$ such that $x_t \in Z_{x_t} \leq h^{-1}(Q)$. x_t beings arbitrary, taking union, $h^{-1}(Q) = \vee\{x_t : x_t \in h^{-1}(Q)\} \leq \vee\{Z_{x_t} : x_t \in h^{-1}(Q)\} \leq h^{-1}(Q)$. So, $h^{-1}(Q) = \vee\{Z_{x_t} : x_t \in h^{-1}(Q)\}$ and $h^{-1}(Q) \in ps\text{-}(\alpha, U_{\sigma_1})$. Hence, h is *ps-ro* fuzzy strongly α-continuous.
$(a) \Rightarrow (c)$ For $Q \in ps\text{-}(s^c, V_{\sigma_2})$, $(1-Q) \in ps\text{-}(s, V_{\sigma_2})$ and $h^{-1}(1-Q) \in ps\text{-}(\alpha, U_{\sigma_1})$, which shows that $h^{-1}(Q) \in ps\text{-}(\alpha^c, U_{\sigma_1})$.
$(c) \Rightarrow (a)$ Let $Q \in ps\text{-}(s, U_{\sigma_1})$. Then, $h^{-1}(1-Q) \in ps\text{-}(\alpha^c, U_{\sigma_1})$ and $h^{-1}(Q) \in ps\text{-}(\alpha, U_{\sigma_1})$. Hence, h is *ps-ro* fuzzy strongly α-continuous.
$(c) \Rightarrow (d)$ For every fuzzy set Q on V, $ps\text{-}scl(Q) \in ps\text{-}(s^c, V_{\sigma_2})$. So, $h^{-1}(ps\text{-}scl(Q)) \in ps\text{-}(\alpha^c, U_{\sigma_1})$. Thus, $h^{-1}(ps\text{-}scl(Q)) \geq ps\text{-}cl(ps\text{-}int(ps\text{-}cl(h^{-1}(ps\text{-}scl(Q)))))$. Also, $ps\text{-}cl(ps\text{-}int(ps\text{-}cl(h^{-1}(ps\text{-}scl(Q))))) \geq ps\text{-}cl(ps\text{-}int(ps\text{-}cl(h^{-1}(Q))))$.
This gives, $ps\text{-}cl(ps\text{-}int(ps\text{-}cl(h^{-1}(Q)))) \leq h^{-1}(ps\text{-}scl(Q))$.
$(d) \Rightarrow (e)$ For any fuzzy set P, $h(P)$ is fuzzy set on V and taking $h(P) = Q$ we have,

$P \leq h^{-1}(Q)$. So, $ps\text{-}cl(ps\text{-}int(ps\text{-}cl(h^{-1}(Q)))) \leq h^{-1}(ps\text{-}scl(Q))$. So, $ps\text{-}cl(ps\text{-}int(ps\text{-}cl(P))) \leq ps\text{-}cl(ps\text{-}int(ps\text{-}cl(h^{-1}(Q))) \leq h^{-1}(ps\text{-}scl(Q)) = h^{-1}(ps\text{-}scl(h(P))$. Thus, $h(ps\text{-}cl(ps\text{-}int(ps\text{-}cl(P)))) \leq ps\text{-}scl(h(P))$.
$(e) \Rightarrow (c)$ Let $Q \in ps\text{-}(s^c, V_{\sigma_2})$. Then, $h^{-1}(Q)$ is fuzzy set on U and taking $h^{-1}(Q) = P$, we have $h(P) \leq Q$. Now, $h(ps\text{-}cl(ps\text{-}int(ps\text{-}cl(h^{-1}(Q)))) \leq ps\text{-}scl(h(h^{-1}(Q))) \leq ps\text{-}scl(Q) = Q$. So, $h^{-1}(h(ps\text{-}cl(ps\text{-}int(ps\text{-}cl(h^{-1}(Q))))) \leq h^{-1}(Q)$ and $ps\text{-}cl(ps\text{-}int(ps\text{-}cl(h^{-1}(Q)))) \leq h^{-1}(Q)$, showing that $h^{-1}(Q) \in ps\text{-}(\alpha^c, U_{\sigma_1})$.
$(c) \Rightarrow (f)$ Let P be any fuzzy set on U. Since, $P \leq h^{-1}(h(P))$, $P \leq h^{-1}(ps\text{-}scl(h(P)))$. Since, $ps\text{-}scl(h(P)) \in ps\text{-}(s^c, V_{\sigma_2})$, $h^{-1}(ps\text{-}scl(h(P))) \in ps\text{-}(\alpha^c, U_{\sigma_1})$. Now, $ps\text{-}\alpha cl(P) \leq h^{-1}(ps\text{-}scl(h(P)))$ and $h(ps\text{-}\alpha cl(P)) \leq h(h^{-1}(ps\text{-}scl(h(P)))) \leq ps\text{-}scl(h(P))$. So, $ps\text{-}\alpha cl(P) \leq ps\text{-}scl(h(P))$.
$(f) \Rightarrow (c)$ Let $Q \in ps\text{-}(s^c, V_{\sigma_2})$. Then, $ps\text{-}\alpha cl(h^{-1}(Q)) \leq ps\text{-}scl(h(h^{-1}(Q))) \leq ps\text{-}scl(Q) = Q$ and $ps\text{-}\alpha cl(h^{-1}(Q)) \leq h^{-1}(Q)$. Now, $h^{-1}(Q) \leq ps\text{-}\alpha cl(h^{-1}(Q))$. So, $h^{-1}(Q) = ps\text{-}\alpha cl(h^{-1}(Q))$ which shows that $h^{-1}(Q) \in ps\text{-}(\alpha^c, V_{\sigma_2})$.
$(f) \Rightarrow (g)$ Let us take $P = h^{-1}(Q)$ corresponding to fuzzy set Q on V. Then, $h(ps\text{-}\alpha cl(h^{-1}(Q))) \leq ps\text{-}scl(h(h^{-1}(Q))) \leq ps\text{-}scl(Q)$. Now, $ps\text{-}\alpha cl(h^{-1}(Q)) \leq h^{-1}(h(ps\text{-}\alpha cl(h^{-1}(Q)))) \leq h^{-1}(ps\text{-}scl(Q))$. So, $ps\text{-}\alpha cl(h^{-1}(Q))) \leq h^{-1}(ps\text{-}scl(Q))$.
$(g) \Rightarrow (f)$ Corresponding to every fuzzy set P on U, taking $h(P) = Q$, $ps\text{-}\alpha cl(h^{-1}(h(P))) \leq h^{-1}(ps\text{-}scl(h(P)))$. So, $ps\text{-}\alpha cl(P) \leq ps\text{-}\alpha cl(h^{-1}(h(P))) \leq h^{-1}(ps\text{-}scl(h(P)))$. Thus, $h(ps\text{-}\alpha cl(P)) \leq ps\text{-}scl(h(P))$.
$(a) \Rightarrow (h)$ Corresponding to every fuzzy set Q on V, as $ps\text{-}sint(Q) \in ps\text{-}(s, V_{\sigma_2})$ and so, $h^{-1}(ps\text{-}sint(Q)) \in ps\text{-}(\alpha, U_{\sigma_1})$. Thus, $h^{-1}(ps\text{-}sint(Q)) = ps\text{-}\alpha int(h^{-1}(ps\text{-}sint(Q))) \leq ps\text{-}\alpha int(h^{-1}(Q))$. Therefore, $h^{-1}(ps\text{-}sint(Q)) \leq ps\text{-}\alpha int(h^{-1}(Q))$.
$(h) \Rightarrow (a)$ Consider $Q \in ps\text{-}(s, V_{\sigma_2})$. Then, $ps\text{-}sint(Q) = Q$, $h^{-1}(ps\text{-}sint(Q)) \leq ps\text{-}\alpha int(h^{-1}(Q))$. So, $h^{-1}(Q) \leq ps\text{-}\alpha int(h^{-1}(Q))$. Also, $ps\text{-}\alpha int(h^{-1}(Q)) \leq h^{-1}(Q)$. Therefore, $h^{-1}(Q) = ps\text{-}\alpha int(h^{-1}(Q))$, which shows that $h^{-1}(Q) \in ps\text{-}(\alpha, U_{\sigma_1})$. Hence, h is *ps-ro* fuzzy strongly α-continuous.

Theorem 2.10. *A bijective mapping h between fts (U, σ_1) and (V, σ_2) is ps-ro fuzzy strongly α-continuous iff for each fuzzy set P on U, $ps\text{-}sint(h(P)) \leq h(ps\text{-}\alpha int(P))$.*
Proof: Let $h : U \to V$ be a bijective *ps-ro* fuzzy strongly α-continuous. Let P be any fuzzy set on U. $h(P)$ is also a fuzzy set on V and $ps\text{-}sint(h(P)) \in ps\text{-}(s, V_{\sigma_2})$. Again, $h^{-1}(ps\text{-}sint(h(P))) \in ps\text{-}(\alpha, U_{\sigma_1})$ and by Theorem (2.9), $h^{-1}(ps\text{-}sint(h(P))) \leq ps\text{-}\alpha int(h^{-1}(h(P)))$. Since, h is one-to-one, $ps\text{-}\alpha int(h^{-1}(h(P))) = ps\text{-}\alpha int(P)$, $h^{-1}(ps\text{-}sint(h(P))) \leq ps\text{-}\alpha int(P)$. Again, h being onto, $ps\text{-}sint(h(P)) = h(h^{-1}(ps\text{-}sint(h(P)))) \leq h(ps\text{-}\alpha int(P))$. Hence, $ps\text{-}sint(h(P)) \leq h(ps\text{-}\alpha int(P))$. Conversely, let $Q \in ps\text{-}(s, V_{\sigma_2})$. Then, $ps\text{-}sint(Q) = Q$ and $h^{-1}(Q)$ is any fuzzy set on U. As, h is onto, $h(h^{-1}(Q)) = Q$ and $Q = ps\text{-}sint(Q) = ps\text{-}sint(h(h^{-1}(Q)))$. Thus, $ps\text{-}sint(h(h^{-1}(Q))) \leq h(ps\text{-}\alpha int(h^{-1}(Q)))$ which gives, $Q \leq h(ps\text{-}\alpha int(h^{-1}(Q)))$. Since, h is one-to-one, $ps\text{-}\alpha int(h^{-1}(Q)) = h^{-1}(h(ps\text{-}\alpha int(h^{-1}(Q))))$. Therefore, $h^{-1}(Q) \leq h^{-1}(h(ps\text{-}\alpha int(h^{-1}(Q)))) = ps\text{-}\alpha int(h^{-1}(Q))$. As, $ps\text{-}\alpha int(h^{-1}(Q)) \leq h^{-1}(Q)$, $h^{-1}(Q) = ps\text{-}\alpha int(h^{-1}(Q))$, thus $h^{-1}(Q) \in ps\text{-}(\alpha, U_{\sigma_1})$. Hence, h is *ps-ro* fuzzy strongly α-continuous.

Theorem 2.11. *Let (U, σ_1), (V, σ_2) and (W, σ_3) be fts and $h : U \to V$, $g : V \to W$ be two functions then the following holds:*
(a) If h is ps-ro fuzzy α-irresolute and g is ps-ro strongly α-continuous then $g \circ h$ is ps-ro fuzzy strongly α-continuous.
(b) If h is ps-ro strongly α-continuous and g is ps-ro fuzzy irresolute then $g \circ h$ is ps-ro

fuzzy strongly α-continuous.
Proof: (a) For $P \in ps\text{-}(s, W_{\sigma_3})$, $g^{-1}(P) \in ps\text{-}(\alpha, V_{\sigma_2})$. Now, $(g \circ h)^{-1}(P) = h^{-1}(g^{-1}(P))$. Since, $g^{-1}(P) \in ps\text{-}(\alpha, V_{\sigma_2})$, $h^{-1}(g^{-1}(P)) \in ps\text{-}(\alpha, U_{\sigma_1})$ i.e., $(g \circ h)^{-1}(P) \in ps\text{-}(\alpha, U_{\sigma_1})$. So, $g \circ h$ is *ps-ro* fuzzy strongly α-continuous.
(b) Corresponding to $P \in ps\text{-}(s, W_{\sigma_3})$, $g^{-1}(P) \in ps\text{-}(s, V_{\sigma_2})$. Now, $(g \circ h)^{-1}(P) = h^{-1}(g^{-1}(P))$. Since, $g^{-1}(P) \in ps\text{-}(s, V_{\sigma_2})$, $(g \circ h)^{-1}(P) \in ps\text{-}(\alpha, U_{\sigma_1})$. Hence, $g \circ h$ is *ps-ro* fuzzy strongly α-continuous.

Theorem 2.12. *Let (U, σ_1), (V, σ_2) and (W, σ_3) be fts and $h : U \to V$, $g : V \to W$ be two functions. If h is ps-ro fuzzy strongly α-continuous and g is ps-ro fuzzy semicontinuous, then $g \circ h$ is ps-ro fuzzy α-continuous.*
Proof: Let $Q \in ps\text{-}(o, W_{\sigma_3})$ then $g^{-1}(Q) \in ps\text{-}(s, V_{\sigma_2})$. Now, $(g \circ h)^{-1}(Q) = h^{-1}(g^{-1}(Q))$. Since, $g^{-1}(Q) \in ps\text{-}(s, V_{\sigma_2})$, $(g \circ h)^{-1}(Q) \in ps\text{-}(\alpha, U_{\sigma_1})$. Hence, $g \circ h$ is *ps-ro* fuzzy α-continuous.

Corollary 2.13. *Let (U, σ_1), (V, σ_2) and (W, σ_3) be fts and $h : U \to V$, $g : V \to W$ be two functions. If h is ps-ro fuzzy strongly α-continuous and g is ps-ro fuzzy continuous, then $g \circ h$ is ps-ro fuzzy α-continuous.*

Theorem 2.14. *Let U_i and V_i ; $i = 1, 2$ be fts such that U_1 is a product related to U_2 and V_1 is a product related to V_2. If $h_1 \times h_2 : U_1 \times U_2 \to V_1 \times V_2$ is ps-ro fuzzy strongly α-continuous, then $h_1 : U_1 \to U_2$ and $h_2 : V_1 \to V_2$ are ps-ro fuzzy strongly α-continuous.*
Proof: Let Q be any *ps-ro* semiopen fuzzy set on V_1. Then, $Q \times 1$ is *ps-ro* semiopen fuzzy set on $V_1 \times V_2$ as V_1 is a product related to V_2. Since, $h_1 \times h_2$ is *ps-ro* fuzzy strongly α-continuous, $(h_1 \times h_2)^{-1}(Q \times 1)$ is *ps-ro* α-open fuzzy set on U i.e., $(h_1 \times h_2)^{-1}(Q \times 1) \leq$ *ps-int*(*ps-cl*(*ps-int*($(h_1 \times h_2)^{-1}(Q \times 1)$))). Now, $(h_1 \times h_2)^{-1}(Q \times 1) = h_1^{-1}(Q) \times 1$. As U_1 is a product related to U_2, we have *ps-int*(*ps-cl*(*ps-int*($h_1^{-1}(Q) \times 1$))) $=$ *ps-int*(*ps-cl*(*ps-int*($h_1^{-1}(Q)$))) $\times 1 \geq h_1^{-1}(Q) \times 1$. So, $h_1^{-1}(Q) \times 1 \leq$ *ps-int*(*ps-cl*(*ps-int*($h_1^{-1}(Q)$))) $\times 1$. $h_1^{-1}(Q)$ is *ps-ro* α-open fuzzy set on U_1. Hence, h_1 is *ps-ro* fuzzy strongly α-continuous. Similarly, it can be shown that h_2 is *ps-ro* fuzzy strongly α-continuous.

Theorem 2.15. *Let a function h between fts (U, σ_1) and (V, σ_2) be ps-ro fuzzy strongly α-continuous, then ps-cl(ps-int(ps-cl($h^{-1}(Q)$))) $\leq h^{-1}$(ps-cl(Q)) and ps-αcl($h^{-1}(Q)$) $\leq h^{-1}$(ps-cl(Q)), for every fuzzy set Q on V.*
Proof: Let h be *ps-ro* fuzzy strongly α-continuous and Q be any fuzzy set on V. Then, *ps-cl*(Q) $\in ps\text{-}(s^c, V_{\sigma_2})$ and h^{-1}(*ps-cl*(Q)) $\in ps\text{-}(\alpha^c, U_{\sigma_1})$. Thus, *ps-cl*(*ps-int*(*ps-cl*(h^{-1}(*ps-cl*(Q))))) $\leq h^{-1}$(*ps-cl*(Q). Since, $P \leq$ *ps-cl*(P) for any fuzzy set P on U, *ps-cl*(*ps-int*(*ps-cl*($h^{-1}(Q)$))) $\leq h^{-1}$(*ps-cl*(Q))). Again, for any fuzzy set Q on V, $Q \leq$ *ps-cl*(Q) and $h^{-1}(Q) \leq h^{-1}$(*ps-cl*(Q)). So, *ps-αcl*($h^{-1}(Q)$) $\leq$ *ps-αcl*(h^{-1}(*ps-cl*(Q))) $= h^{-1}$(*ps-cl*(Q)) (as h^{-1}(*ps-cl*(Q)) $\in ps\text{-}(\alpha^c, U_{\sigma_1})$. Hence, *ps-$\alpha$cl*($h^{-1}(Q)$) $\leq h^{-1}$(*ps-cl*(Q)).

Theorem 2.16. *Let a function h between fts (U, σ_1) and (V, σ_2) be ps-ro fuzzy strongly α-continuous, then h(ps-cl(ps-int(ps-cl(P)))) $\leq$ ps-cl($h(P)$) and h(ps-αcl(P)) $\leq$ ps-cl($h(P)$), for every fuzzy set P on U.*
Proof: For any fuzzy set P on U, taking $h(P) = Q$, $P \leq h^{-1}(Q)$. From Theorem (2.15), *ps-cl*(*ps-int*(*ps-cl*(P))) $\leq$ *ps-cl*(*ps-int*(*ps-cl*($h^{-1}(Q)$))) $\leq h^{-1}$(*ps-cl*(Q)) $= h^{-1}$(*ps-cl*($h(P)$)). So, h(*ps-cl*(*ps-int*(*ps-cl*(P)))) $\leq h(h^{-1}$(*ps-cl*($h(P)$))) $\leq$ *ps-cl*($h(P)$) and h(*ps-cl*(*ps-int*(*ps-cl*(P)))) $\leq$ *ps-cl*($h(P)$). Again, $P \leq h^{-1}(h(P)) \leq h^{-1}$(*ps-cl*($h(P)$)). Now,

$ps\text{-}cl(h(P)) \in ps\text{-}(s^c, V_{\sigma_2})$, $h^{-1}(ps\text{-}cl(h(P))) \in ps\text{-}(\alpha^c, U_{\sigma_1})$ and $ps\text{-}\alpha cl(P)) \leq h^{-1}(ps\text{-}cl(h(P))$, $h(ps\text{-}\alpha cl(P)) \leq h(h^{-1}(ps\text{-}cl(h(P)))) \leq ps\text{-}cl(h(P))$. So, $h(ps\text{-}\alpha cl(P))) \leq ps\text{-}cl(h(P))$.

Theorem 2.17. *Let a function h between fts (U, σ_1) and (V, σ_2) be ps-ro fuzzy strongly α-continuous, then $h^{-1}(ps\text{-}int(Q)) \leq ps\text{-}\alpha int(h^{-1}(Q))$, for every fuzzy set Q on V.*
Proof: Let a function h be *ps-ro* fuzzy strongly α-continuous and Q be a fuzzy set on V. Then, $ps\text{-}int(Q) \in ps\text{-}(s, V_{\sigma_2})$, $h^{-1}(ps\text{-}int(Q)) \in ps\text{-}(\alpha, U_{\sigma_1})$. Now, $h^{-1}(ps\text{-}int(Q)) = ps\text{-}\alpha int(h^{-1}(ps\text{-}int(Q))) \leq ps\text{-}\alpha int(h^{-1}(Q))$, since $ps\text{-}int(Q) \leq Q$. Hence, $h^{-1}(ps\text{-}int(Q)) \leq ps\text{-}\alpha int(h^{-1}(Q))$.

Lemma 2.18. *: Let $g : U \to U \times V$ be the graph of a function $h : U \to V$ .i.e. $g(u) = (u, h(u)), \forall u \in U$. If P and Q are fuzzy sets on U and V, then, $g^{-1}(P \times Q) = P \wedge h^{-1}(Q)$.*

Theorem 2.19. *Let h be a function between fts (U, σ_1) and (V, σ_2). h is ps-ro fuzzy strongly α-continuous if the graph $g : U \to U \times V$ is ps-ro fuzzy strongly α-continuous.*
Proof: Let $Q \in ps\text{-}(s, V_{\sigma_2})$, by Lemma (2.18), $h^{-1}(Q) = 1 \wedge h^{-1}(Q) = g^{-1}(1 \times Q)$. As, $1 \in ps\text{-}(s, U_{\sigma_1})$ and $Q \in ps\text{-}(s, V_{\sigma_2})$ we have $(1 \times Q)$ is *ps-ro* semiopen fuzzy set on $U \times V$. Now, $g^{-1}(1 \times Q) \in ps\text{-}(\alpha, U_{\sigma_1})$ and $h^{-1}(Q) \in ps\text{-}(\alpha, U_{\sigma_1})$. Hence, h is *ps-ro* fuzzy strongly α-continuous.

Theorem 2.20. *Let (U, σ_1) and (V, σ_2) be two fts and if a function h from (U, σ_1) to (V, σ_2) be ps-ro fuzzy strongly α-continuous then for any nowhere ps-ro fuzzy dense set P on V, preimage of $P \in ps\text{-}(\alpha^c, U_{\sigma_1})$.*
Proof: For any any nowhere *ps-ro* fuzzy dense set P on V, $1 - ps\text{-}int(ps\text{-}cl(P)) = 1$. So, $ps\text{-}cl(1-(ps\text{-}cl(P))) = 1$ and $ps\text{-}cl(ps\text{-}int(1-P)) = 1$ Hence, $1-P \leq ps\text{-}cl(ps\text{-}int(1-P))$, proving that $(1 - P) \in ps\text{-}(s, V_{\sigma_2})$. Now, $h^{-1}(1 - P) = 1 - h^{-1}(P) \in ps\text{-}(\alpha, U_{\sigma_1})$. Thus, $h^{-1}(P) \in ps\text{-}(\alpha^c, U_{\sigma_1})$.

Theorem 2.21. *For a function h between fts (U, σ_1) and (V, σ_2), the following are equivalent:*
(a) h is ps-ro fuzzy strongly α-continuous.
(b) For each x_t on U, the preimage of each ps-ro fuzzy semi-nbd Q of $h(x_t)$ on V is a ps-ro fuzzy α-nbd of x_t on U.
(c) For each x_t on U and each ps-ro fuzzy semi-nbd Q of $h(x_t)$ on V, $\exists$ a ps-ro fuzzy α-nbd P of x_t on U such that $h(P) \leq Q$.
(d) For each x_t on U and each $Q \in ps\text{-}(s, V_{\sigma_2})$ with $h(x_t) \leq Q$, $\exists$ $P \in ps\text{-}(\alpha, U_{\sigma_1})$ with $x_t \leq P$ and $h(P) \leq Q$.
Proof: $(a) \Rightarrow (b)$ For x_t on U and *ps-ro* fuzzy semi-nbd Q of $h(x_t)$ on V, $\exists$ $W \in ps\text{-}(s, V_{\sigma_2})$ with $h(x_t) \leq W \leq Q$. Now, $h^{-1}(W) \in ps\text{-}(\alpha, U_{\sigma_1})$ with $x_t \leq h^{-1}(W)$. Then, $x_t \leq h^{-1}(W) \leq h^{-1}(Q)$, which shows that $h^{-1}(Q)$ is a *ps-ro* α-nbd of x_t on U.
$(b) \Rightarrow (c)$ For x_t on U and *ps-ro* fuzzy semi-nbd Q of $h(x_t)$ on V, $h^{-1}(Q)$ is a *ps-ro* fuzzy α-nbd of x_t on U. Let $h^{-1}(Q) = P$. Then $h(h^{-1}(Q)) = h(P)$. Since, $h(h^{-1}(Q)) \leq Q$, $h(P) \leq Q$.
$(c) \Rightarrow (d)$ For x_t on U and $Q \in ps\text{-}(s, V_{\sigma_2})$ such that $h(x_t) \leq Q$, $\exists$ a *ps-ro* fuzzy α-nbd W of x_t on U such that $h(W) \leq Q$. So, $\exists$ $P \in ps\text{-}(\alpha, U_{\sigma_1})$ such that $x_t \leq P \leq W$, which gives $h(x_t) \leq h(P) \leq h(W) \leq Q$. Hence, $h(P) \leq Q$.
$(d) \Rightarrow (a)$ Let $Q \in ps\text{-}(s, V_{\sigma_2})$ x_t be a fuzzy point on $h^{-1}(Q)$. Then, $x_t \leq h^{-1}(Q)$ and $h(x_t) \leq h(h^{-1}(Q)) \leq Q$. Now, $\exists$ $P \in ps\text{-}(\alpha, U_{\sigma_1})$ such that $x_t \leq P$ and $h(P) \leq Q$ which gives $P \leq h^{-1}(Q)$. As, $P \in ps\text{-}(\alpha, U_{\sigma_1})$, $P \leq ps\text{-}int(ps\text{-}cl(ps\text{-}int(P)))$. So,

$x_t \leq P \leq ps\text{-}int(ps\text{-}cl(ps\text{-}int(P))) \leq ps\text{-}int(ps\text{-}cl(ps\text{-}int(h^{-1}(Q))))$. Now, x_t being arbitrary, taking union, $h^{-1}(Q) = \vee\{x_t : x_t \in h^{-1}(Q)\} \leq \vee\{P : x_t \in h^{-1}(Q)\} \leq h^{-1}(Q)$. So, $\vee\{P : x_t \in h^{-1}(Q)\} = h^{-1}(Q)$. Thus, $h^{-1}(Q) \in ps\text{-}(\alpha, U_{\sigma_1})$, proving h is *ps-ro* fuzzy strongly α-continuous.

Acknowledgements

We sincerely thank the referees for their valuable comments and suggestions on the manuscript.

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