Thai Journal of **Math**ematics
Volume 20 Number 2 (2022)
Pages 669–677

http://thaijmath.in.cmu.ac.th

ISSN 1686-0209

A Model of Hepatitis C and Blood Pressure via Nano Topological Spaces

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Abstract Rough sets concept are important in real life applications. A nano topological space became a new type of modern topology in terms of rough sets. In this paper, we aim to analyze some real life problems using nano topology, especially, in medicine. We find that the key factors is necessary to decide whether a patient has Hepatitis C or not. Also, we conclude the key attribute has close connection to the disease of high blood pressure or hypertension.

MSC: 54B05; 54C05

Keywords: Nano topology; Rough sets; Lower Approximation; Upper Approximation; Hepatitis C; Blood pressure; Core

Submission date: 15.07.2019 / Acceptance date: 02.11.2019

1. Introductory Remarks and Review of Literature

The Nobel prize 2016 in physics was gifted to three scientists in phase transitions and topological phases of matter, this event has directed the attentions to the need of more knowledge about topology. The presented paper aims to introduce a new decision for medical problems with the help of rough mathematics of Pawlak [6, 7]. A topological reduction of attributes applied in set-valued ordered information systems in finding the key foods which is suitable for two age groups in order to be healthy [11] in terms of basis of nano topology. It is also a tool suitable for analyzing not only quantitative attributes but also qualitative ones. The results of the nano open set model are easy to understand, while the results from other methods need an interpretation of the technical parameters. Thus it is advantageous to use nano topology in real life situations. There are some recent studies in 2017 that also involve topological investigation for medical diagnosis, see [2, 13].

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Published by The Mathematical Association of Thailand.

In our study, we have shown that real world problems can be dealt with the nano topology. The concept of basis has been applied to find the deciding factors of a recent outbreak "Hepatitis C" which had been reported, especially, in Egypt and a chronic disease "High Blood Pressure". We could find that 'Yellow skin and eyes' and 'Fever' are the deciding factors for Hepatitis C and rapid heartbeat is the only deciding symptom for blood pressure.

1.1. Lower and Upper Approximations

Rough set theory, first introduced by Pawlak in 1982 [6], helps to make decision rules from a decision system. A decision system is a special case of an information system. In a medical decision system, upper and lower approximations are used to describe these uncertain concepts roughly. Formally, for a medical decision system, we can approximate X using only the information contained in a nonempty finite set of objects called the universe U by constructing the medical lower and medical upper approximations of X , denoted by $MedL_R(X)$ and $MedH_R(X)$, respectively, where $MedL_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by x and $MedH_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$. The set $MedH_R(X) - MedL_R(X)$ is called the medical boundary region of X and denoted by $MedB_R(X)$. According to Pawlak$'$s definition X is called a rough set as shown in Figure 1. In this figure, the whole liver can be illustrated by a nonempty set X. The cell with Hepatitis C virus can be illustrated by $MedL_R(X)$. Equivalence classes are indeed in the core of the definition of rough sets. As a generalization of equivalence classes, coverings can also be used to define lower and upper approximations. The union of equivalence classes which intersects with liver is denoted by $MedH_R$. The difference between $MedH_R$ and $MedL_R(X)$ is $MedB_R(X)$.

If $MedH_R(X) \neq MedL_R(X)$, then X is a Pawlak rough set. Elements belonging to the same equivalence class are said to be indiscernible with one another. The results in the following proposition is equivalent to Pawlak's approximations ([6],[7]).

Proposition 1.1. *If (U, R) is an approximation space and $X, Y \subseteq U$, then we have the following properties:*
(1) $MedL_R(X) \subseteq X \subseteq MedH_R(X)$ (Contractions, Extensions);
(2) $MedL_R(\phi) = MedH_R(\phi) = \phi$ (Normality) and, $MedL_R(U) = MedH_R(U) = U$ (Co-normality) ;
(3) $MedH_R(X \cup Y) = MedH_R(X) \cup MedH_R(Y)$ (Addition);
(4) $MedH_R(X \cap Y) \subseteq MedH_R(X) \cap MedH_R(Y)$ (Multiplication);
(5) $MedL_R(X \cup Y) \supseteq MedL_R(X) \cup MedL_R(Y)$;
(6) $MedL_R(X \cap Y) = MedL_R(X) \cap MedL_R(Y)$ (Multiplication);
(7) $MedL_R(X) \subseteq MedL_R(Y)$ and $MedH_R(X) \subseteq MedH_R(Y)$ whenever $X \subseteq Y$ (Monotone);
(8) $MedH_R(X^c) = [MedL_R(X)]^c$ and $MedL_R(X^c) = [MedH_R(X)]^c$ (Complementary operation);
(9) $MedH_R(MedH_R(X)) = MedL_R(MedH_R(X)) = MedH_R(X)$ (Idempotency);
(10) $MedL_R(MedL_R(X)) = MedH_R(MedL_R(X)) = MedL_R(X)$ (Idempotency).

1.2. Fundamentals of Nano Topology

Lellis Thivagar et al. [10] introduced a nano topological space with respect to a subset X of an universe set U which is defined in terms of lower and upper approximations

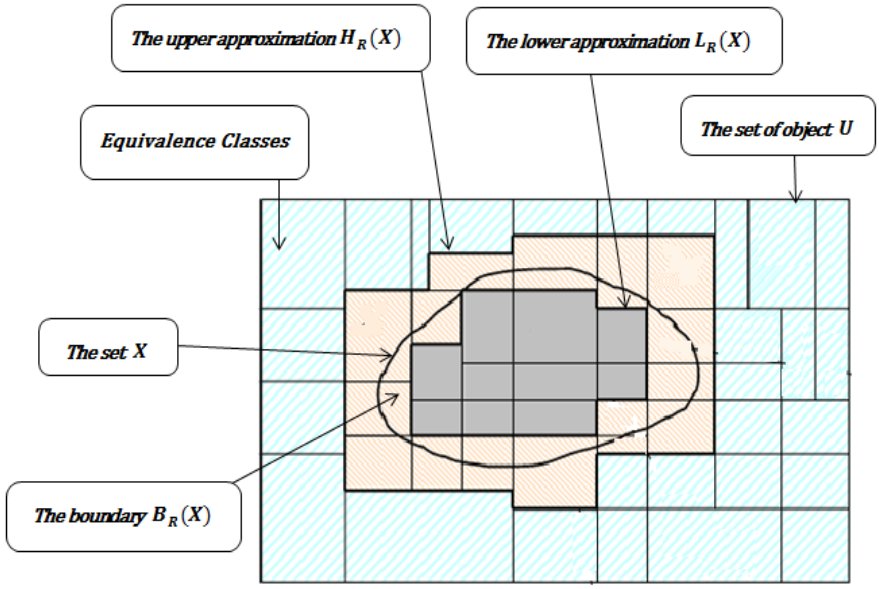

FIGURE 1. A rough set.

and boundary region of X. The elements of a new topological space are called the nano open sets. But certain nano terms are satisfied simply to mean "draft" in its modern scientific sense in order to magnititude-one billionth of something. Nano car is an example. The topology recommended here is named so because of its size, since it has at most five elements in it. The following definition is equivalent to Lellis Thivagar and Carmel Richard for a nano topological space [5, 8, 12, 13] and more recently, see [9]. Lashin et al. in [3] induced the topology by binary relations which used to generalize the basic rough set concepts. The suggested topological structure opened up the way for applying rich amount of topological facts and methods in the process of granular computing.

Definition 1.2. *Let U be the universe set and R be an equivalence relation on U, then for $X \subseteq U$, $\tau_R(X) = \{U, \phi, MedL_R(X), MedH_R(X), MedB_R(X)\}$ is called a nano topology on U and satisfies the following axioms:*
(i) U and $\phi \in \tau_R(X)$;
(ii) The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$;
(iii) The intersection of the elements of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$. We call $(U, \tau_R(X))$ a nano topological space. The elements of $\tau_R(X)$ are called nano open sets, $[\tau_R(X)]^c$ called nano closed sets. The basis of $\tau_R(X)$ is $\beta(\tau_R(X)) = \{U, MedL_R(X), MedB_R(X)\}$.

Definition 1.3. *[9, 10] If τ_R is the nano topology on U with respect to X, then the class $\beta(\tau_R(X)) = \{U, MedL_R(X), MedB_R(X)\}$ is the basis for $\tau_R(X)$.*

2. Methodology

Let us note that the inclusions in Proposition 1.1 can not be in general substituted by the equalities. This has some important algorithmic and logical consequences. Also, We investigate more new properties of Proposition 1.1 through the following examples.

Example 2.1. *If $U = \{p_1, p_2, p_3, p_4, p_5\}$ is a universe set and an equivalence relation R with an equivalence classes $U/R = \{\{p_1\}, \{p_2\}, \{p_5\}, \{p_3, p_4\}\}$, then*
(1) $MedH_R(X) \cap MedH_R(Y) \nsubseteq MedH_R(X \cap Y)$. Take $X = \{p_3, p_5\}$ and $Y = \{p_4, p_5\}$. $MedH_R(X \cap Y) = MedH_R(\{p_5\}) = \{p_5\}$, and $MedH_R(X) \cap MedH_R(Y) = \{p_3, p_4, p_5\}$.
(2) $MedL_R[MedH_R(X)] \nsubseteq X$. Take $X = \{p_3\}$, then $MedH_R(X) = \{p_3, p_4\}$ and $MedL_R[MedH_R(X)] = \{p_3, p_4\}$.
(3) $X \nsubseteq MedH_R[MedL_R(X)]$. Take $X = \{p_3\}$, then $Med\ L_R(X) = \phi$ and so $MedH_R[MedL_R(X)] = \phi$.

Example 2.2. *If $U = \{p_1, p_2, p_3, p_4, p_5\}$ is a universe set and a general binary relation R with $U/R = \{\{p_1, p_3\}, \{p_3, p_4\}, \{p_3, p_5\}, \{p_1, p_2, p_3\}, \{p_1, p_4, p_5\}\}$, then*
(1) $MedL_R(X \cup Y) \nsubseteq MedL_R(X) \cup MedL_R(Y)$. Because if $X = \{p_1, p_2\}$ and $Y = \{p_3\}$, then $X \cup Y = \{p_1, p_2, p_3\}$, $MedL_R(\{p_1, p_2, p_3\}) = \{p_1, p_2, p_3\}$. But $MedL_R(\{p_1, p_2\}) = \phi$ and $MedL_R(\{p_3\}) = \phi$. Then, $MedL_R(X) \cup MedL_R(Y) = \phi$.
(2) $MedL_R(X)^c \neq [MedH_R(X)]^c$. We take $X = \{p_1\}$, then $[X]^c = \{p_2, p_3, p_4, p_5\}$, and $MedL_R(\{p_2, p_3, p_4, p_5\}) = \{p_3, p_4, p_5\}$. Also, $MedH_R(X) = U$ and $[MedH_R(X)]^c = \phi$.
(3) $MedH_R(X)^c \neq [MedL_R(X)]^c$. Because if $X = \{p_1, p_2, p_3\}$, then $[X]^c = \{p_4, p_5\}$, and $MedH_R(\{p_4, p_5\}) = \{p_1, p_3, p_4, p_5\}$. Also, $MedL_R(X) = \{p_1, p_2, p_3\}$ and $[MedL_R(X)]^c = \{p_4, p_5\}$.
(4) $MedH_R[Med\ H_R(X)] \nsubseteq Med\ H_R(X)$. Take $X = \{p_4, p_5\}$, then $Med\ H_R(X) = \{p_1, p_3, p_4, p_5\}$ and $MedH_R[MedH_R(X)] = U$.
(5) $MedH_R[MedL_R(X)] \nsubseteq Med\ L_R(X)$. Take $X = \{p_1, p_3\}$, then $MedL_R(X) = \{p_1, p_3\}$ and so $MedH_R\ [MedL_R(X)] = U$.

Definition 2.3. *In terms of basis of a nano topological space $(X, \tau_R(X))$, a criterion reduction of an equivalence relationship information system is a minimal attribute subset B of R such that $\beta(\tau_B(X)) = \beta(\tau_R(X))$ and $CORE(R) = \{a \in R : \beta(\tau_R(X)) \neq \beta(\tau_{R-(a)}(X))\}$.*

The following algorithm enables us to determine the CORE(R).

Algorithm 2.4. *We have the following procedures:*
Input: *Information system for patients.*
Output: *The core of equivalence relation R.*
Step (1): *If R is an equivalence relationship information system (U, R), where R is divided into two classes of condition attributes and decision attribute. The class U/R on U corresponding to R and a subset X of U, represent the data as an information table, columns of which are labelled by attributes, rows by objects and entries of the table are attribute values.*
Step (2): *Find the lower approximation $MedL_R(X)$, upper approximation $MedH_R(X)$ and the boundary region $MedB_R(X)$ of X with respect to R.*
Step (3): *Generate the nano topology $\tau_R(X)$ and its basis $\beta(\tau_R(X))$.*
Step (4): *Remove an attribute x from U/R and find the lower and upper approximations and the boundary region of X with respect to the dominance relation on $U/R - (x)$.*
Step (5): *Generate the nano topology $\tau_{R-(x)}(X)$ and the corresponding basis $\beta(\tau_{R-(x)}(X))$.*

Table 1: Infection information about 8 patients.

Patients	Yellow skin and eyes(Y)	Dark urine(D)	Joint and abdominal pains(J)	Fever(F)	Hepatitis C
P_1	1	1	1	+	Yes
P_2	1	0	0	+	No
P_3	1	0	0	+	Yes
P_4	0	0	0	++	No
P_5	0	1	1	+	No
P_6	1	1	0	++	Yes
P_7	1	1	0	-	No
P_8	1	1	0	++	Yes

Step (6): *Repeat steps 4 and 5 for all attributes in U/R.*
Step (7): *Those attributes in U/R for which $\beta(\tau_R(X)) \neq \beta(\tau_{R-(x)}(X))$ form the criterion reduction.*
Step (8): *If there is more than one criterion reduction, their intersection gives CORE(R).*

3. Hepatitis C in a View Point of Nano Topology

Liver is responsible for many important things in the human body, it works to purify the body and blood of toxins, but neglects much health and safety of the liver even sometimes exposed to cancer diseases that threaten human life. Hepatitis C virus (HCV)[1] infection started as an epidemic between the 1960s and 1980s. Hepatitis C is an infectious disease caused by the Hepatitis C virus (HCV) that primarily affects the liver. During the initial infected people often have mild or no symptoms and sometimes a fever, a dark urine, joint and abdominal pains and yellow skin and eyes occurs. In recent decades the disease has spread to the world, especially, in Egypt which has a good experience in treatment of Hepatitis C. In Table 1, consider the following information giving data about 8 patients.

The symbol '1' means the patient has the symptom and '0' otherwise. Also the symbol '+' means the patient has high fever, '++' means the patient has very high fever and '-' means the patient has no fever. The columns of the table represent the attributes (the symptoms for Hepatitis C) and the rows represent the objects (the patients). The entries in the table are the attribute values. The patient P_1 is characterized by the value set (Yellow skin and eyes, 1), (Dark urine, 1), (Joint and abdominal pains, 1), (Fever, +) and (Hepatitis C, 1), which gives information about the patient P_1. In the table, the patients P_1, P_2, P_3, P_6, P_7 and P_8 are indiscernible with respect to the attribute 'Yellow skin and eyes'. The attribute 'Yellow skin and eyes' generates two equivalence classes, namely, $\{P_1, P_2, P_3, P_6, P_7, P_8\}$ and $\{P_4, P_5\}$. The attributes "Yellow skin and eyes" and "Dark urine" generate the equivalence classes $\{P_1, P_6, P_7, P_8\}$, $\{P_2, P_3\}$, $\{P_4\}$ and $\{P_5\}$. The equivalence classes for the attributes "Yellow skin and eyes", "Dark urine", "Joint and abdominal pains" and "Fever" are $\{P_1\}$, $\{P_2, P_3\}$, $\{P_4\}$, $\{P_5\}$, $\{P_6, P_8\}$ and $\{P_7\}$. For the set of patients having hepatitis C, the lower approximation $= \{P_1, P_6, P_8\}$ and the upper approximation $= \{P_1, P_2, P_3, P_6, P_8\}$ and hence the boundary region $= \{P_2, P_3\}$. Hence the patients P_2 and P_3 can not be uniquely classified in view of the available knowledge. The patients P_1, P_6 and P_8 display symptoms which enable us to classify them with certainty as having Hepatitis C. In our case, the symptoms Yellow skin and eyes, Dark

urine, Joint and abdominal pains and Fever are considered as the condition attributes and the disease Hepatitis C is considered as the decision attribute. Not all condition attributes in an information system are necessary to depict the decision attribute before decision rules are generated. It may happen that the decision attribute depends not on the whole set of condition attributes but on a subset of it and hence we are interested to find this subset which is given by the core. Here $U = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}$.

Case 1: (Patients with Hepatitis C) Let $X = \{P_1, P_3, P_6, P_8\}$ be the set of patients having Hepatitis C. Let R be the equivalence relation on U with respect to the condition attributes. The family of equivalence classes corresponding to R is given by:

$U/R = \{\{P_1\}, \{P_4\}, \{P_5\}, \{P_7\}, \{P_2, P_3\}, \{P_6, P_8\}\}\}$.

The medical lower and medical upper approximations of X with respect to R are given by $MedL_R(X) = \{P_1, P_6, P_8\}$ and $MedH_R(X) = \{P_1, P_2, P_3, P_6, P_8\}$. So the medical boundary of X with respect to R is $MedB_R(X) = \{P_2, P_3\}$. Therefore, the nano topology on U with respect to X is given by:

$\tau_R(X) = \{\phi, U, \{P_2, P_3\}, \{P_1, P_6, P_8\}, \{P_1, P_2, P_3, P_6, P_8\}\}$.

The basis for the topology $\tau_R(X)$ is given by $\beta(\tau_R(X)) = \{U, \{P_2, P_3\}, \{P_1, P_6, P_8\}\}$.
Step 1: If we remove the attribute 'Yellow skin and eyes' from the set of condition attributes, the family of equivalence classes corresponding to the resulting set of attributes is given by $U/R - (Y) = \{\{P_1, P_5\}, \{P_2, P_3\}, \{P_4\}, \{P_6, P_8\}, \{P_7\}\}$. The corresponding medical lower and medical upper approximations are given by $MedL_{R-(Y)}(X) = \{P_6, P_8\}$ and $MedH_{R-(Y)}(X) = \{P_1, P_2, P_3, P_5, P_6, P_8\}$. So $MedB_{R-(Y)}(X) = \{P_1, P_2, P_3, P_5\}$. Hence
$\tau_{R-(Y)}(X) = \{\phi, U, \{P_6, P_8\}, \{P_1, P_2, P_3, P_5, P_6, P_8\}, \{P_1, P_2, P_3, P_5\}\}$ and its basis $\beta(\tau_{R-(Y)}(X)) = \{U, \{P_6, P_8\}, \{P_1, P_2, P_3, P_5\}\} \neq \beta(\tau_R(X))$.
Step 2: If we remove the attribute 'Dark urine' from the set of condition attributes, the family of equivalence classes corresponding to the resulting set of attributes is given by $U/R - (D) = \{\{P_1\}, \{P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_8\}, \{P_7\}\}$ which is the same of U/R and hence $\tau_{R-(D)}(X) = \tau_R(X)$ and $\beta(\tau_{R-(D)}(X)) = \beta(\tau_R(X))$. On removal of the attribute 'Joint and abdominal pains', we get $U/R-(J) = \{\{P_1\}, \{P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_8\}, \{P_7\}\} = U/R$ and hence $\tau_{R-(J)}(X) = \tau_R(X)$ and $\beta(\tau_{R-(J)}(X)) = \beta(\tau_R(X))$.
Step 3: When the attribute 'Fever' is omitted,
$U/R-(F) = \{\{P_1\}, \{P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_7, P_8\}\}$. Then, $MedL_{R-(F)}(X) = \{P_1\}$ and $MedH_{R-(F)}(X) = \{P_1, P_2, P_3, P_6, P_7, P_8\}$. So, $MedB_{R-(F)}(X) = \{P_2, P_3, P_6, P_7, P_8\}$. Therefore, $\tau_{R-(F)}(X) = \{\phi, U, \{P_1\}, \{P_1, P_2, P_3, P_6, P_7, P_8\}, \{P_2, P_3, P_6, P_7, P_8\}\}$ and its basis $\beta(\tau_{R-(F)}(X)) = \{U, \{P_1\}, \{P_2, P_3, P_6, P_7, P_8\} \neq \beta(\tau_R(X))$. If $M = \{Y, F\}$, then $U/r = \{\{P_1, P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_8\}, \{P_7\}\}$, $MedL_r(X) = \{P_6, P_8\}$, $Med\ H_r(X) = \{P_1, P_2, P_3, P_6, P_8\}$, $MedB_r(X) = \{P_1, P_2, P_3\}$. Therefore, the basis for the nano topology corresponding to M is given by $\beta(\tau_M(X)) = \{U, \{P_6, P_8\}, \{P_1, P_2, P_3\}\}$. Also $\beta(\tau_M(X)) \neq \beta(\tau_{R-(x)}(X))$ for all $x \in \{Y, F\}$. Therefore **CORE(R)** $= \{Y, F\}$.

Case 2: (Patients not with Hepatitis C) Let $X = \{P_2, P_4, P_5, P_7\}$ be the set of patients not having Hepatitis C. Then, $U/R = \{\{P_1\}, \{P_4\}, \{P_5\}, \{P_7\}, \{P_2, P_3\}, \{P_6, P_8\}\}\}$. Then, $MedL_R(X) = \{P_4, P_5, P_7\}$, $MedH_R(X) = \{P_2, P_3, P_4, P_5, P_7\}$

and $MedB_R(X) = \{P_2, P_3\}$. Therefore, $\tau_R(X) = \{\phi, U, \{P_4, P_5, P_7\}, \{P_2, P_3, P_4, P_5, P_7\}, \{P_2, P_3\}\}$ and $\beta(\tau_R(X)) = \{U, \{P_4, P_5, P_7\}, \{P_2, P_3\}\}$.
Step 1: Omitting the attribute 'Yellow skin and eyes',

$U/R - (Y) = \{\{P_1, P_5\}, \{P_2, P_3\}, \{P_4\}, \{P_6, P_8\}, \{P_7\}\}$,

$Med\ L_{R-(Y)}(X) = \{P_4, P_7\}$, $Med\ H_{R-(Y)}(X) = \{P_1, P_2, P_3, P_4, P_5, P_7\}$, $Med\ B_{R-(Y)}(X) = \{P_1, P_2, P_3, P_5\}$, $\tau_{R-(Y)}(X) = \{\phi, U, \{P_4, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_7\}, \{P_1, P_2, P_3, P_5\}\}$ and its basis $\beta(\tau_{R-(Y)}(X)) = \{U, \{P_4, P_7\}, \{P_1, P_2, P_3, P_5\}\} \neq \beta(\tau_R(X))$.
Step 2: If the attribute 'Dark urine' is removed,
$U/R - (D) = \{\{P_1\}, \{P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_8\}, \{P_7\}\}$ which is the same of U/R and hence $\beta(\tau_{R-(D)}(X)) = \beta(\tau_R(X))$.
Step 3: When the attribute 'Joint and abdominal pains' is removed, we get

$U/R - (J) = \{\{P_1\}, \{P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_8\}, \{P_7\}\} = U/R$

and hence $\beta(\tau_{R-(J)}(X)) = \beta(\tau_R(X))$.
Step 4: When the attribute 'Fever' is omitted, we get
$U/R-(F) = \{\{P_1\}, \{P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_7, P_8\}\}$. Then, $Med\ L_{R-(F)}(X) = \{P_4, P_5\}$ and $Med\ H_{R-(F)}(X) = \{P_2, P_3, P_4, P_5, P_6, P_7, P_8\}$. So, $Med\ B_{R-(F)}(X) = \{P_2, P_3, P_6, P_7, P_8\}$. Therefore,

$\tau_{R-(F)}(X) = \{\phi, U, \{P_4, P_5\}, \{P_2, P_3, P_4, P_5, P_6, P_7, P_8\}, \{P_2, P_3, P_6, P_7, P_8\}\}$

and its basis $\beta(\tau_{R-(F)}(X)) = \{U, \{P_4, P_5\}, \{P_2, P_3, P_6, P_7, P_8\} \neq \beta(\tau_R(X))$.
Step 5: If $M = \{Y, F\}$, then $U/r = \{\{P_1, P_2, P_3\}, \{P_4\}, \{P_5\}, \{P_6, P_8\}, \{P_7\}\}$, $Med\ L_r(X) = \{P_4, P_5, P_7\}$, $Med\ H_r(X) = \{P_1, P_2, P_3, P_4, P_5, P_7\}$, $Med\ B_r(X) = \{P_1, P_2, P_3\}$. Therefore, $\tau_r(X) = \{\phi, U, \{P_1, P_2, P_3\}, \{P_4, P_5, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_7\}\}$ and its basis

$\beta(\tau_r(X)) = \{U, \{P_1, P_2, P_3\}, \{P_4, P_5, P_7\}\} \neq \beta(\tau_{R-(x)}(X))$

for every $x \in M$ and r is the equivalence relation on U with respect to M. Therefore, here again, **CORE(R)** $= \{Y, F\}$.

Observation 3.1. *From both cases we conclude that both of 'Yellow skin and eyes' and 'Fever' are the key attributes necessary to decide whether a patient has Hepatitis C or not. In this case the patient must analyze HCV which declare if he has virus C or not. If the result is positive, the patient must analyze PCR which confirm the existence of virus C and liver function tests.*

4. High Blood Pressure in View Point of Nano Topology

High blood pressure is also known as hypertension. Blood pressure is the amount of force exerted against the walls of the arteries as blood flows through them. If a person has high blood pressure it means that the walls of the arteries are receiving too much pressure repeatedly. This high blood pressure produces the classical symptoms of rapid heartbeat, headaches and difficulty urinating. Consider the following table giving information about six patients as shown in Table 2.

Consider $U = \{P_1, P_2, P_3, P_4, P_5, P_6\}$ and rapid heartbeat (T) and headaches(E)and difficulty urinating(D) form the condition attributes. Let $X = \{P_1, P_2, P_3\}$ be the set of

Table 2 : Infection information about 6 patients.				
Patients	Rapid heartbeat(T)	Headaches(E)	Difficulty urinating(D)	High Blood Pressure
P_1	1	1	0	Yes
P_2	1	0	1	Yes
P_3	1	0	0	Yes
P_4	0	1	1	No
P_5	0	1	0	No
P_6	0	0	1	No

patients having high blood pressure and the quotient set
$U/R = \{\{P_1\},\{P_2\},\{P_3\},\{P_4\},\{P_5\}\},\{P_6\}\}$. The medical lower and medical upper approximations of X with respect to R are given by $MedL_R(X) = \{P_1, P_2, P_3\}$, $MedH_R(X) = \{P_1, P_2, P_3\}$ and $MedB_R(X) = \phi$. The nano topology on U is $\tau_R(X) = \{\phi, U, \{P_1, P_2, P_3\}\}$ and its basis is $\beta(\tau_R(X)) = \{U, \phi, \{P_1, P_2, P_3\}\}$. If the attribute 'Rapid heartbeat' is removed from the set of condition attributes, then $U/R-(T) = \{\{P_1, P_5\}, \{P_3\}, \{P_4\}, \{P_2, P_6\}\}$, $MedL_{R-(T)}(X) = \{P_3\}$, $MedH_{R-(T)}(X) = \{P_1, P_2, P_3, P_5, P_6\}$, $MedB_{R-(T)}(X) = \{P_1, P_2, P_5, P_6\}$ and hence

$$\tau_{R-(T)}(X) = \{\phi, U, \{P_3\}, \{P_1, P_2, P_5, P_6\}, \{P_1, P_2, P_3, P_5, P_6\}\}$$

and its basis $\beta(\tau_{R-(T)}(X)) = \{U, \{P_3\}, \{P_1, P_2, P_5, P_6\}\} \neq \beta(\tau_R(X))$.
If the attribute 'Headaches' is removed, then $U/R-(E) = \{\{P_2\}, \{P_5\}, \{P_1, P_3\}, \{P_4, P_6\}\}$, $MedL_{R-(E)}(X) = \{P_1, P_2, P_3\}$, $MedH_{R-(E)}(X) = \{P_1, P_2, P_3\}$, $MedB_{R-(E)}(X) = \phi$ and hence $\tau_{R-(E)}(X) = \{\phi, U, \{P_1, P_2, P_3\}\}$ and its basis $\beta(\tau_{R-(E)}(X)) = \{U, \phi, \{P_1, P_2, P_3\}\} = \beta(\tau_R(X))$. If the attribute 'Difficulty urinating' is removed, then $U/R-(D) = \{\{P_1\}, \{P_2, P_3\}, \{P_4, P_5\}, \{P_6\}\}$, $MedL_{R-(D)}(X) = \{P_1, P_2, P_3\}$, $MedH_{R-(D)}(X) = \{P_1, P_2, P_3\}$, $MedB_{R-(D)}(X) = \phi$. Therefore, $\tau_{R-(D)}(X) = \tau_R(X)$ and its basis $\beta(\tau_{R-(D)}(X)) = \beta(\tau_R(X))$.
Now, if we put $M = \{T\}$, then we remark that $\beta(\tau_M(X)) \neq \beta(\tau_{R-(T)}(X))$. Therefore **CORE(R)** $= \{T\}$. Similarly if X is taken as the set of patients not having high blood pressure, then again **CORE(R)** $= \{T\}$.

Observation 4.1. *The core of R has $\{T\}$ as its only element. So 'Rapid heartbeat' is the key attribute that has close connection to the disease high blood pressure.*

5. Conclusions

We have applied the equivalence relationship information systems in attribute reduction using the basis of nano topology in two real life situations. Here we have shown by means of topological reduction the decision factors of a recent outbreak "Hepatitis C" which had been reported, especially, in Egypt and a chronic disease "High Blood Pressure". We could find that 'Yellow skin and eyes' and 'Fever' are the deciding factors for Hepatitis C and Rapid heartbeat is the only deciding symptom for High Blood Pressure.

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