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Application of Inverse Possibility Fuzzy Soft Sets over Parameters on Some Semigroups

Peerapong Suebsan[1,*] **and Manoj Siripitukdet**[2,3]

[1] *Department of Mathematics, School of Science, University of Phayao, Phayao 56000, Thailand*
e-mail : peerapong.su@up.ac.th

[2] *Department of Mathematics, Faculty of Science, Naresuan University, Phitsanulok 65000, Thailand*

[3] *Research Center for Academic Excellence in Mathematics, Naresuan University*
e-mail : manojs@nu.ac.th

Abstract The main purpose of this paper is to define inverse possibility fuzzy soft sets over parameters on some semigroups. We present a new algorithm for solving some investment decision making problems based on a comparison table of inverse possibility fuzzy soft sets over parameters on some semigroups.

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1. Introduction

The solution to real-world problems in environment, computer science, economics, engineering, medical science and many other fields involve data that contain uncertainties. While fuzzy sets [1], rough sets [2], interval mathematics [3] and other mathematics methods are often useful approaches to explaining uncertainty. In 1999, Molodtsov [4] defined soft set theory, such that a new mathematical method approach to uncertainty, which is free from the parameterization inadequacy condition of fuzzy set theory, probability theory, rough set theory, etc. The soft set theory has been developed by many researchers [5–8].

The soft sets are extended to fuzzy soft sets by Maji et al. [9] (2001). They introduced fuzzy soft sets, the union and intersection, fuzzy soft subsets and investigate their properties. In 2007, Roy and Maji [10] discussed the application of the algorithm of fuzzy soft sets in decision making problems. The score value of fuzzy soft sets in decision making problems was computed using the comparison table in the algorithm. In 2010, Majumdar and Samanta [11] defined and studied the generalised fuzzy soft sets where the degree is attached with the parameterization of fuzzy sets while defining a fuzzy soft sets. Next, Kalaichelvi and Malini [12] studied application of fuzzy soft sets to investment decision

*Corresponding author.

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making problem based on the data collected from female employees working in India. Next, Alkhazaleh et al. [13] introduced the concept of possibility fuzzy soft set and its operation and studied some of its properties. They presented applications of this theory in solving a decision making problem. Later, Kalaichelvi and Seenivasan [14] presented a decision making with the help of possibility fuzzy soft set initiated by Alkhazaleh et al. [13], to select a best player for hockey team.

The fuzzy soft sets are developed to fuzzy soft semigroups by Yang [15] (2011). He defined fuzzy soft [left, right] ideals over semigroups and fuzzy soft semigroups, and studied sufficient and necessary conditions for α-level set, intersection and union of fuzzy soft [left, right] ideals. Next, Suebsan and Siripitukdet [16] defined extended averages of fuzzy soft sets over some semigroups. They discussed the application of fuzzy soft sets in decision making problems. The score value of fuzzy soft sets in decision making problems was computed using the comparison table based on extended averages in the algorithm. In 2019, Khalil and Hassan [17] introduced an inverse fuzzy soft set, along with its properties, characteristics, and operations. They constructed an algorithm using max-min and min-max decision of inverse fuzzy soft set for a fuzzy decision making problem.

When it comes to existing researches of fuzzy soft sets in decision making problems, they are limited to some extent. In a case where the maximum score more than one value to fuzzy soft sets, the above algorithms are unable to make a decision in decision making problems. Motivated and inspired by the works above, we are interested in the inverse possibility fuzzy soft sets over parameters on some semigroups.

In this paper, we define an inverse possibility fuzzy soft sets over parameters on some semigroups. We investigate some operations such as inverse possibility fuzzy soft subsets, complement inverse possibility fuzzy soft sets with supported examples. We construct an algorithm using the comparison table of inverse possibility fuzzy soft sets to investment decision making problems.

2. Preliminaries

In this section, we shall give some of basic definitions and results that will be used.

Suebsan and Siripitukdet [16] defined some semigroups as follows.

Let S be a set and let $\alpha : S \to \mathbb{R}$ be a 1-1 function.

Define operations $\triangle$ and $\bigtriangledown$ on S as follows: For any $x, y \in S$, define

$$x \triangle y = \begin{cases} x & \text{if} \quad \alpha(x) \geq \alpha(y), \\ y & \text{if} \quad \alpha(x) < \alpha(y) \end{cases}$$

and

$$x \bigtriangledown y = \begin{cases} x & \text{if} \quad \alpha(x) < \alpha(y), \\ y & \text{if} \quad \alpha(x) \geq \alpha(y). \end{cases}$$

Then $(S, \triangle)$ and $(S, \bigtriangledown)$ are semigroups. These semigroups are called **semigroups induced by a function** α and denote S_α. We write $S_\alpha = \{x(\alpha(x))|x \in S\}$.

In real-world problems, we can construct semigroups induced by 1-1 functions.

Example 2.1. [16] A family is looking to purchase a water purifier. Let $S_\alpha = \{o_1(3), o_2(2), o_3(4), o_4(1), o_5(5)\}$ be a set of five water purifiers with a limited time warranty (year) under consideration. Then $(S_\alpha, \triangle)$ is a semigroup.

Example 2.2. [16] A corporation is evaluating the decision of an investment opportunity. Let $S_\alpha = \{o_1(2), o_2(4), o_3(3), o_4(5), o_5(1)\}$ be a set of five investment avenues with risk assessments under consideration, where o_1:Bank Deposit, o_2:Shares, o_3:Mutual Fund, o_4:Stocks, o_5:Government Bonds and the levels of risk are 1:very low, 2:low, 3:moderate, 4:high, 5:very high. Then $(S_\alpha, \bigtriangledown)$ is a semigroup.

Definition 2.3. [1] Let X be a nonempty set. A function f from X to the unit interval $[0,1]$ is called a *fuzzy set* on X.

Definition 2.4. [4] Let U be an initial universal set, E be a set of parameters and let $\emptyset \neq A \subseteq E$. A pair $(\widetilde{F}, A)$ is called a *soft set* over U, where $\widetilde{F}$ is a mapping $\widetilde{F} : A \to P(U)$, where $P(U)$ denotes the power set of U.

Definition 2.5. [7] Let U be an initial universal set, E be a set of parameters and let $\emptyset \neq A \subseteq E$. A pair (F, A) is called a *fuzzy soft set* over U, where F is a mapping given by $F : A \to \mathrm{Fuz}(U)$ and $\mathrm{Fuz}(U)$ is the set of all fuzzy sets on U.

Note that let (F, A) be a fuzzy soft set over U. For $p \in A, F(p) \in \mathrm{Fuz}(U)$. Set $F_p := F(p)$. Then $F_p \in \mathrm{Fuz}(U)$.

Definition 2.6. [13] Let U be an initial universal set, E be a set of parameters and let $\emptyset \neq A \subseteq E$. Let $F : A \to \mathrm{Fuz}(U)$ where $\mathrm{Fuz}(U)$ is the set of all fuzzy sets on U. Let μ be a fuzzy set on A, that is $\mu : A \to \mathrm{Fuz}(U)$. Define $F^\mu : A \to \mathrm{Fuz}(U) \times \mathrm{Fuz}(U)$ given by

$$F^\mu_p = (F_p(u), \mu_p(u)) \text{ for all } u \in U.$$

F^μ is called a *possibility fuzzy soft set* over U.

$F^\mu_{p_i}$ may be written as follows:

$$F^\mu_{p_i} = \left\{ \left(\frac{u_1}{F_{p_i}(u_1)}, \mu_{p_i}(u_1) \right), \left(\frac{u_2}{F_{p_i}(u_2)}, \mu_{p_i}(u_2) \right), ..., \left(\frac{u_n}{F_{p_i}(u_n)}, \mu_{p_i}(u_n) \right) \right\}.$$

The following example is the example of the possibility fuzzy soft set over a universe.

Example 2.7. Let $U = \{u_1, u_2, u_3\}$ be a set of three cars under consideration. Let $E = \{e_1\{\text{fuel efficiency}\}, e_2\{\text{beautiful}\}, e_3\{\text{ecofriendly}\}, e_4\{\text{deluxe}\}, e_5\{\text{high speed}\}\}$ be a set of parameters and $A = \{e_1, e_3, e_5\}$. Let $\mu : A \to \mathrm{Fuz}(U)$.
Define $F^\mu : A \to \mathrm{Fuz}(U) \times \mathrm{Fuz}(U)$ as

$$F^\mu_{e_1} = \Big\{ \big(\frac{u_1}{0.4}, 0.5\big), \big(\frac{u_2}{0.3}, 0.8\big), \big(\frac{u_3}{0.4}, 0.9\big) \Big\}.$$
$$F^\mu_{e_3} = \Big\{ \big(\frac{u_1}{0.6}, 0.7\big), \big(\frac{u_2}{0.4}, 0.7\big), \big(\frac{u_3}{0.1}, 0.5\big) \Big\}.$$
$$F^\mu_{e_5} = \Big\{ \big(\frac{u_1}{0.2}, 0.5\big), \big(\frac{u_2}{0.6}, 0.7\big), \big(\frac{u_3}{0.5}, 0.7\big) \Big\}.$$

Then F^μ is a possibility fuzzy soft set over U. This can expressed as Table 1.

TABLE 1. F^μ in tabular form

$F^\mu =$

A \ U	u_1	u_2	u_3
e_1	$0.4, 0.5$	$0.3, 0.8$	$0.4, 0.9$
e_3	$0.6, 0.7$	$0.4, 0.7$	$0.1, 0.5$
e_5	$0.2, 0.5$	$0.6, 0.7$	$0.5, 0.7$

The concept of the inverse fuzzy soft set over an initial universal set are introduced by Khalil et al. [17].

Definition 2.8. [17] Let U be an initial universal set and let E be a set of parameters. A pair $(\widehat{F}, U)$ is called an *inverse soft set* over E, where $\widehat{F}$ is a mapping given by $\widehat{F} : U \to P(E)$ and $P(E)$ is the power set of E.

Definition 2.9. [17] Let U be an initial universal set and let E be a set of parameters. A pair $(\mathcal{F}, U)$ is called an *inverse fuzzy soft set* over E, where $\mathcal{F}$ is a mapping given by $\mathcal{F} : U \to \mathrm{Fuz}(E)$ is a mapping and $\mathrm{Fuz}(E)$ is the set of all fuzzy parameter set on E.

In 2007, Roy and Maji [10] used the comparison table approach in decision making problems. Let $U = \{o_1, o_2, ..., o_n\}$ be an object set and let $E = \{e_1, e_2, ..., e_k\}$ be a set of parameters.

The comparison table is a square table in which the number of rows and columns are equal, rows and columns both are labelled by the object names $o_1, o_2, ..., o_n$ of U, and the entries are $c_{ij}, i, j \in \{1, 2, ..., n\}$ given by

c_{ij} = the number of parameters for which the membership value of o_i exceeds or equal to the membership value of o_j.

Obviously, $0 \leq c_{ij} \leq k$, and $c_{ii} = k$, for all i, j where, k is the number of all parameters in a fuzzy soft set. Thus, c_{ij} indicates a numerical measure, which is an integer number and o_i dominates o_j in c_{ij} number of parameters out of k parameters.

3. Inverse Possibility Fuzzy Soft Sets in Decision Making Problems

In this section, we define inverse possibility fuzzy soft sets which are a new idea of fuzzy soft sets over parameters on some semigroups and present an algorithm for identification of objects.

Let S_α be a semigroup and let E be a set of parameters.

Definition 3.1. Let $\emptyset \neq A \subseteq E$ and μ be a fuzzy set on S_α, that is $\mu : S_\alpha \to \mathrm{Fuz}(A)$. $\widetilde{\mathcal{F}}^\mu$ is called an *inverse possibility fuzzy soft set* over A, where $\widetilde{\mathcal{F}}^\mu$ is a mapping $\widetilde{\mathcal{F}}^\mu : S_\alpha \to \mathrm{Fuz}(A) \times \mathrm{Fuz}(A)$ as

$$\widetilde{\mathcal{F}}^\mu_u = (\widetilde{\mathcal{F}}_u(p), \mu_u(p)) \text{ for all } p \in A,$$

where $\mathrm{Fuz}(A)$ is the set of all fuzzy parameter set on A.

$\widetilde{\mathcal{F}}^\mu_{u_i}$ may be written as follows:

$$\widetilde{\mathcal{F}}^\mu_{u_i} = \left\{ \left(\frac{p_1}{\widetilde{\mathcal{F}}_{u_i}(p_1)}, \mu_{u_i}(p_1) \right), \left(\frac{p_2}{\widetilde{\mathcal{F}}_{u_i}(p_2)}, \mu_{u_i}(p_2) \right), ..., \left(\frac{p_n}{\widetilde{\mathcal{F}}_{u_i}(p_n)}, \mu_{u_i}(p_n) \right) \right\}.$$

The following example is the example of the inverse possibility fuzzy soft set over a parameter set on some semigroups.

Example 3.2. Let $S_\alpha = \{u_1(1), u_2(2), u_3(3)\}$ be a set of three cars with a limited time warranty (year) under consideration. Let S_α be a semigroup with a binary operation $\triangle$ defined by Table 2.

TABLE 2. The Multiplication Table of a Semigroup S_α

$\triangle$	u_1	u_2	u_3
u_1	u_1	u_2	u_3
u_2	u_2	u_2	u_3
u_3	u_3	u_3	u_3

Let $E=\{e_1\{\text{fuel efficiency}\}, e_2\{\text{beautiful}\}, e_3\{\text{ecofriendly}\}, e_4\{\text{deluxe}\}, e_5\{\text{high speed}\}\}$ be a set of parameters and $A=\{e_1,e_3,e_5\}$. Let $\mu : S_\alpha \to \mathrm{Fuz}(A)$. Define $\widetilde{\mathcal{F}}^\mu : S_\alpha \to \mathrm{Fuz}(A)\times \mathrm{Fuz}(A)$ as

$$\widetilde{\mathcal{F}}^\mu_{u_1} = \Big\{(\frac{e_1}{0.4}, 0.5), (\frac{e_3}{0.3}, 0.8), (\frac{e_5}{0.4}, 0.9)\Big\}.$$

$$\widetilde{\mathcal{F}}^\mu_{u_2} = \Big\{(\frac{e_1}{0.6}, 0.7), (\frac{e_3}{0.4}, 0.7), (\frac{e_5}{0.1}, 0.5)\Big\}.$$

$$\widetilde{\mathcal{F}}^\mu_{u_3} = \Big\{(\frac{e_1}{0.2}, 0.5), (\frac{e_3}{0.6}, 0.7), (\frac{e_5}{0.5}, 0.7)\Big\}.$$

Then $\widetilde{\mathcal{F}}^\mu$ is an inverse possibility fuzzy soft set over A. This can expressed as Table 3.

TABLE 3. $\widetilde{\mathcal{F}}^\mu$ in tabular form

$\widetilde{\mathcal{F}}^\mu =$

S_α \ A	e_1	e_3	e_5
u_1	$0.4, 0.5$	$0.3, 0.8$	$0.4, 0.9$
u_2	$0.6, 0.7$	$0.4, 0.7$	$0.1, 0.5$
u_3	$0.2, 0.5$	$0.6, 0.7$	$0.5, 0.7$

Definition 3.3. Let $\widetilde{\mathcal{F}}^\mu$ and $\widetilde{\mathcal{G}}^\delta$ be two inverse possibility fuzzy soft sets over a parameter A. Then $\widetilde{\mathcal{F}}^\mu$ is called an *inverse possibility fuzzy soft subset* of $\widetilde{\mathcal{G}}^\delta$, denoted by $\widetilde{\mathcal{F}}^\mu \widetilde{\subseteq} \widetilde{\mathcal{G}}^\delta$, if
(i) μ_u is fuzzy subset of δ_u for all $u \in S_\alpha$.
(ii) $\widetilde{\mathcal{F}}_u$ is also fuzzy subset of $\widetilde{\mathcal{G}}_u$ for all $u \in S_\alpha$.

Example 3.4. From Example 3.2, define a function $\widetilde{\mathcal{G}}^\delta : S_\alpha \to \mathrm{Fuz}(A)\times \mathrm{Fuz}(A)$ as

$$\widetilde{\mathcal{G}}^\delta_{u_1} = \Big\{(\frac{e_1}{0.5}, 0.6), (\frac{e_3}{0.4}, 0.8), (\frac{e_5}{0.5}, 0.9)\Big\}.$$

$$\widetilde{\mathcal{G}}^\delta_{u_2} = \Big\{(\frac{e_1}{0.6}, 0.7), (\frac{e_3}{0.5}, 0.7), (\frac{e_5}{0.3}, 0.6)\Big\}.$$

$$\widetilde{\mathcal{G}}^\delta_{u_3} = \Big\{(\frac{e_1}{0.4}, 0.7), (\frac{e_3}{0.6}, 0.8), (\frac{e_5}{0.6}, 0.8)\Big\}.$$

Then $\widetilde{\mathcal{F}}^\mu$ is an inverse possibility fuzzy soft subset of $\widetilde{\mathcal{G}}^\delta$.

Definition 3.5. Let $\widetilde{\mathcal{F}}^\mu$ be an inverse possibility fuzzy soft sets over a parameter A. Then $1-\widetilde{\mathcal{F}}^\mu$ is called a *complement* of $\widetilde{\mathcal{F}}^\mu$ is defined by $1-\widetilde{\mathcal{F}}^\mu = \widetilde{\mathcal{G}}^\delta$ such that $\delta_u = (1-\mu)_u$ and $\widetilde{\mathcal{G}}_u = (1-\widetilde{\mathcal{F}})_u$ for all $u \in S_\alpha$.

Example 3.6. From Example 3.2, $1-\widetilde{\mathcal{F}}^{\mu}=\widetilde{\mathcal{G}}^{\delta}$ is given by

$$\widetilde{\mathcal{G}}^{\mu}_{u_1}=\Big\{\big(\frac{e_1}{0.6},0.5\big),\big(\frac{e_3}{0.7},0.2\big),\big(\frac{e_5}{0.6},0.1\big)\Big\}.$$
$$\widetilde{\mathcal{G}}^{\mu}_{u_2}=\Big\{\big(\frac{e_1}{0.4},0.3\big),\big(\frac{e_3}{0.6},0.3\big),\big(\frac{e_5}{0.9},0.5\big)\Big\}.$$
$$\widetilde{\mathcal{G}}^{\mu}_{u_3}=\Big\{\big(\frac{e_1}{0.8},0.5\big),\big(\frac{e_3}{0.4},0.3\big),\big(\frac{e_5}{0.5},0.3\big)\Big\}.$$

This can expressed as Table 4.

TABLE 4. $1-\widetilde{\mathcal{F}}^{\mu}$ in tabular form

$1-\widetilde{\mathcal{F}}^{\mu}=$

S_α \ A	e_1	e_3	e_5
u_1	$0.6, 0.5$	$0.7, 0.2$	$0.6, 0.1$
u_2	$0.4, 0.3$	$0.6, 0.3$	$0.9, 0.5$
u_3	$0.8, 0.5$	$0.4, 0.3$	$0.5, 0.3$

Definition 3.7. Let $\widetilde{\mathcal{F}}^{\mu}$ and $\widetilde{\mathcal{G}}^{\delta}$ be two inverse possibility fuzzy soft sets over a parameter A. Then $\widetilde{\mathcal{F}}^{\mu}$ and $\widetilde{\mathcal{G}}^{\delta}$ are called an *equal*, written as $\widetilde{\mathcal{F}}^{\mu}=\widetilde{\mathcal{G}}^{\delta}$ if $\widetilde{\mathcal{F}}^{\mu}\widetilde{\subseteq}\widetilde{\mathcal{G}}^{\delta}$ and $\widetilde{\mathcal{G}}^{\delta}\widetilde{\subseteq}\widetilde{\mathcal{F}}^{\mu}$. In other words, $\widetilde{\mathcal{F}}^{\mu}=\widetilde{\mathcal{G}}^{\delta}$ if the following conditions are satisfied
(i) μ_u is equal to δ_u for all $u\in S_\alpha$.
(ii) $\widetilde{\mathcal{F}}_u$ is equal to $\widetilde{\mathcal{G}}_u$ for all $u\in S_\alpha$.

In the following algorithm, we use the inverse possibility fuzzy soft sets over parameters in decision making problems.

Let $S_\alpha=\{u_1,u_2,...,u_n\}$ be a set of a semigroup and let $E=\{e_1,e_2,...,e_k\}$ be a set of parameters.

Algorithm

Step 1. Input the inverse possibility fuzzy soft sets $\widetilde{\mathcal{F}}^{\mu}$ in tabular form.
Step 2. Compute the choice value table.
Step 3. Construct the comparison table of $\widetilde{\mathcal{F}}^{\mu}$.
Step 4. Compute the row sum (r_i), the column sum (c_i) and the score value $(t_i=r_i-c_i)$ of u_i for i.
Step 5. The decision is T_k.

5.1.If $T_k=\max_i t_i$ then we choose T_k.

5.2.If T_k has more than one object then we choose o_k corresponding to operation of the semigroup.

The choice table is obtained by multiplying each entry of the table representing the inverse possibility fuzzy soft sets by the corresponding value of $\widetilde{\mathcal{F}}^{\mu}$.

4. APPLICATIONS

In this section, we apply the concept of the inverse possibility fuzzy soft sets in the new algorithm to investment decision making problems.

The following example, we apply the example in [12] to the inverse possibility fuzzy soft sets over parameters in some semigroups.

Factors Influencing Investment Decision

e_1 :Safety of funds:

It is certainty of return on capital and the assurance of protection to the funds invested under changing conditions.
e_2 :Liquidity of funds:
It refers to the easy conversion of investment into liquid cash to meet any finical requirements of the investor without loss of time.
e_3 :High returns:
It is the basic objective of an investor. He aims higher return that facilitates rapid growth of funds invested.
e_4 :Maximum profit in minimum period:
The choice of investment is influenced by the relation between period of investment and rate of return. Investors choose investment avenues in which higher return is possible in shorter period of time.
e_5 :Stable return:
It refers to the consistent return from investment. If the return from investment is volatile in nature, the choice of investor may prove to be wrong when he could realize only a low rate of return.
e_6 :Easy accessibility:
It refers to the physical location of the institutions offering investment avenues and also the simplicity of procedures and formalities involved in the process of investment.
e_7 :Tax concession:
Certain investments and returns from investments are eligible for deduction under income tax. An investor who is particular to avail tax concession prefers such eligible investments.

Investment Avenues:

I_1 :Bank Deposit
I_2 :Gold
I_3 :Mutual Fund
I_4 :Real Estate
I_5 :Shares and Stocks

Example 4.1. Let $S_\alpha = \{I_1(1), I_2(2), I_3(3), I_4(4), I_5(5)\}$ be a set of seven investment avenues with risk assessments under consideration and the levels of risk are 1:very low, 2:low, 3:moderate, 4:high, 5:very high. Let S_α be a semigroup with a binary operation $\triangledown$ defined by Table 5.

Table 5. The Multiplication Table of a Semigroup S_α

$\triangledown$	I_1	I_2	I_3	I_4	I_5
I_1	I_1	I_1	I_1	I_1	I_1
I_2	I_1	I_2	I_2	I_2	I_2
I_3	I_1	I_2	I_3	I_3	I_3
I_4	I_1	I_2	I_3	I_4	I_4
I_5	I_1	I_2	I_3	I_4	I_5

Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ be a set of parameters.

We developed a decision making model using inverse possibility fuzzy soft sets by considering a set of factors preferred by an investor to identify the investment avenue that suits best requirements of the said investor.

Case 1: Preference of investor factors by investor X. Let $A = \{e_2, e_4, e_7\}$ where e_2 :liquidity of funds, e_4 :maximum profit in minimum period and e_7 :tax concession.

Step 1. Let $\widetilde{\mathcal{F}}^{\mu}$ be an inverse possibility fuzzy soft set as in Table 6.

TABLE 6. $\widetilde{\mathcal{F}}^{\mu}$ in tabular form

$\widetilde{\mathcal{F}}^{\mu} =$

S_α \ A	e_2	e_4	e_7
I_1	$0.4, 0.5$	$0.3, 0.8$	$0.4, 0.9$
I_2	$0.4, 0.6$	$0.6, 0.8$	$0.2, 0.6$
I_3	$0.3, 0.5$	$0.5, 0.7$	$0.3, 0.5$
I_4	$0.4, 0.5$	$0.6, 0.7$	$0.2, 0.5$
I_5	$0.5, 0.7$	$0.8, 0.9$	$0.4, 0.6$

Step 2. Compute the choice value table as in Table 7.

TABLE 7. The choice value table of $\widetilde{\mathcal{F}}^{\mu}$

S_α \ A	e_2	e_4	e_7	Choice value
I_1	0.20	0.24	0.36	0.80
I_2	0.24	0.48	0.12	0.84
I_3	0.15	0.35	0.15	0.65
I_4	0.20	0.42	0.10	0.72
I_5	0.35	0.72	0.24	1.31

Step 3. The comparison table of $\widetilde{\mathcal{F}}^{\mu}$ is given in Table 8.

TABLE 8. The comparison table of $\widetilde{\mathcal{F}}^{\mu}$

c_{ij}	I_1	I_2	I_3	I_4	I_5
I_1	3	1	2	2	1
I_2	2	3	2	3	0
I_3	1	1	3	1	0
I_4	2	0	2	3	0
I_5	2	3	3	3	3

Step 4. The score value is shown in Table 9.

TABLE 9. The score value table

	Row sum	Column sum	Score value
I_1	9	10	-1
I_2	10	7	3
I_3	6	12	-6
I_4	7	12	-5
I_5	14	4	10

Step 5. The decision is I_5 which has the highest score value. Thus the I_5:Shares and Stocks is the best investment avenues to be selected by the investor X.

Case 2: Preference of investor factors by investor Y. Let $B = \{e_1, e_3, e_5, e_6\}$ where e_1 :safety of funds:, e_3 :high returns, e_5 :stable return and e_6 :easy accessibility.

Step 1. Let $\widetilde{\mathcal{F}}^{\mu}$ be an inverse possibility fuzzy soft set as in Table 10.

TABLE 10. $\widetilde{\mathcal{F}}^{\mu}$ in tabular form

$\widetilde{\mathcal{F}}^{\mu} =$

S_α \ B	e_1	e_3	e_5	e_6
I_1	$0.4, 0.5$	$0.3, 0.8$	$0.4, 0.9$	$0.5, 0.8$
I_2	$0.4, 0.6$	$0.6, 0.8$	$0.3, 0.6$	$0.3, 0.6$
I_3	$0.3, 0.5$	$0.5, 0.7$	$0.4, 0.5$	$0.4, 0.5$
I_4	$0.4, 0.5$	$0.6, 0.7$	$0.2, 0.5$	$0.3, 0.6$
I_5	$0.5, 0.7$	$0.2, 0.3$	$0.3, 0.4$	$0.1, 0.2$

Step 2. Compute the choice value table as in Table 11.

TABLE 11. The choice value table of $\widetilde{\mathcal{F}}^{\mu}$

S_α \ B	e_1	e_3	e_5	e_6	Choice value
I_1	0.20	0.24	0.36	0.40	1.20
I_2	0.24	0.48	0.18	0.18	1.08
I_3	0.15	0.35	0.20	0.20	0.90
I_4	0.20	0.42	0.10	0.18	0.90
I_5	0.35	0.06	0.12	0.02	0.55

Step 3. The comparison table of $\widetilde{\mathcal{F}}^{\mu}$ is given in Table 12.

TABLE 12. The comparison table of $\widetilde{\mathcal{F}}^{\mu}$

c_{ij}	I_1	I_2	I_3	I_4	I_5
I_1	4	2	3	3	3
I_2	2	4	2	4	3
I_3	1	2	4	2	3
I_4	2	1	2	4	2
I_5	1	1	1	2	4

Step 4. The score value is shown in Table 13.

Table 13. The score value table

	Row sum	Column sum	Score value
I_1	15	10	5
I_2	15	10	5
I_3	12	12	0
I_4	11	14	-3
I_5	10	15	-5

Step 5. The decision are I_1 and I_2 which have the highest score value. Thus the I_1:Bank Deposit is the best investment avenues to operation of the semigroup (Table 5) to be selected by the investor Y.

Remark 4.2. The advantage of the new algorithm can be decided in case which has the maximum score more than one value for inverse possibility fuzzy soft sets. The semigroup plays a key role in dealing with problems specified above.

5. Conclusion

In this paper, inverse possibility fuzzy soft sets over parameters are introduced. The new algorithm in decision making problems based on the comparison table from inverse possibility fuzzy soft sets over parameters on some semigroups are presented. The examples presented in this paper demonstrate that the new algorithm is practical for solving to investment decision making problems.

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