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Intuitionistic Fuzzy F-Normed Space

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Abstract In this paper, we introduce intuitionistic fuzzy F-norm and study its relations with α-norms. Intuitionistic fuzzy F-norm is a generalization of intuitionistic fuzzy pseudo norm. We show that intuitionistic fuzzy F-norm furnished with a topology satisfying some conditions is a topological vector space. We also show that in an intuitionistic fuzzy F-space, an open ball with center at origin is balanced, absorbing and convex. By illustrative examples we also explain that the topology induced by F-norm and the topology induced by intuitionistic fuzzy F-norm are the same.

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1. Introduction

In the study of topological vector spaces J. van Neumann [1] has first studied the notion of pseudo metric, which can be defined in locally convex topological spaces. It is real valued function and has some properties of norm. D.H. Hyers proposed a pseudo norm [2], which is generalization of the pseudo metric[1] and previously proposed pseudo norm [3]. Fréchet spaces are the locally convex metrizable topological vector spaces, but F-spaces are complete metrizable topological vector spaces. An F-norm can be the topology of an F-space.

Definition 1.1. [4] A pseudo norm on a vector space V over the field $\mathbb{K}$ (field of real/complex numbers) is a real function $\|\cdot\| : V \to \mathbb{R}$ defined on V such that for any $v_1, v_2 \in V$ and for all $c \in \mathbb{K}$ with $|c| \leq 1$,
$(PN.1)$ $\|v_1\| \geq 0$
$(PN.2)$ $\|v_1\| = 0$ if and only if $v_1 = \theta$
$(PN.3)$ $\|c\,v_1\| \leq \|v_1\|$
$(PN.4)$ $\|v_1 + v_2\| \leq \|v_1\| + \|v_2\|$.

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Definition 1.2. [4] Let τ be a topology on a vector space V over a field $\mathbb{K}$(field of real/complex numbers). The pair (V, τ) is said to be a topological vector space if the following two axioms are satisfied:
$(TVS.1)$ $(v_1, v_2) \to v_1 + v_2$ is continuous on $V \times V$ into V,
$(TVS.2)$ $(k, v_1) \to kv_1$ is continuous on $\mathbb{K} \times V$ into V.

Clearly, a given pseudo norm on V, generated by a translation-invariant metric $d(v_1, v_2) = \|v_1 - v_2\|$, defines a topology on V satisfying $(TVS.1)$; but $(TVS.2)$ may not be satisfied.

Definition 1.3. [4] Suppose the pseudo norm satisfies the following two extra conditions:
(PN.5) $k_n \to 0 \Rightarrow \|k_n v\| \to 0, \ \forall \, v \in V.$
(PN.6) $\|v_n\| \to 0 \Rightarrow \|kv_n\| \to 0, \ \forall \, k \in \mathbb{K}.$
then $(TVS.2)$ is satisfied and a topological vector space is obtained.
The pseudo norm which satisfies (PN.5) and (PN.6) is called F-norm on V.

After Zadeh's introduction of fuzzy set [5]; many research investigations, by mathematicians, engineers, scientists, computer and management scientists all over the world, have been made in fuzzy set theory and its applications. The notion of fuzzy norm was studied by many mathematicians, namely, A. Katsaras [6, 7], C. Felbin [8], Cheng-Moderson [9], Bag-Samanta [10, 11], I. Golet [12], C. Alegre and S. Romaguera [13], etc. As a generalization of the works of Bag-Samanta [10, 11], S. Nădăban introduced fuzzy pseudo-norm [14].
An extension of fuzzy sets is known as intuitionistic fuzzy sets, developed by Krassimir T. Atanassov in 1986 [15]. R. Saadati and J.H. Park [16] defined intuitionistic fuzzy norm on a linear space. The study of intuitionistic fuzzy norm received a lot of interest in the past years.
In this paper, a more extensive concept than intuitionistic fuzzy pseudo norm [17–21] has been introduced as intuitionistic fuzzy F-norm. In section 3, we have introduced intuitionistic fuzzy F-norm as a generalization of intuitionistic fuzzy pseudo norm and studied decomposition theorems of intuitionistic fuzzy F-norm into two family of α-norms. We have also established that intuitionistic fuzzy F-normed spaces are intuitionistic fuzzy topological vector spaces. In section 4, we have investigated some of the basic properties of topological vector space in intuitionistic fuzzy F-normed space.

2. Preliminaries

Definition 2.1. [17] Let V be linear space over the field $\mathbb{K}$ (field of real/complex numbers). An intuitionistic fuzzy subset (μ, ν) of $(V \times \mathbb{R}, V \times \mathbb{R})$ is said to be an intuitionistic fuzzy pseudo norm on V if $\forall \, v_1, v_2 \in V$
(IFP.1) $\forall s \in \mathbb{R}, \ \mu(v_1, s) + \nu(v_1, s) \leq 1;$
(IFP.2) $\forall s \in \mathbb{R}$ with $s \leq 0$, $\mu(v_1, s) = 0;$
(IFP.3) $\forall s \in \mathbb{R}^+$, $\mu(v_1, s) = 1$ if and only if $v_1 = \theta;$
(IFP.4) $\forall s \in \mathbb{R}^+$, $\mu(cv_1, s) \geq \mu(v_1, s)$ if $|c| \leq 1, \forall c \in \mathbb{K};$
(IFP.5) $\mu(v_1 + v_2, s + t) \geq \min\{\mu(v_1, s), \mu(v_2, t)\}, \ \forall s, t \in \mathbb{R}^+;$
(IFP.6) $\lim_{s \to \infty} \mu(v_1, s) = 1;$
(IFP.7) if there exists $\alpha \in (0, 1)$ such that $\mu(v_1, s) > \alpha, \forall s \in \mathbb{R}^+$ then $v_1 = \theta;$
(IFP.8) $\forall v_1 \in V$, $\mu(v_1, \cdot)$ is left continuous on $\mathbb{R};$
(IFP.9) $\forall s \in \mathbb{R}$ with $s \leq 0$, $\nu(v_1, s) = 1;$
(IFP.10) $\forall s \in \mathbb{R}^+$, $\nu(v_1, s) = 0$ if and only if $v_1 = \theta;$

(IFP.11) $\forall s \in \mathbb{R}^+$, $\nu(cv_1, s) \leq \nu(v_1, s)$ if $|c| \leq 1$, $\forall c \in \mathbb{K}$;
(IFP.12) $\nu(v_1 + v_2, s + t) \leq \max\{\nu(v_1, s), \nu(v_2, t)\}$, $\forall s, t \in \mathbb{R}^+$;
(IFP.13) $\lim_{s \to \infty} \nu(v_1, s) = 0$;
(IFP.14) if there exists $\alpha \in (0,1)$ such that $\nu(v_1, s) < \alpha$, $\forall s \in \mathbb{R}^+$ then $v_1 = \theta$;
(IFP.15) $\forall v_1 \in V$, $\nu(v_1, \cdot)$ is left continuous on $\mathbb{R}$.

Here (V, μ, ν) is called intuitionistic fuzzy pseudo normed linear space.

Remark 2.2. [22] If $*$ is a t-norm and *diamond* is a t-co-norm then $a * a = a$ and $a \diamond a = a$, $\forall a \in [0,1]$ is satisfied only when $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$.

Definition 2.3. [17] A sequence $\{a_n\}_{n \in \mathbb{N}}$ in an intuitionistic fuzzy pseudo normed linear space (V, μ, ν) is said to be α-convergent if there exists $a \in V$ such that $\forall s > 0$, $\lim_{n \to \infty} \mu(a_n - a, s) > \alpha$, $\lim_{n \to \infty} \nu(a_n - a, s) < \alpha$. Then a is called α-limit of a_n in (V, μ, ν).

From [16] and Remark 2.2 the definition of intuitionistic fuzzy metric space is as follows:

Definition 2.4. [16] Let V be an arbitrary set. An intuitionistic fuzzy subset $(\mathcal{M}, \mathcal{N})$ of $(V \times V \times \mathbb{R}^+, V \times V \times \mathbb{R}^+)$ is said to be intuitionistic fuzzy metric on V, $\forall v_1, v_2, v_3 \in V$; $s, t > 0$ if the following conditions hold: $\forall v_1, v_2, v_3 \in V$ and for each $s, t > 0$,
(ifm.I) $\mathcal{M}(v_1, v_2, s) + \mathcal{N}(v_1, v_2, s) \leq 1$;
(ifm.II) $\mathcal{M}(v_1, v_2, 0) = 0$;
(ifm.III) $\mathcal{M}(v_1, v_2, s) = 1$ iff. $v_1 = v_2$;
(ifm.IV) $\mathcal{M}(v_1, v_2, s) = \mathcal{M}(v_2, v_1, s)$;
(ifm.V) $\mathcal{M}(v_1, v_3, s + t) \geq \min\{\mathcal{M}(v_1, v_2, s), \mathcal{M}(v_2, v_3, t)\}$;
(ifm.VI) $\mathcal{M}(v_1, v_2, \cdot) : \mathbb{R}^+ \to [0,1]$ is left-continuous;
(ifm.VII) $\mathcal{N}(v_1, v_2, 0) = 1$;
(ifm.VIII) $\mathcal{N}(v_1, v_2, s) = 0$ iff. $v_1 = v_2$;
(ifm.IX) $\mathcal{N}(v_1, v_2, s) = \mathcal{N}(v_2, v_1, s)$;
(ifm.X) $\mathcal{N}(v_1, v_3, s + t) \leq \max\{\mathcal{N}(v_1, v_2, s), \mathcal{N}(v_2, v_3, t)\}$;
(ifm.XI) $\mathcal{N}(v_1, v_2, \cdot) : \mathbb{R}^+ \to [0,1]$ is left-continuous.

3. Intuitionistic fuzzy F-space

Definition 3.1. Let V be a linear space over the field $\mathbb{K}$ (field of real/complex numbers) and $\{v_n\}$ and $\{k_n\}$ be two sequences in V and $\mathbb{K}$ respectively. An Intuitionistic fuzzy pseudo norm (μ, ν) on V is said to be an intuitionistic fuzzy F-norm if $\forall v \in V$ and $\forall s \in \mathbb{R}^+$ the following conditions satisfy :
(IFP.16) $k_n \to 0 \Rightarrow \lim_{n \to \infty} \mu(k_n v, s) = 1$.
(IFP.17) $\lim_{n \to \infty} \mu(v_n, s) > \alpha \Rightarrow \lim_{n \to \infty} \mu(k v_n, s) > \alpha$, $\forall k \in \mathbb{K}$, $\alpha \in (0,1)$.
(IFP.18) $k_n \to 0 \Rightarrow \lim_{n \to \infty} \nu(k_n v, s) = 0$.
(IFP.19) $\lim_{n \to \infty} \nu(x_n, t) < \alpha \Rightarrow \lim_{n \to \infty} \nu(k v_n, s) < \alpha$, $\forall k \in \mathbb{K}$, $\alpha \in (0,1)$.
Here (V, μ, ν) is called intuitionistic fuzzy F-normed linear space.

Remark 3.2. An intuitionistic fuzzy F-norm is an intuitionistic fuzzy pseudo norm.

Example 3.3. Let $(V, \|\cdot\|)$ be a F-normed linear space. Define $\mu, \nu : V \times \mathbb{R} \to [0,1]$ by

$$\mu(v, s) = \begin{cases} 1 & \text{if } s > \|v\|, \\ 0 & \text{if } s \leq \|v\|. \end{cases}$$

$$\nu(v,s)=\begin{cases}0 & \text{if } \ s > \|\,v\,\|,\\ 1 & \text{if } \ s \le \|\,v\,\|.\end{cases}$$

Then $(\mu,\,\nu)$ is an intuitionistic fuzzy F-norm on V and (V,μ,ν) is an intuitionistic fuzzy F-normed linear space.

Proof. (IFP.16,IFP.18) If $k_n \to 0$ then $\|k_n v\| \to 0$, [by Definition 1.3].
Therefore, $\exists\, m_0 \in \mathbb{N}$ such that $\|k_n v\| < s,\ \forall\, n \ge m_0,\ \forall\, s > 0$.
Thus $\mu(k_n v, s) = 1$ and $\nu(k_n v, s) = 0,\ \forall\, n \ge m_0$.
Hence $\lim\limits_{n\to\infty} \mu(k_n v, s) = 1$ and $\lim\limits_{n\to\infty} \nu(k_n v, s) = 0$.
(IFP.17,IFP.19) If $\lim\limits_{n\to\infty} \mu(v_n, s) > \alpha$ and $\lim\limits_{n\to\infty} \nu(v_n, s) < \alpha$, then $\exists m_0 \in \mathbb{N}$ such that $\mu(\,v_n, s\,) > \alpha$ and $\nu(\,v_n, s\,) < \alpha,\ \forall\, n \ge m_0,\ \forall\, s > 0$.
Therefore, $\mu(v_n, s) = 1$ and $\nu(v_n, s) = 0,\ \forall\, n \ge m_0, \forall\, s > 0$.
Hence $\|v_n\| < s,\ \forall\, n \ge m_0,\ \forall\, s > 0$. Therefore $\|v_n\| \to 0$ as $n \to \infty$, hence $\|k\, v_n\| \to 0$ as $n \to \infty,\ \forall\, k \in \mathbb{K}$, [by Definition 1.3].
Thus $\exists\, m_0 \in \mathbb{N}$ such that $\|k\, v_n\| < s,\ \forall\, n \ge m_0,\ \forall\, s > 0$. Therefore $\exists\, m_0 \in \mathbb{N}$ such that $\mu(\,kv_n, s\,) = 1 > \alpha$ and $\nu(kv_n, s) = 0 < \alpha, \forall\, n \ge m_0, \forall s > 0$. Hence $\lim\limits_{n\to\infty} \mu(kv_n, s) > \alpha$ and $\lim\limits_{n\to\infty} \nu(kv_n, t) < \alpha,\ \forall\, s > 0$. ■

Example 3.4. Let $(V,\,\|\cdot\|)$ be a F-normed linear space. Define $\mu, \nu : V \times \mathbb{R} \to [0,1]$ by

$$\mu\,(\,v\,,\,s\,)\;=\begin{cases}1 & \text{if } \ s > \|\,v\,\|,\ s > 0,\\ \dfrac{s}{s+\|v\|} & \text{if } \ s \le \|\,v\|\,,\ s > 0,\\ 0 & \text{if } \ s \le 0.\end{cases}$$

$$\nu\,(\,v\,,\,s\,)\;=\begin{cases}0 & \text{if } \ s > \|\,v\,\|,\ s > 0,\\ \dfrac{\|v\|}{s+\|v\|} & \text{if } \ s \le \|\,v\|\,,\ s > 0,\\ 1 & \text{if } \ s \le 0.\end{cases}$$

Then $(\mu,\,\nu)$ is an intuitionistic fuzzy F-norm on V and (V,μ,ν) is an intuitionistic fuzzy F-normed linear space.

Proof. (IFP.16, IFP.18) If $k_n \to 0$ then $\|k_n v\| \to 0$ [by Definition 1.3].
Therefore, $\exists\, m_0 \in \mathbb{N}$ such that $\|k_n v\| < s,\ \forall\, n \ge m_0,\ \forall\, s > 0$.
Thus $\mu(k_n v, s) = 1$ and $\nu(k_n v, s) = 0,\ \forall\, n \ge m_0$.
Hence $\lim\limits_{n\to\infty} \mu(k_n v, s) = 1$ and $\lim\limits_{n\to\infty} \nu(k_n v, s) = 0$.
(IFP.17,IFP.19) If $\lim\limits_{n\to\infty} \mu(v_n, s) > \alpha$ and $\lim\limits_{n\to\infty} \nu(v_n, s) < \alpha$, then $\exists\, m_0 \in \mathbb{N}$ such that $\mu(\,v_n, s\,) > \alpha$ and $\nu(v_n, s) < \alpha,\ \forall\, n \ge m_0,\ \forall\, s > 0$. Hence two cases arises.
Case-1: $\mu(v_n, s) = 1$ and $\nu(v_n, s) = 0$. Then the proof is same as proof of Example 3.3.
Case-2: $\mu(v_n, s) = \frac{s}{s+\|v\|}$ and $\nu(v_n, s) = \frac{\|v\|}{s+\|v\|}$. Therefore,
$\exists\, m_0 \in \mathbb{N}$ such that $\frac{s}{s+\|v_n\|} > \alpha,\ \forall\, n \ge m_0$ and $s > 0$.
$\Rightarrow\ \exists\, m_0 \in \mathbb{N}$ such that $\|v_n\| <\ s(\frac{1-\alpha}{\alpha})\,,\ \forall\, n \ge m_0\,, \forall\, s > 0$.
Since s is arbitrary $\|v_n\| \to 0\ \Rightarrow\ \|kv_n\| \to 0,\ \forall\, k \in \mathbb{K}$, [by Definition 1.3].
and $\exists\, m_0 \in \mathbb{N}$ such that $\frac{\|v_n\|}{s+\|v_n\|} < \alpha,\ \forall\, n \ge m_0$ and $s > 0$.
$\Rightarrow\ \exists\, m_0 \in \mathbb{N}$ such that $\|v_n\| <\ s(\frac{\alpha}{1-\alpha})\,,\ \forall\, n \ge m_0\,, \forall\, s > 0$.
Since s is arbitrary $\|v_n\| \to 0\ \Rightarrow\ \|kv_n\| \to 0,\ \forall\, k \in \mathbb{K}$, [by Definition 1.3].

Hence $\exists\, m_0 \in \mathbb{N}$ such that $\|k_n v\| < s,\ \forall\, n \geq m_0,\ \forall\, s > 0$.
$\Rightarrow\ \exists m_0 \in \mathbb{N}$ such that $\mu(kv_n, s) = 1 > \alpha$ and $\nu(kv_n, s) = 0 < \alpha,\ \forall n \geq m_0\,, \forall\, s > 0$
$\Rightarrow\ \lim\limits_{n \to \infty} \mu(kv_n, s) > \alpha$ and $\lim\limits_{n \to \infty} \nu(kv_n, s) < \alpha,\ \forall\, s > 0$. ■

Proposition 3.5. *Let (V, μ, ν) be an intuitionistic fuzzy F-normed linear space. If a sequences $\{v_n\}_n \in V$ converges to v then it is also α-convergent to v.*

Proof. Here $v_n\,(\in V)$ converges to v in an intuitionistic fuzzy F-normed linear space. Therefore for any $\alpha \in (0,1)$ we have $\lim\limits_{n \to \infty} \mu(v_n - v, s) = 1 > \alpha$ and $\lim\limits_{n \to \infty} \nu(v_n - v, s) = 0 < \alpha,\ \forall\, s > 0$. Hence the proof. ■

Theorem 3.6. *Let (V, μ, ν) be a intuitionistic fuzzy F-normed linear space. Now for any $\alpha \in (0,1)$ the functions $\|\cdot\|_\alpha\,,\ \|\cdot\|_\alpha^* : V \to [0, \infty)$ defined by*

$$\|v\|_\alpha = \bigwedge \{\, s > 0 \,:\, \mu\,(\,v, s\,) > \,\alpha\}$$

$$\|\,v\,\|_\alpha^* = \bigwedge \{\, s > 0 \,:\, \nu\,(\,v, s\,) < \,\alpha\}$$

Then the family of functions $\|\cdot\|_\alpha : V \to [0, \infty)$ with α varying in $(0,1)$ is an ascending family of F-norms on V. And the family of functions $\|\cdot\|_\alpha^* : V \to [0, \infty)$ with α varying in $(0,1)$ is a descending family of F-norms on V.
These norms are called α-F-norms.

Proof. For proving the theorem we have only to verify (PN.5) and (PN.6) of Definition 1.3.
(PN.1), (PN.2), (PN.3), (PN.4), ascending property, and descending property follows from Theorem 3.2 of [17].
(PN.5) Let $k_n \to 0$, then $\|k_n v\| \to 0, \forall\, v \in V$.
i.e., $\lim\limits_{n \to \infty} \mu(k_n v, s) = 1$ and $\lim\limits_{n \to \infty} \nu(k_n v, s) = 0,\ \forall\, s > 0$.
$\Rightarrow\ \exists m_0 \in \mathbb{N}$ such that $\mu(k_n v, s) > \alpha$ and $\nu(k_n v, s) < \alpha,\ \forall n \geq m_0\,, \forall\, s > 0,\ \forall \alpha \in (0,1)$.
$\Rightarrow\ \exists m_0 \in \mathbb{N}$ such that $\|\,k_n\, v\,\|_\alpha < s$ and $\|\,k_n\, v\,\|_\alpha^* < s,\ \forall n \geq m_0\,, \forall\, s > 0,\ \forall \alpha \in (0,1)$.
$\Rightarrow\ \|\,k_n\, v\,\|_\alpha \to 0$ and $\|\,k_n\, v\,\|_\alpha^* \to 0$ as $n \to \infty$, for any $\alpha \in (0,1)$.
(PN.6) We take a fixed α. Let $v_n \to 0$ then we have $v_n \overset{\alpha}{\to} 0$.
i.e., $\lim\limits_{n \to \infty} \mu(x_n, s) > \alpha$ and $\lim\limits_{n \to \infty} \nu(v_n, s) < \alpha$.
Then form (IFP.17) and (IFP.19) we have
$\lim\limits_{n \to \infty} \mu(kv_n, t) > \alpha$ and $\lim\limits_{n \to \infty} \nu(kv_n, s) < \alpha\,, \forall\, k \in \mathbb{K},\ s > 0$.
Hence $\exists\, m_0 \in \mathbb{N}$ such that $\|\,k\, v_n\,\|_\alpha < s$ and $\|\,k\, v_n\,\|_\alpha^* < s,\ \forall n \geq m_0\,, \forall\, s > 0,\ \forall \alpha \in (0,1)$.
$\Rightarrow\ \|\,k\, v_n\,\|_\alpha \to 0$ and $\|\,k\, v_n\,\|_\alpha^* \to 0$ as $n \to \infty$, for any $\alpha \in (0,1)$. ■

Theorem 3.7. *Let $\|\cdot\|_\alpha\,,\ \|\cdot\|_\alpha^*$ be α-F-norms defined in Theorem 3.6. Now $\mu'', \nu'' : V \times \mathbb{R} \to [\,0\,,\,1\,]$ be defined by*

$$\mu''(\,v\,,\,s\,) = \begin{cases} \bigvee \{\, \alpha \in (0,1) \,:\, \|\,v\,\|_\alpha < s\}, & \text{if } s > 0. \\ 0, & \text{if } s \leq 0. \end{cases}$$

$$\nu''(\,v\,,\,s\,) = \begin{cases} \bigwedge \{\, \alpha \in (0,1) \,:\, \|\,v\,\|_\alpha^* < s\}, & \text{if } s > 0. \\ 1, & \text{if } s \leq 0. \end{cases}$$

Then (i) (μ'', ν'') is an intuitionistic fuzzy F-norm on V.
(ii) $\mu'' = \mu$ and $\nu'' = \nu$.

Proof. From Theorem 3.10 of [14] it follows that (μ'', ν'') is an intuitionistic fuzzy pseudo-norm on V and we have only to verify (IFP.16) to (IFP.19).
(IFP.16,IFP.18) Let $k_n \to 0$ then $\|k_n v\| \to 0$ [by Definition 1.3], therefore, $\|k_n v\|_\alpha \to 0$ and $\|k_n v\|^*_\alpha \to 0$.
Hence $\exists\, m_0 \in \mathbb{N}$ such that $\|k_n v\|_\alpha < s$ and $\|k_n v\|^*_\alpha < s$, $\forall n \geq m_0$, $\forall s > 0$, $\forall \alpha \in (0,1)$.
$\Rightarrow$ $\exists\, m_0 \in \mathbb{N}$ such that $\mu''(k_n v, s) > \alpha$ and $\nu''(k_n v, s) < \alpha$, $\forall n \geq m_0$, $\forall s > 0$, $\forall \alpha \in (0,1)$.
$\Rightarrow$ $\lim_{n\to\infty} \mu''(k_n v, s) = 1$ and $\lim_{n\to\infty} \nu''(k_n v, s) = 0,\ s > 0$.
(IFP.17,IFP.19) Let $\lim_{n\to\infty} \mu''(v_n, s) > \alpha$ and $\lim_{n\to\infty} \nu''(v_n, s) < \alpha$.
$\Rightarrow$ $\exists\, m_0 \in \mathbb{N}$ such that $\mu''(v_n, s) > \alpha$ and $\nu''(v_n, s) < \alpha$, $\forall n \geq m_0$, $\forall s > 0$, $\forall \alpha \in (0,1)$.
$\Rightarrow$ $\exists\, m_0 \in \mathbb{N}$ such that $\|v_n\|_\alpha < s$ and $\|v_n\|^*_\alpha < s$, $\forall n \geq m_0$, $\forall s > 0$, $\forall \alpha \in (0,1)$.
$\Rightarrow$ $\|v_n\|_\alpha \to 0$ and $\|v_n\|^*_\alpha \to 0$.
Since $\|\cdot\|_\alpha$ and $\|\cdot\|^*_\alpha$ are F-norms on V, from (PN.6) of Definition 1.3, we have $\|k v_n\|_\alpha \to 0$ and $\|k v_n\|^*_\alpha \to 0$, $\forall k \in \mathbb{K}$.
$\Rightarrow$ $\exists\, m_0 \in \mathbb{N}$ such that $\|k v_n\|_\alpha < s$ and $\|k v_n\|^*_\alpha < s$, $\forall n \geq m_0$, $\forall s > 0$, $\forall \alpha \in (0,1)$.
$\Rightarrow$ $\exists\, m_0 \in \mathbb{N}$ such that $\mu''(k v_n, s) > \alpha$ and $\nu''(k v_n, s) < \alpha$, $\forall n \geq m_0$, $\forall s > 0$, $\forall \alpha \in (0,1)$.
$\Rightarrow$ $\lim_{n\to\infty} \mu''(k v_n, s) > \alpha$ and $\lim_{n\to\infty} \nu''(k v_n, s) < \alpha$. ■

Theorem 3.8. *Let (V, μ, ν) be an intuitionistic fuzzy F-normed linear space. Then (V, μ, ν) furnished with the topology τ satisfying the following conditions:*
(i) $(v, u) \to v + u$ is continuous from $V \times V$ to V.
(ii) $(k, v) \to k v$ is continuous from $\mathbb{K} \times V$ to V.
is a topological vector space.

Proof. Let $\{v_n\}$ and $\{u_n\}$ be two sequences in V converges to a and b respectively. Then $\lim_{n\to\infty} \mu(v_n - a, s) = 1$, $\lim_{n\to\infty} \nu(v_n - a, s) = 0$ and $\lim_{n\to\infty} \mu(u_n - b, t) = 1$, $\lim_{n\to\infty} \nu(u_n - b, t) = 0$.
Now, $\mu((v_n + u_n) - (a + b), s + t) \geq \min\{\mu(v_n - a, s), \mu(u_n - b, t)\}$.
Hence $\lim_{n\to\infty} \mu((v_n + u_n) - (a + b), s + t) = 1$.
Also $\nu((v_n + u_n) - (a + b), s + t) \leq \max\{\nu(v_n - a, s), \nu(u_n - b, t)\}$.
Hence $\lim_{n\to\infty} \nu((v_n + u_n) - (a + b), s + t) = 0$.
Therefore, $v_n + u_n \to a + b$. Thus $(v, u) \to v + u$ is continuous with respect to μ and ν.
Let $v_n \to a$ in V and $k_n \to k$ in $\mathbb{K}$. Then $\lim_{n\to\infty} \mu(v_n - a, s) = 1$, $\lim_{n\to\infty} \nu(v_n - a, s) = 0$ and $\lim_{n\to\infty} \mu(k_n - k, s) = 1$, $\lim_{n\to\infty} \nu(k_n - k, s) = 0$.
Now, $k_n v_n - ka = (k_n - k)(v_n - a) + a(k_n - k) + (v_n - a)k$. Therefore,

$$\lim_{n\to\infty} \mu(k_n v_n - ka, s_0 + s_1 + s_2)$$
$$\geq \min\{\lim_{n\to\infty} \mu(k_n - k)(v_n - a), s_0),\ \lim_{n\to\infty} \mu(a(k_n - k), s_1),\ \lim_{n\to\infty} \mu(k(v_n - a), s_2\}$$
$$\lim_{n\to\infty} \nu(k_n v_n - ka, s_0 + s_1 + s_2)$$
$$\leq \max\{\lim_{n\to\infty} \nu(k_n - k)(v_n - a), s_0),\ \lim_{n\to\infty} \nu(a(k_n - k), s_1),\ \lim_{n\to\infty} \nu(k(v_n - a), s_2\}.$$

Since $k_n \to k$ we have $(k_n - k)a \to 0$. i.e., $\lim_{n\to\infty} \mu(a(k_n - k), s_1) = 1$ and $\lim_{n\to\infty} \nu(a(k_n - k), s_1) = 0$, $\forall\, s_1 > 0$.
$v_n \to a$ therefore $v_n \overset{\alpha}{\to} a$.
$\Rightarrow \lim_{n\to\infty} \mu(v_n - a, s_2) > \alpha$ and $\lim_{n\to\infty} \nu(v_n - a, s_2) < \alpha$, $\forall\, \alpha \in (0,1)$.
$\Rightarrow \lim_{n\to\infty} \mu(k(v_n - a), s_2) > \alpha$ and $\lim_{n\to\infty} \nu(k(v_n - a), s_2) < \alpha$, $\forall\, \alpha \in (0,1)$. [by (IFP.17) and (IFP.19)].
$\Rightarrow \lim_{n\to\infty} \mu(k(v_n - a), s_2) = 1$ and $\lim_{n\to\infty} \nu(k(v_n - a), s_2) = 0$.
Now $\mu((k_n - k)(v_n - a), s_0) \geq \mu(v_n - a, s_0)$ and $\nu((k_n - k)(v_n - a), s_0) \leq \nu(v_n - a, s_0)$, [by (IFP.4) and (IFP.11)].
Therefore, $\lim_{n\to\infty} \mu((k_n - k)(v_n - a), s_0) = 1$ and $\lim_{n\to\infty} \nu((k_n - k)(v_n - a), s_0) = 0$.
Hence $\lim_{n\to\infty} \mu(k_n v_n - ka, s_0 + s_1 + s_2) = 1$ and $\lim_{n\to\infty} \nu(k_n v_n - ka, s_0 + s_1 + s_2) = 0$.
Thus $(k, v) \to k\,v$ is continuous. ∎

4. Some properties of topological vector spaces in intuitionistic fuzzy F-normed spaces

Theorem 4.1. *Let (V, μ, ν) be an intuitionistic fuzzy pseudo normed linear space, then $(V, \mathcal{M}, \mathcal{N})$ is an intuitionistic fuzzy metric space, where $\mathcal{M}, \mathcal{N}$ are defined by $\mathcal{M}(v_1, v_2, s) = \mu(v_1 - v_2, s)$ and $\mathcal{N}(v_1, v_2, s) = \nu(v_1 - v_2, s)$, $\forall\, v_1, v_2 \in V$, $s \geq 0$.*

Proof. (ifm.I) $\mathcal{M}(v_1, v_2, s) + \mathcal{N}(v_1, v_2, s) = \mu(v_1 - v_2, s) + \nu(v_1 - v_2, s) \leq 1$.
(ifm.II,ifm.VII) $\mathcal{M}(v_1, v_2, 0) = \mu(v_1 - v_2, 0) = 0$ and $\mathcal{N}(v_1, v_2, 0) = \nu(v_1 - v_2, 0) = 1$.
(ifm.III,ifm.VIII) $\mathcal{M}(v_1, v_2, s) = 1 \Leftrightarrow \mu(v_1 - v_2, s) = 1 \Leftrightarrow v_1 - v_2 = \theta \Leftrightarrow v_1 = v_2$ and $\mathcal{N}(v_1, v_2, s) = 0 \Leftrightarrow \nu(v_1 - v_2, s) = 0 \Leftrightarrow v_1 - v_2 = \theta \Leftrightarrow v_1 = v_2$.
(ifm.IV,ifm.IX) From (IFP.4) and (IFP.11) taking $c = -1$ we obtain $\mu(-v_1, s) \geq \mu(v_1, s)$ and $\nu(-v_1, s) \leq \nu(v_1, s)$. Swaping v_1 with $-v_1$ we have $\mu(v_1, s) \geq \mu(-v_1, s)$ and $\nu(v_1, s) \leq \nu(-v_1, s)$. Hence $\mu(-v_1, s) = \mu(v_1, s)$ and $\nu(-v_1, s) = \nu(v_1, s)$. Therefore $\mu(v_1 - v_2, s) = \mu(v_2 - v_1, s)$ and $\nu(v_1 - v_2, s) = \nu(v_2 - v_1, s)$. Thus $\mathcal{M}(v_1, v_2, s) = \mathcal{M}(v_2, v_1, s)$ and $\mathcal{N}(v_1, v_2, s) = \mathcal{N}(v_2, v_1, s)$, $\forall\, v_1, v_2 \in V$, $s \in \mathbb{R}^+$.
(ifm.V,ifm.X) $\mathcal{M}(v_1, v_3, s + t) = \mu(v_1 - v_3, s + t) \geq \min\{\mu(v_1 - v_2, s),\ \mu(v_2 - v_3, t)\}$ $= \min\{\mathcal{M}(v_1, v_2, s),\ \mathcal{M}(v_2, v_3, t)\}$ and $\mathcal{N}(v_1, v_3, s + t) = \nu(v_1 - v_3, s + t) \leq \max\{\nu(v_1 - v_2, s),\ \nu(v_2 - v_3, t)\} = \max\{\mathcal{N}(v_1, v_2, s),\ \mathcal{N}(v_2, v_3, t)\}$, $\forall\, v_1, v_2, v_3 \in V$, $\forall\, s, t > 0$.
(ifm.VI,ifm.XI) From (IFP.8) and (IFP.15) we have $\mu(v_1 - v_2, \cdot)$ and $\nu(v_1 - v_2, \cdot)$ are left continuous on $\mathbb{R}$ respectively. Therefore $\mathcal{M}(v_1, v_2, \cdot)$ and $\mathcal{N}(v_1, v_2, \cdot)$ are left continuous on $\mathbb{R}$. ∎

Definition 4.2. Let (V, μ, ν) be an intuitionistic fuzzy F-normed linear space. An open ball $B(v, r, s)$ with center at v, radius $0 < r < 1$ and $s \in \mathbb{R}^+$ is defined by $B(v, r, s) = \{a \in V : \mu(v - a, s) > 1 - r,\ \nu(v - a, s) < r\}$.

Theorem 4.3. *Let (V, μ, ν) be an intuitionistic fuzzy F-normed linear space. Then $B(0, r, s)$ is balanced for all $s > 0$ and $r \in (0,1)$.*

Proof. Let $v \in B(0, r, s)$ then $\exists\, s_0 > 0$ such that $\mu(v, s) > 1 - r$ and $\nu(v, s) < r$, $\forall\, s > s_0$ and $0 < r < 1$. Then by (IFP.4) and (IFP.11) we have $\mu(cv, s) \geq \mu(v, s) > 1 - r$ and $\nu(cv, s) \leq \nu(v, s) < r, \forall\, c \in \mathbb{K}$ such that $|c| \leq 1$. Thus $cv \in B(0, r, s)\,, \forall\, |c| \leq 1$. ∎

Theorem 4.4. *Let (V, μ, ν) be an intuitionistic fuzzy F-normed linear space. Then $B(0, r, s)$ is absorbing for all $s > 0$ and $r \in (0,1)$.*

Proof. From the Theorem 4.3 if we take a positive real number λ such that $\lambda > 1$ then $cv \in B(0, r, s), \forall |c| < \lambda$. ■

Theorem 4.5. *Let (V, μ, ν) be an intuitionistic fuzzy F-normed linear space and let $\mathfrak{B}$ be the family of open balls with center at origin. Then*
(i) If $Y \in \mathfrak{B}$ then there is $X \in \mathfrak{B}$ such that $X + X \subseteq Y$.
(ii) If $X, Y \in \mathfrak{B}$ then there is $Z \in \mathfrak{B}$ such that $Z \subseteq Y \cap X$.

Proof. (i) Let $x + y \in X + X$ then $x, y \in X(\in \mathfrak{B})$.
Therefore $\mu(x, s) > 1 - r$, $\nu(x, s) < r$ and $\mu(y, t) > 1 - r$, $\nu(y, t) < r$. Now,
$\mu(x + y, s + t) \geq \min\{\mu(x, s), \mu(y, t)\} > 1 - r$ and
$\nu(x + y, s + t) \leq \max\{\nu(x, s), \nu(y, t)\} < r$.
Hence $x + y \in B(0, r, s + t)$. Thus $x + y \in Y$.
(ii) Let $r = \min\{r_1, r_2\}$. Then
$B(0, r, s) = \{x \in V : \mu(x, s) > 1 - r, \nu(x, s) < r\}$
$\subseteq \{x \in V : \mu(x, s) > 1 - r_1, \nu(x, s) < r_1\} \cap \{x \in V : \mu(x, s) > 1 - r_2, \nu(x, s) < r_2\}$
$= B(0, r_1, s) \cap B(0, r_2, s)$.
Again since $\mu(x, \cdot)$ is non-decreasing and $\nu(x, \cdot)$ is non-increasing therefore by taking $s = \min\{s_1, s_2\}$ we have $B(0, r, s) \subseteq B(0, r, s_1) \cap B(0, r, s_2)$. Hence $B(0, r, s) \subseteq B(0, r_1, s_1) \cap B(0, r_2, s_2)$. ■

Theorem 4.6. *Let (V, μ, ν) be an intuitionistic fuzzy F-normed linear space and then the open balls with center at origin are convex.*

Proof. Let $v_1, v_2 \in B(0, r, s)$ and $\lambda \in [0, 1]$. Then $\exists\, s_0 > 0$ such that $\mu(v_1, s) > 1 - r$, $\mu(v_2, s) > 1 - r$ and $\nu(v_1, s) < r$, $\nu(v_2, s) < r$, $\forall\, s > s_0$.
$\mu(\lambda v_1 + (1 - \lambda)v_2,\ s)$
$= \mu(\lambda v_1 + (1 - \lambda)v_2,\ \lambda s + (1 - \lambda) s)$
$\geq \min\{\mu(\lambda v_1,\ \lambda s),\ \mu((1 - \lambda)v_2,\ (1 - \lambda) s)\}$, [by (IFP.5)]
$\geq \min\{\mu(v_1,\ \lambda s),\ \mu(v_2,\ (1 - \lambda) s)\}$, [by (IFP.4)]
$= \min\{\mu(v_1, s_1),\ \mu(v_2, s_2)\}$, where $s_1 = \lambda s > s_0$ and $s_2 = (1 - \lambda)s > s_0$ for some $s_0 > 0$
$> \min\{1 - r, 1 - r\}$, since $\exists\, s_0 > 0$ such that $s_1 > s_0$ and $s_2 > s_0$
$= 1 - r$. and
$\nu(\lambda v_1 + (1 - \lambda)v_2, s)$
$= \nu(\lambda v_1 + (1 - \lambda)v_2, \lambda s + (1 - \lambda) s)$
$\leq \max\{\nu(\lambda v_1,\ \lambda s),\ \nu((1 - \lambda)v_2,\ (1 - \lambda) s)\}$, [by (IFP.12)]
$\leq \max\{\nu(v_1,\ \lambda s),\ \nu(v_2,\ (1 - \lambda) s)\}$, [by (IFP.11)]
$= \max\{\nu(v_1, s_1), \nu(v_2, s_2)\}$, where $s_1 = \lambda s > s_0$ and $s_2 = (1 - \lambda)s > s_0$ for some $s_0 > 0$
$< \max\{r, r\}$, since $\exists\, s_0 > 0$ such that $s_1 > s_0$ and $s_2 > s_0$
$= r$.
Thus, $\lambda v_1 + (1 - \lambda)v_2 \in B(0, r, s)$. ■

Remark 4.7. Let $\tau_{\mu,\nu} = \{A \subseteq V : v \in A \Leftrightarrow B(v, r, s) \subseteq A\}$. Then $\tau_{\mu,\nu}$ is a topology on V.

Proof. From the Theorem 4.1 and Theorem 3.3 of [23] the remark follows. ■

Remark 4.8. Let $(V, \|\cdot\|)$ be a F-normed linear space. Define μ, $\nu : V \times \mathbb{R} \to [0,1]$ by

$$\mu(v, s) = \begin{cases} 1 & \text{if } s > \|v\|,\ s > 0, \\ \dfrac{s}{s + \|v\|} & \text{if } s \leq \|v\|,\ s > 0, \\ 0 & \text{if } s \leq 0. \end{cases}$$

$$\nu(v, s) = \begin{cases} 0 & \text{if } s > \|v\|,\ s > 0, \\ \dfrac{\|v\|}{s + \|v\|} & \text{if } s \leq \|v\|,\ s > 0, \\ 1 & \text{if } s \leq 0. \end{cases}$$

Then the topology τ induced by the F-norm $\|\cdot\|$ and the topology $\tau_{\mu,\nu}$ induced by the intuitionistic fuzzy F-norm are equivalent.

Proof. We first show that $T \in \tau \Rightarrow T \in \tau_{\mu,\nu}$. i.e., $\forall\, v \in T$, $s > 0$, $\exists\, 0 < r < 1$ such that $B(v, r, s) \subseteq T$.
Let $v \in T$ then $\exists\, \epsilon > 0$ such that $B(v, \epsilon) \subseteq T$. Let $r = \dfrac{\epsilon}{1+\epsilon} \in (0,1)$ and $s = 1$.
If $a \in B(v, r, s)$ then

$$\mu(v-a, t) > 1-r = 1-\frac{\epsilon}{1+\epsilon} = \frac{1}{1+\epsilon} \Rightarrow \frac{1}{1+\|v-a\|} > \frac{1}{1+\epsilon} \Rightarrow \|v-a\| < \epsilon,$$

$$\nu(v-a, t) < r = \frac{\epsilon}{1+\epsilon} \Rightarrow \frac{\|v-a\|}{1+\|v-a\|} < \frac{\epsilon}{1+\epsilon} \Rightarrow \|v-a\| < \epsilon.$$

Hence $a \in B(v, \epsilon) \subseteq T$.
Conversely, we show that $T \in \tau_{\mu,\nu} \Rightarrow T \in \tau$ i.e., $\forall\, v \in T$, $\exists\, \epsilon > 0$ such that $B(v, \epsilon) \subseteq T$.
Let $v \in T$. Since $T \in \tau_{\mu,\nu}$, $s > 0$, $\exists\, 0 < r < 1$ such that $B(v, r, s) \subseteq T$.
Let $\epsilon = \dfrac{sr}{1-r} > 0$.
Let $a \in B(v, \epsilon)$ then $\|v - a\| < \epsilon$. Now,

$$\mu(v-a, s) = \frac{s}{s + \|v-a\|} > \frac{s}{s+\epsilon} = \frac{s}{s + \frac{sr}{1-r}} = \frac{s(1-r)}{s} = 1 - r$$

$$\nu(v-a, s) = \frac{\|v-a\|}{s + \|v-a\|} < \frac{\epsilon}{1+\epsilon} = \frac{\frac{sr}{1-r}}{s + \frac{sr}{1-r}} = \frac{sr}{s - sr + sr} = r.$$

Hence $a \in B(v, r, s) \subseteq T$. ■

Theorem 4.9. *Let (V, μ, ν) be an intuitionistic fuzzy F-normed linear space then $(V, \mathcal{M}, \mathcal{N})$ is an intuitionistic fuzzy metrizable topological vector space, where $(\mathcal{M}, \mathcal{N})$ are defined by Theorem 4.1.*

Proof. Let $\mathcal{M}$, $\mathcal{N} : V \times V \times \mathbb{R}^+ \to [0,1]$ be defined by $\mathcal{M}(v_1, v_2, s) = \mu(v_1 - v_2, s)$ and $\mathcal{N}(v_1, v_2, s) = \nu(v_1 - v_2, s)$. Then $(\mathcal{M}, \mathcal{N})$ is a intuitionistic fuzzy metric by Theorem 4.1, and this metric is compatible with the topology of V, by Theorem 3.8. Also, $\mathcal{M}(v_1 + v_3, v_2 + v_3, s) = \mu(v_1 - v_2, s) = \mathcal{M}(v_1, v_2, s)$ and $\mathcal{N}(v_1 + v_3, v_2 + v_3, s) = \nu(v_1 - v_2, s) = \mathcal{N}(v_1, v_2, s)$. Therefore $(\mathcal{M}, \mathcal{N})$ is translation-invariant intuitionistic fuzzy metric. ■

Example 4.10. Let l_p be the linear space of all sequence $\{a_n\}$ such that $\sum_{n=1}^{\infty} \|a_n\|^p < \infty$, for $a_n \in \mathbb{K}(= \mathbb{R}$ or $\mathbb{C})$. Let $a_n, b_n \in l_p$ and take $s > 0$.
Let $d(a_n, b_n) = \sum_{n=1}^{\infty} \|a_n - b_n\|^p$, and

$$\mu(a_n, s) = \frac{s}{s + \sum_{n=1}^{\infty} \|a_n\|^p}, \ \nu(a_n, s) = \frac{\sum_{n=1}^{\infty} \|a_n\|^p}{s + \sum_{n=1}^{\infty} \|a_n\|^p}.$$

Then by Example 3.3 of [17], (μ, ν) is an intuitionistic fuzzy F-norm. And the topology τ_d induced by the invariant metric d and the topology $\tau_{\mu,\nu}$ induced by the intuitionistic fuzzy F-norm (μ, ν) are the same.

Proof. Let $a_n \in A(\tau_d)$. Then $\exists\, \epsilon > 0$ such that $B(a_n, \epsilon) \subseteq A$. Let $r = \frac{\epsilon}{1+\epsilon} \in (0, 1)$ and $s = 1$.
If $b_n \in B(a_n, r, s)$ then

$$\mu(a_n - b_n, s) > 1 - r = 1 - \frac{\epsilon}{1+\epsilon} = \frac{1}{1+\epsilon}$$
$$\Rightarrow \frac{1}{1 + \sum_{n=1}^{\infty} \|a_n - b_n\|^p} > \frac{1}{1+\epsilon}$$
$$\Rightarrow \sum_{n=1}^{\infty} \|a_n - b_n\|^p < \epsilon,$$

$$\nu(a_n - b_n, s) < r = \frac{\epsilon}{1+\epsilon}$$
$$\Rightarrow \frac{\sum_{n=1}^{\infty} \|a_n - b_n\|^p}{1 + \sum_{n=1}^{\infty} \|a_n - b_n\|^p} < \frac{\epsilon}{1+\epsilon}$$
$$\Rightarrow \sum_{n=1}^{\infty} \|a_n - b_n\|^p < \epsilon.$$

Hence $b_n \in B(a_n, \epsilon)$.
Conversely, let $a_n \in A(\tau_{\mu,\nu})$. Then $s > 0$, $\exists\, 0 < r < 1$ such that $B(a_n, r, s) \subseteq A$. Let us take $\epsilon = \frac{sr}{1-r} > 0$. Let $b_n \in B(a_n, \epsilon)$ then $\sum_{n=1}^{\infty} \|a_n - b_n\|^p < \epsilon$. Therefore

$$\mu(a_n - b_n, s) = \frac{s}{s + \sum_{n=1}^{\infty} \|a_n - b_n\|^p} > \frac{s}{s+\epsilon} = \frac{s}{s + \frac{sr}{1-r}} = \frac{s(1-r)}{s} = 1 - r\,,$$

$$\nu(a_n - b_n, s) = \frac{\sum_{n=1}^{\infty} \|a_n - b_n\|^p}{s + \sum_{n=1}^{\infty} \|a_n - b_n\|^p} < \frac{\epsilon}{1+\epsilon} = \frac{\frac{sr}{1-r}}{s + \frac{sr}{1-r}} = \frac{sr}{s - sr + sr} = r.$$

Hence $b_n \in B(a_n, r, s)$. ∎

5. Conclusion

Intuitionistic fuzzy F-norm is a generalization of intuitionistic fuzzy norm as well as intuitionistic fuzzy pseudo norm. In this paper, we characterize metrizable topological vector space by intuitionistic fuzzy F-norm. It is observed that a topological vector space is intuitionistic fuzzy metrizable iff. it is metrizable.
Even though the structure of intuitionistic fuzzy F-norm is more complected (than the

structure of intuitionistic fuzzy norm), it is a very rich and more general structure. With a suitable adaptation, the notion of intuitionistic fuzzy F-norm deserves the attention in the extension of classical functional analysis in fuzzy environment.

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