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Finite-Time Stability of a Class of Uncertain Switched Nonlinear Systems with Time-Varying Delay

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Abstract In this paper, we investigate finite-time stability (FTS) of a class of uncertain switched nonlinear systems with time-varying delay. By using the average dwell time method and Gronwall-Bellman inequality, novel FTS criteria are derived. The FTS criteria of uncertain switched nonlinear criteria are delays-dependent and given in terms of linear matrix inequalities (LMIs) which can be solved by various available algorithms. Numerical example is given to illustrate effectiveness of our proposed methods.

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1. Introduction

Time-varying delay systems have received considerable attention over the past few decades. The main reason is that the real processes in our world always involve time-delay systems, such as transportation systems, electrical power systems, communication systems, economic systems and so on. Several dynamic system often depend on time-delay; namely, the present state depends on the states, which is the main sources of instability and less capable performance of the systems, see [1], [2–5], [6] and [7] Therefore, the study of time-delay should be highlighted, especially those with time-varying delay.

It is well known that a switched system is a particular type of hybrid system that contains several subsystems and a switching law, the assignment at any time as soon as the subsystem is active. A different switching rules cause different system behaviors and hence lead to a different system display. Therefor, the switched system has been

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attracted attention in stable and stabilization. The result related to the switched system are reported in the literature (see [8],[9],[10],[11], [12] and [13]).

Generally, most of the result of switched system is involved with stability analysis and design of switching rules, see [14–16], ([12], [13], [17] and [18]) which is defined over an infinite time interval. nevertheless, in many real world applications, the main aim is concerned with the behavior of the system over a fixed finite time interval. For instance, the problem of sending a rocket from the neighborhood of a point A to the neighborhood of a point B over a fixed interval [15]. In this example, the concept of FTS is proposed. Some early results on FTS of switched system can be found in [14] and [16], Lyapunov function technique has been used in these work. It should be noted that, there are a few results on FTS of switched system with time-varying delay. In [19], FTS of switched system with time-varying delay has been investigated by using Lyapunov-Krasovskii functional and average dwell time (ADT) approach. The problem of switched system with time-varying delay via Gronwall-Bellman inequality has been studied in [8]. By using linear matrix inequality technique the researcher have obtained the feasible condition guaranteeing stability of such a system, see [19], [12] and [13].

Motivated by the above-mentioned discussion and the practical background, we shall derive the new FTS for switched uncertain nonlinear system with time-varying delay. The main contribution of our studies are as follows. (i) By employing average dwell time method and Gronwall-Bellman inequality technique, we derive new and less conservative FTS for switched uncertain nonlinear system with time-varying delay in terms of LMIs. (ii) The time-delay function are only required to be continuous but not necessarily differentiable.

The rest of the paper is organized as follows. Section 2, present notations, definitions and auxiliary lemmas required for the proof of the main results. In section 3, the FTS of switched uncertain nonlinear system with time-varying delay is obtained. Illustrative numerical example is presented in section 4. Section 5 concludes the paper.

2. Problem Formulation and Preliminaries

Consider the following uncertain switched nonlinear systems with time-varying delay;

$$\begin{aligned} \dot{x}(t) &= (A_\sigma + \Delta A_\sigma(t))x(t) + (B_\sigma + \Delta B_\sigma(t))x(t-d(t)) + f_\sigma(x(t),t) - f_\sigma(0), \\ x(t) &= \varphi(s), \in [-d, 0], \end{aligned} \tag{2.1}$$

where $x(t) \in R^n$ is the state vector, d(t) denotes the time-varying delay.Which satisfies $0 < d(t) \leq d$, $\varphi(t)$ is a continuous vector-valued initial function on $[-d, 0]$ for a known constant $d > 0$. $\Delta A_i(t)$ and $\Delta B_i(t)$ are the time-varying uncertain matrices which given in the following terms :

$$\Delta A_i(t) = E_a F_a(t) H_a, \Delta B_i(t) = E_b F_b(t) H_b \tag{2.2}$$

and E_a, E_b, H_a, H_b are known constant matrices with appropriate dimensions, $F_a(t), F_b(t)$ are unknown uncertain matrices and satisfy

$$F_a^T(t) F_a(t) \leq I, F_b^T(t) F_b(t) \leq I; t \in \mathbb{R}. \tag{2.3}$$

The function $\sigma(t) : \mathbb{R}^+ \cup \{0\} \to \underline{N}, \underline{N} = 1, 2, ..., N$ is the switching signal which is piecewise constant and right continuous.The switching sequence,
$\sigma(t) : (t_0, \sigma(t_0)), (t_1, \sigma(t_1)), ..., (t_k, \sigma(t_k)), \sigma(t_k)\epsilon\underline{N}, k = 0, 1, ...,$ is called switched sequence, $t_0 = 0$ is the initial time and t_k denotes the k^{th} switching instant. Moreover, $\sigma(t) = i$ means that the i^{th} subsystem is activated. N denotes the number of the subsystem. For $i \in \underline{N}, A_i$ and B_i are know real constant matrices. $f_i(\bullet) : \mathbb{R}^n \to \mathbb{R}^n$ is an unknown nonlinear function satisfying

$$\|f_i(x(t), t) - f_i(\hat{x}(t), t)\| \le \|U_i(x(t) - \hat{x}(t))\|, \tag{2.4}$$

$x(t)$, $\hat{x}(t) \in \mathbb{R}^n$ and U_i are known real constant matrices.

Definition 2.1. [20] For given positive number c_1, c_2, T and a symmetric positive definite matrix M , the uncertain switched nonlinear systems (2.1) is finite-time stable (FTS) with respect to(c_1, c_2, T, M) if the following condition hold

$$\sup_{-d\le\ \theta\le 0} \{\phi(s)^T M\phi(s), \dot{\phi}(s)^T M\dot{\phi}(s)\} < c_1 \Rightarrow x^T(t)Mx(t) < c_2; \forall t \in [0, T]. \tag{2.5}$$

Definition 2.2. [8] For any $T_2 > T_1 > 0$, let $N_\sigma(T_1, T_2)$ denote the switching number of $\sigma(t)$ on an interval (T_1, T_2), if

$$N_\sigma(T_1, T_2) \le N_0 + (T_1, T_2)/\tau_a \tag{2.6}$$

holds for given $N_0 \ge 0, \tau_a > 0$. Then the constant τ_a is called the average dwell time (ADT) and N_0 is the chatter bound. Without loss of generality,we choose $N_0 = 0$ in this paper.Before concluding this section, the following lemmas are given which will be used in the main results.

Lemma 2.3. [8] (Gronwall- Bellman inequality). *Let x(t), y(t) be real - valued non-negative continuous function with domain $\{t \mid t \ge t_0\}$, a is a nonnegative scalar, if the following inequality*

$$x(t) \le a + \int_{t_0}^{t} x(s)y(s) \tag{2.7}$$

holds, for $t \ge t_0$, then $x(t) \le a\ exp(\int_{t_0}^{t} y(s)ds)$.

Proposition 2.4. [20] (Schur complement lemma). *Given matrices X,Y,Z, where $Y = Y^T > 0$ and $X = X^T, X + Z^T Y^{-1} Z < 0$ if and only if*

$$\begin{bmatrix} X & Z^T \\ Z & -Y \end{bmatrix} < 0. \tag{2.8}$$

Proposition 2.5. [21] *Let E, H and F be any constant matrices of appropriate dimensions and $F^T F \le I$. For any $\epsilon < 0$, we have*

$$EFH + H^T F^T E^T \le \epsilon EE^T + \epsilon^{-1} H^T H. \tag{2.9}$$

3. Main Result

Theorem 3.1. *For a given positive numbers c_1, c_2, T, positive constant $\alpha > 0$, a symmetric positive definite matrix M and any matrices U_i, if there exist positive definite matrices P_i, R_i and any matrices K_i, W_i, X_i, Y_i, Z_i such that the following matrix inequality holds:*

$$\Psi_i = \begin{bmatrix} \Psi_{11i} & \Psi_{12i} & \Psi_{13i} & \Psi_{14i} & \Psi_{15i} & \Psi_{16i} & 0 & 0 & \Psi_{19i} & \Psi_{110i} \\ * & \Psi_{22i} & \Psi_{23i} & \Psi_{24i} & 0 & 0 & 0 & 0 & dX_i & 0 \\ * & * & \Psi_{33i} & \Psi_{34i} & 0 & 0 & 0 & 0 & dY_i & 0 \\ * & * & * & \Psi_{44i} & 0 & 0 & K_iE_a & K_iE_b & dZ_i & 0 \\ * & * & * & * & \frac{-1I}{\epsilon} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \frac{-1I}{\epsilon} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \frac{-1I}{\epsilon} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & \frac{-1I}{\epsilon} & 0 & 0 \\ * & * & * & * & * & * & * & * & -dR_i & 0 \\ * & * & * & * & * & * & * & * & * & -I \end{bmatrix} < 0 \quad (3.1)$$

the average dwell time satisfies

$$\tau_a > \tau_a^* = \frac{ln\mu}{(ln(\frac{c_2 a}{c_1^2 b}) \cdot \frac{1}{T}) - \alpha}, \quad (3.2)$$

$$\frac{e^{\alpha T} c_1^2 b}{a} < c_2, \quad (3.3)$$

then the system (2.1) is FTS with respect to (c_1, c_2, T, M), where
$\Psi_{11i} = A_i^T P_i + P_i A_i - \alpha P_i + W_i^T + W_i + 2\epsilon^{-1} H_a^T H_a$,
$\Psi_{12i} = P_i B_i + X_i^T - W_i, \Psi_{13i} = P_i + Y_i^T, \Psi_{14i} = A_i^T K_i^T + Z_i^T$,
$\Psi_{15i} = P_i E_a, \Psi_{16i} = P_i E_b, \Psi_{19i} = dW_i, \Psi_{110i} = U_i^T$,
$\Psi_{22i} = -X_i - X_i^T + 2\epsilon^{-1} H_b^T H_b, \Psi_{23i} = -Y_i^T, \Psi_{24i} = B_i^T K_i^T - Z_i^T$,
$\Psi_{33i} = -I, \Psi_{34i} = K_i^T, \Psi_{44i} = dR_i - 2K_i$ *and* $\mu \geq 1$ *satisfying*

$$P_i \leq \mu P_j, R_i \leq \mu R_j, \forall_{i,j} \in \underline{N}. \quad (3.4)$$

Proof Assume that the i^{th} subsystem is activated during $[t_k, t_{k+1})$ and j^{th} subsystem is activated during $[t_{k-1}, t_k)$, respectively.

For the i^{th} subsystem, we introduce a Lyapunov-Krasovskii functional candidates of the form

$$V_i(t) = V_{1i}(t) + V_{2i}(t) \quad (3.5)$$

where
$V_{1i}(t) = x^T(t) P_i x(t)$,
$V_{2i}(t) = \int_{-d}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) R_i \dot{x}(s) ds d\theta$.
We have

$$\begin{aligned} V_{1i} &= x^T(t) P_i x(t) \\ &= x^T(t) M^{\frac{1}{2}} M^{-\frac{1}{2}} P_i M^{-\frac{1}{2}} M^{\frac{1}{2}} x(t) \\ &= x^T(t) M^{\frac{1}{2}} P_i M^{\frac{1}{2}} x(t) \\ &\geq \lambda_{min}(\bar{P}_i) x^T M(t) x(t), \quad where; \bar{P}_i = M^{-\frac{1}{2}} P_i M^{-\frac{1}{2}}. \end{aligned}$$

By taking the derivative along the trajectory of system (2.1), we have

$$\begin{aligned}\dot V_{1i}(t) &= 2x^T(t)P_i\dot x(t)\\ &= x^T(t)[(A_\sigma+\Delta A_\sigma(t))^TP_i+P_i(A_\sigma+\Delta A_\sigma(t))]x(t)\\ &\quad +x^T(t)P_i(B_\sigma+\Delta B_\sigma(t))x(t-d(t))+x^T(t-d(t))(B_\sigma+\Delta B_\sigma(t))^TP_ix(t)\\ &\quad +x^T(t)P_i[f_\sigma(x(t),t)-f_\sigma(0)]+[f_\sigma(x(t),t)-f_\sigma(0)]^TP_ix(t).\end{aligned}$$

The inequality (2.4) can be written as :

$$[f_i(x(t),t)-f_i(0)]^T[f_i(x(t),t)-f_i(0)]\le x^T(t)U_i^TU_ix(t), \tag{3.6}$$

thus

$$\begin{aligned}\dot V_{1i}(t) \;\le\;& x^T(t)A_i^TP_ix(t)+x^T(t)P_iA_ix(t)+x^T(t)U_i^TU_ix(t)+x^TP_iB_ix(t-d(t))\\ &+x^T(t-d(t))B_i^TP_ix(t)+x^T(t)P_i[f_i(x(t),t)-f_i(0)]\\ &+[f_i(x(t),t)-f_i(0)]^TP_ix(t)-[f_i(x(t),t)-f_i(0)]^T[f_i(x(t),t)-f_i(0)]\\ &+\epsilon x^T(t)P_iE_aE_a^TP_ix(t)+\epsilon^{-1}x^T(t)H_a^TH_ax(t)+\epsilon x^T(t)P_iE_bE_b^TP_ix(t)\\ &+\epsilon^{-1}x^T(t-d(t))H_b^TH_bx(t-d(t)).\end{aligned} \tag{3.7}$$

$$\begin{aligned}\dot V_{2i}(t) &= d\dot x^T(t)R_i\dot x(t)-\int_{t-d}^{t}\dot x^T(s)R_i\dot x(s)d\theta\\ &\le d\dot x^T(t)R_i\dot x(t)-\int_{t-d(t)}^{t}\dot x^T(s)R_i\dot x(s)ds.\end{aligned} \tag{3.8}$$

On the other hand, by using Newton - Leibniz formula, we have

$$x(t)-x(t-d(t))=\int_{t-d(t)}^{t}\dot x(s)ds. \tag{3.9}$$

Then, for any appropriately dimensioned matrices $\psi_i=[W_i^T,X_i^T,Y_i^T,Z_i^T]^T$, we obtain

$$2\xi(t)\psi_i[x(t)-x(t-d(t))-\int_{t-d(t)}^{t}\dot x(s)ds]=0, \tag{3.10}$$

where $\xi(t)=[x^T(t)x^T(t-d(t))[f_i(x(t),t)-f_i(0)]^T\dot x^T(t)]^T$.
By using the following identity relation, we obtain

$$\begin{aligned}&\dot x^T(t)(-2K_i)(\dot x(t)-((A_\sigma+\Delta A_\sigma(t))x(t)+(B_\sigma+\Delta B_\sigma(t))x(t-d(t))\\ &+f_\sigma(x(t),t)-f_\sigma(0)))=0.\end{aligned} \tag{3.11}$$

From (3.7), (3.8), (3.10) and (3.11), we have

$$\begin{aligned}\dot V_i(t)-\alpha V_{1i} \;\le\;& \xi^T(t)(\Pi_i+d\psi_iR_i^{-1}\psi_i^T)\xi(t)-\int_{t-d(t)}^{t}[\xi^T(t)\psi_iR_i^{-1}\psi_i^T\xi(t)\\ &+\xi^T(t)\psi_i\dot x(s)+\dot x^T(s)\psi_i^T\xi(t)+\dot x^T(s)R_i\dot x(s)]ds,\end{aligned} \tag{3.12}$$

where

$$\Pi_i=\begin{bmatrix}\Phi_{11i} & P_iB_i+X_i^T-W_i & P_i+Y_i^T & A_i^TK_i^T+Z_i^T\\ * & \Phi_{21i} & -Y_i^T & B_i^TK_i^T-Z_i^T\\ * & * & -I & K_i^T\\ * & * & * & \Phi_{44i}\end{bmatrix},$$

$$\Phi_{11i}=A_i^TP_i+P_iA_i-\alpha P_i+W_i^T+W_i+U_i^TU_i+\epsilon P_iE_aE_a^TP_i$$

$$+\epsilon P_i E_b E_b^T P_i + 2\epsilon^{-1} H_a^T H_a^T,$$
$$\Phi_{21i} = -X_i - X_i^T + 2\epsilon^{-1} H_b^T H_b^T,$$
$$\Phi_{44i} = dR_i - 2K_i + \epsilon K_i E_a E_a^T K_i^T + \epsilon K_i E_b E_b^T K_i^T.$$

Note that

$$\int_{t-d(t)}^{t} [\psi_i^T \xi(t) + R_i \dot{x}(s)]^T R_i^{-1} [\psi_i^T \xi(t) + R_i \dot{x}(s)] ds > 0. \tag{3.13}$$

From **Proposition 2.4.**, $\Pi_i < 0$ is equivalent to $\Omega_i < 0$
where

$$\Omega_i = \begin{bmatrix} \hbar_{11i} & \hbar_{12i} & P_i + Y_i^T & A_i^T K_i + Z_i^T & P_i E_a & P_i E_b & 0 & 0 \\ * & \hbar_{21i} & -Y_i^T & B_i^T K_i - Z_i^T & 0 & 0 & 0 & 0 \\ * & * & -I & K_i & 0 & 0 & 0 & 0 \\ * & * & * & dR_i - 2K_i & 0 & 0 & K_i E_a & K_i E_b \\ * & * & * & * & \frac{-1I}{\epsilon} & 0 & 0 & 0 \\ * & * & * & * & * & \frac{-1I}{\epsilon} & 0 & 0 \\ * & * & * & * & * & * & \frac{-1I}{\epsilon} & 0 \\ * & * & * & * & * & * & * & \frac{-1I}{\epsilon} \end{bmatrix},$$

where

$$\hbar_{11i} = A_i^T P_i + P_i A_i - \alpha P_i + U_i^T U_i + W_i^T + W_i + 2\epsilon^{-1} H_a^T H_a,$$
$$\hbar_{12i} = P_i B_i + X_i^T - W_i$$
$$\hbar_{21i} = -X_i - X_i^T + 2\epsilon^{-1} H_b^T H_b.$$

From **Proposition 2.4.**, we have (3.1) implies

$$\Pi_i + d\psi_i R_i^{-1} \psi_i^T < 0. \tag{3.14}$$

It follows from (3.13)- (3.14), we have

$$\dot{V}_i(t) - \alpha V_{1i}(t) < 0. \tag{3.15}$$

According to (3.4) and (3.5), we get that

$$V_i(t) \le \mu V_j(t) = \mu V_j(t^-), \tag{3.16}$$

$$V_{1i}(t) \le \mu V_{1j}(t), \forall i, j \in \underline{N}. \tag{3.17}$$

From (3.15)-(3.17), for any $t \in [t_k, t_{k+1})$, we have that

$$\begin{aligned} V_{1i(t)} &\le V_i(t) \\ &= V_i(t_k) + \int_{t_k}^{t} \dot{V}_i(s) ds \\ &\le \mu V_j(t_k^-) + \alpha \int_{t_k}^{t} V_{1i}(s) ds \\ &\le \mu V_j(t_{k-1}) + \mu \int_{t_{k-1}}^{t_k} \dot{V}_j(s) ds + \alpha \int_{t_k}^{t} V_{1i}(s) ds \\ &\le \mu V_j(t_{k-1}) + \alpha \int_{t_{k-1}}^{t} V_{1i}(s) ds \\ &\le ... \\ &\le \mu^k V_j(t_0) + \alpha \int_{t_0}^{t} V_{1i}(s) ds. \end{aligned} \tag{3.18}$$

According to **Definition 2.2.** and **Lemma 2.3.**, we obtain

$$\begin{aligned} V_{1i}(t) &\leq \mu^{N_\sigma(t,t_0)} V_j(t_0) e^{+\alpha(t-t_0)} \\ &\leq e^{(\alpha+\frac{ln(\mu)}{\tau_a})(t-t_0)} V_j(t_0) \end{aligned} \tag{3.19}$$

Moreover, we have

$$a x^T(t) M x(t) \leq b \|x(t_0)\|_h^2 e^{(\alpha+\frac{\ln\mu}{\tau_a})(t-t_0)}, \tag{3.20}$$

where a $= \min_{i\in\underline{N}} \lambda_{\min}(\bar{P}_i)$, b $= \max_{i\in\underline{N}} \lambda_{\max}(P_i) + d^2 \max_{i\in\underline{N}} \lambda_{\max}(R_i)$.
From(3.19), we have

$$x^T(t) M x(t) \leq \frac{b}{a} e^{(\alpha+\frac{\ln\mu}{\tau_a})T} c_1^2. \tag{3.21}$$

From(3.3), we get that

$$x^T(t) M x(t) < c_2. \tag{3.22}$$

Hence, system (2.1) is FTS with respect to (c_1, c_2, T, M). ■

4. Numerical Example

In this section, we provide numerical example to illustrate the effectiveness of our theoretical results.
Example 4.1 Consider the uncertain switched nonlinear systems (2.1) with the following parameters :

$$A_1 = \begin{bmatrix} -2.5 & 2.2 \\ -1.8 & -2.2 \end{bmatrix}, B_1 = \begin{bmatrix} -0.3 & 0 \\ 0.1 & -0.4 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} -0.07 & 0.004 \\ 0.005 & 0.075 \end{bmatrix}, F_1 = \begin{bmatrix} sin(t) & 0 \\ 0 & sin(t) \end{bmatrix}, H_1 = \begin{bmatrix} 0.04 & -0.001 \\ 0.002 & -0.05 \end{bmatrix},$$
$$E_2 = \begin{bmatrix} -0.045 & 0.002 \\ 0.001 & 0.04 \end{bmatrix}, F_2 = \begin{bmatrix} sin(t) & 0 \\ 0 & sin(t) \end{bmatrix}, H_2 = \begin{bmatrix} 0.03 & -0.002 \\ 0.001 & 0.06 \end{bmatrix}.$$

$$A_2 = \begin{bmatrix} -1.6 & 0.1 \\ 0.2 & -1.81 \end{bmatrix}, B_2 = \begin{bmatrix} -0.4 & 0.1 \\ 0 & -0.2 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} -0.07 & 0.004 \\ 0.005 & 0.075 \end{bmatrix}, F_1 = \begin{bmatrix} sin(t) & 0 \\ 0 & sin(t) \end{bmatrix}, H_1 = \begin{bmatrix} 0.04 & -0.001 \\ 0.002 & -0.05 \end{bmatrix},$$
$$E_2 = \begin{bmatrix} -0.045 & 0.002 \\ 0.001 & 0.04 \end{bmatrix}, F_2 = \begin{bmatrix} sin(t) & 0 \\ 0 & sin(t) \end{bmatrix}, H_2 = \begin{bmatrix} 0.03 & -0.002 \\ 0.001 & 0.06 \end{bmatrix}.$$

We assume that $\alpha = 1$ and $d(t) = 1 + sin^2(t)$,

$$f_1 = \begin{bmatrix} 0.1cos(0.01x_1) \\ 0.1cos(0.01x_2) \end{bmatrix}, f_2 = \begin{bmatrix} 0.2cos(0.01x_1) \\ 0.2cos(0.01x_2) \end{bmatrix},$$

where $f_1(0) = [0.10.1]^T \neq 0, f_2(0) = [0.20.2]^T \neq 0$.
The Lipschitz matrices are given by

$$U_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, U_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}.$$

By using LMI Control Toolbox in MATLAB, LMIs (3.1) and the condition (3.2), (3.3) are feasible with solutions given by

$$d = 1, \varepsilon = 3, T = 8, c_1 = 0.03, c_2 = 6, M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$P_1 = \begin{bmatrix} 0.8033 & 0.0049 \\ 0.0049 & 0.8171 \end{bmatrix}, P_2 = \begin{bmatrix} 0.8057 & 0.0053 \\ 0.0053 & 0.8158 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 0.2792 & 0.0138 \\ 0.0138 & 0.2996 \end{bmatrix}, R_2 = \begin{bmatrix} 0.2869 & 0.0140 \\ 0.0140 & 0.2883 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} 0.3095 & -0.0049 \\ 0.0203 & 0.3472 \end{bmatrix}, K_2 = \begin{bmatrix} 0.3325 & 0.0209 \\ -0.0024 & 0.3208 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 0.3395 & -0.0107 \\ -0.0265 & 0.3621 \end{bmatrix}, W_2 = \begin{bmatrix} 0.3571 & -0.0156 \\ 0.0012 & 0.3493 \end{bmatrix},$$

$$X_1 = \begin{bmatrix} 0.2495 & -0.0065 \\ 0.0011 & 0.2833 \end{bmatrix}, X_2 = \begin{bmatrix} 0.2764 & 0.0036 \\ -0.0052 & 0.2428 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} -0.0943 & -0.0130 \\ 0.0080 & -0.1127 \end{bmatrix}, Y_2 = \begin{bmatrix} -0.1083 & 0.0061 \\ -0.0139 & -0.0942 \end{bmatrix},$$

$$Z_1 = \begin{bmatrix} 0.0587 & -0.0970 \\ 0.0822 & 0.0721 \end{bmatrix}, Z_2 = \begin{bmatrix} 0.0718 & 0.0851 \\ -0.0957 & 0.0684 \end{bmatrix}.$$

Thus, from **Theorem 3.1.**, the uncertain switched nonlinear systems is FTS with respect to (c_1, c_2, T, M). From (3.2), we get $\tau_a > \tau_a^* = \frac{\ln \mu}{(\ln(\frac{c_2 a}{c_1^2 b}) \cdot \frac{1}{T}) - \alpha} = 1.8589$. In this case, we choose $\tau_a = 2$. The trajectories of solution of the switched is given in Figure 3. In Figure 4, it is shown that if the initial condition satisfies

$$\sup_{-d \leq \theta \leq 0} \{\|x(t_0 + \theta)\|, \|\dot{x}(t_0 + \theta)\|\} < c_1,$$

then we have

$$x^T(t) M x(t) < c_2, \forall t \in [0, 8],$$

where $x(0) = \begin{bmatrix} 0.03 \\ -0.03 \end{bmatrix}, \varphi(s) = \begin{bmatrix} 0.03 + sin(s) \\ -0.03 + sin(s) \end{bmatrix}, s \in [-d, 0)$.

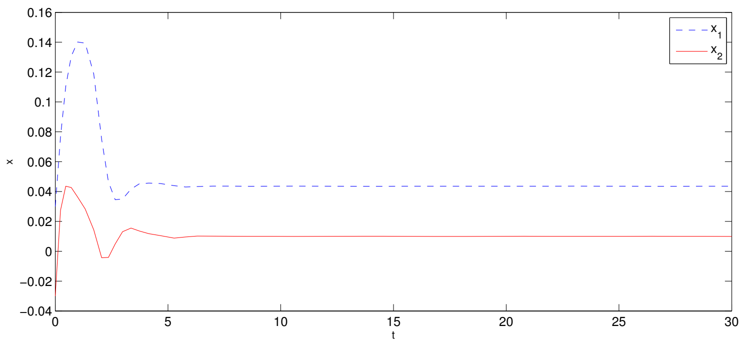

FIGURE 1. The trajectory of solution of subsystem 2.

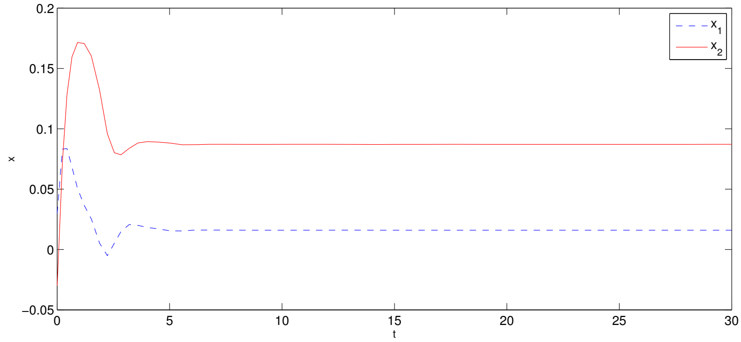

FIGURE 2. The trajectory of solution of subsystem 2.

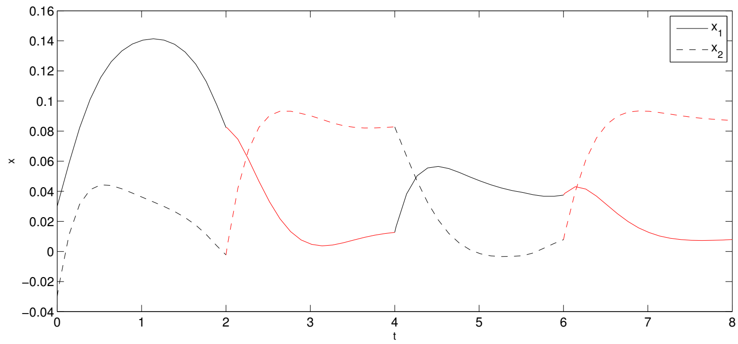

FIGURE 3. The trajectory of solution of switched system.

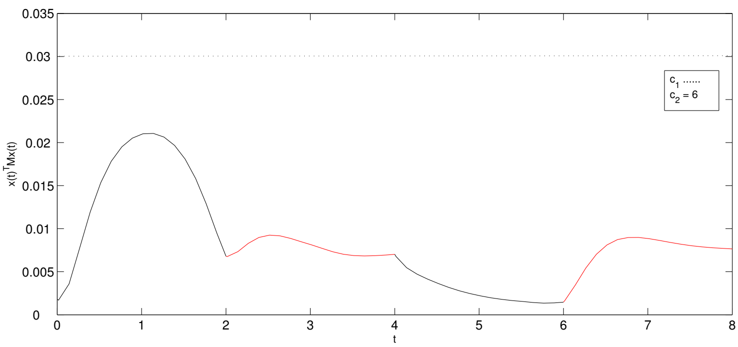

FIGURE 4. Bounds of $x^T(t)Mx(t)$ where $x(t)$ is the state of switched systems.

5. Conclusion

In this paper, the problem of finite- time stability for a class of uncertain switched nonlinear systems with time-varying delay have been studied. By introducing an appropriate Lyapunov-Krasovskii functional and using Gronwall-Bellman inequality, the conditions for FTS of the systems have been established in terms of LMIs which could be solved by various available algorithms.

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