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On Some Properties of the Option Price Related to the Solution of the Black-Scholes Equation

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Abstract In functional analysis, the spectrum of the operators is the one of interests which having interesting properties. In this paper, we applied such spectrum to the option price which is the solution of the Black-Scholes equation. We found that such option price is contained in the spectrum. Moreover we can find the kernel of such the Black-Scholes equation. We also obtain the interesting properties of the kernel such as the expected value and variance. However the results of this paper will useful in the boarding of the research in the area of financial mathematics.

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1. Introduction

Nowadays, the research area in financial mathematics has grown rapidly because of various kinds of investment particularly in the trading in the stock market. The popular one is the option trading. Such option can be obtained from the solution of the Black-Scholes equation was introduced in 1973, see [1, p637-654]. The model of Black-Scholes applies when the limiting distribution is the normal distribution, and explicitly assumes that the price process is continuous and that there are no jumps in asset prices, see [2–4]. We have the Black-Scholes equation is given by

$$\frac{\partial}{\partial t}u(s_t,t)+\frac{1}{2}\sigma^2 s_t^2\frac{\partial^2}{\partial s_t^2}u(s,t)+rs\frac{\partial}{\partial s_t}u(s_t,t)-ru(s_t,t)=0 \tag{1.1}$$

with the terminal condition or the call payoff

$$u(s_T,T)=\max(s_T-p,0)$$

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and denote $\max(s_T - p, 0) = (s_T - p)^+$. Thus

$$u(s_T, T) = (s_T - p)^+ \tag{1.2}$$

for $0 \le t \le T$ where $u(s_t, t)$ is the option price at time t, s_t is the stock price of any time t, T is the expiration time, σ is the volatility of stock, r is the interest rate and p is the strike price. The profit or loss for the holder depends on s_T and p from the call payoff. Suppose $s_T > p$, the holder should exercise his right to get the profit of $s_T - p$ and if $s_T \le p$, we say that the call payoff is worthless, the holder should not exercise his right, otherwise he will get loss.

They obtain the solution $u(s, t)$ of (1.1) that satisfies (1.2) of the form

$$u(s_t, t) = s_t N(d_1) - re^{-r(T-t)} N(d_2) \tag{1.3}$$

where

$$d_1 = \frac{\ln\left({}^{s_t}/_{p}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}, \; d_2 = \frac{\ln\left({}^{s_t}/_{p}\right) + \left(r - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}$$

and

$$N(x) = \frac{1}{2\pi}\int_{-\infty}^{x} e^{-\frac{y^2}{2}}\, dy$$

see [5, p91].

Equation (1.3) is call the Black-Scholes Formula. In this paper, we can find the solution of (1.1) in the other forms which useful to study some properties of the option price. Consequently, we focus on the expected value of the option price returns and consider risk parameters, such as variance. The usefulness of the expected value or mean value as a prediction for the outcome of an experiment is increased when the outcome is not likely to deviate too much from the expected value. In practical, if the variance is known, we can find the standard deviation by taking the square root of the variance. The standard deviation is a more common statistic. It shares the same units as the random variable, so it is easy to interpret. We have looked at the variance and standard deviation as measures of dispersion under the sense on averages. We can also measure the dispersion of random variables in a given distribution by using the variance and standard deviation. This allows us to better understand whatever the distribution represents.

2. Preliminaries

Before reaching the main results, the following definitions and the basic concepts are needs.

Definition 2.1 ([6])**.** Let $f(x) \in L(\mathbb{R})$, the space of integrable function on the set of real number $\mathbb{R}$. The *Fourier transform of* $f(x)$ is defined by

$$\hat{f}(\omega) = \mathcal{F} f(x) = \int_{-\infty}^{\infty} e^{-i\omega x} f(x)\, dx, \tag{2.1}$$

where $\omega,\ x \in \mathbb{R}$. Also the *inverse Fourier transform* is defined by

$$f(x) = \mathcal{F}^{-1}\hat{f}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega x} \hat{f}(\omega)\, d\omega. \tag{2.2}$$

Definition 2.2 ([6])**.** Let f and g be integrable functions on the set of real number $\mathbb{R}$ *the convolution of* f *and* g denoted by $f * g$ and is defined by

$$(f * g)(x) = \int_{-\infty}^{\infty} f(x-t)\, g(t) dt.$$

Definition 2.3 ([7])**.** Let X be a real-valued random variable with density function $f(x)$. *The expected value of* X is defined by

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx,$$

provided the integral

$$\int_{-\infty}^{\infty} |x| f(x)\, dx$$

is finite.

Definition 2.4 ([7])**.** Let X be a real-valued random variable with density function $f(x)$. *The variance of* X is defined by

$$V(X) = E\left((X - E(X))^2\right).$$

Theorem 2.5 ([7])**.** If X be a real-valued random variable with density function $f(x)$ then the variance

$$V(X) = \int_{-\infty}^{\infty} (x - E(X))^2 f(x) dx.$$

Lemma 2.6. Given the Black-scholes equation (1.1) with terminal condition (1.2) for $0 \le t \le T$. By changing the variable s_t to R with $R = \ln s_t$ and $\tau = T - t$ then (1.1) is transformed to

$$\frac{\partial U(R,\tau)}{\partial \tau} - \frac{1}{2}\sigma^2 \frac{\partial^2 U(R,\tau)}{\partial R^2} + (r - \frac{\sigma^2}{2})\frac{\partial U(R,\tau)}{\partial R} + rU(R,\tau) = 0 \tag{2.3}$$

with the initial condition $U(R,0) = (s-p)^+ = (e^R - p)^+$. Let

$$U(R,0) = (e^R - p)^+ = f(R) \tag{2.4}$$

where f is the function of R.

Proof. We have $R = \ln s_t$ and write $u(s_t, t) = U(R,\tau)$ where $\tau = T - t$. Now,

$$\frac{\partial u(s_t,t)}{\partial t} = \frac{\partial U(R,\tau)}{\partial \tau} = \frac{\partial U}{\partial \tau}\frac{\partial \tau}{\partial t} = -\frac{\partial U}{\partial \tau}$$
$$\frac{\partial u(s_t,t)}{\partial t} = \frac{\partial U(R,\tau)}{\partial s_t} = \frac{\partial U(R,\tau)}{\partial R}\frac{\partial R}{\partial s_t} = \frac{1}{s_t}\frac{\partial U(R,\tau)}{\partial R}$$

and

$$\frac{\partial^2 u(s_t,t)}{\partial s_t^2} = \frac{1}{s_t^2}\frac{\partial U^2(R,\tau)}{\partial R^2} - \frac{1}{s_t^2}\frac{\partial U(R,\tau)}{\partial R}.$$

Substitute into (1.1) we obtain (2.3) and (2.4) as required. ∎

Lemma 2.7. The equation (2.3) with the initial condition (2.4) has a solution in convolution form

$$U(R,\tau) = K(R,\tau) * f(R) \tag{2.5}$$

where

$$K(R,\tau) = \frac{e^{-r\tau}}{\sqrt{2\pi\tau\sigma^2}} \exp\left[-\frac{(R-(r-\frac{\sigma^2}{2})\tau)^2}{2\tau\sigma^2}\right] \tag{2.6}$$

is the kernel of (2.3).

Proof. Take the Fourier transform given by (2.1) to both side of (2.3), we obtain

$$\frac{\partial \hat{U}(\omega,\tau)}{\partial\tau} + \frac{1}{2}\sigma^2\omega^2\hat{U}(\omega,\tau) + i\omega(r-\frac{1}{2}\sigma^2)\hat{U}(\omega,\tau) + r\hat{U}(\omega,\tau) = 0. \tag{2.7}$$

Now, for any fixed ω, (2.7) is the ordinary differential equation of variable τ and we obtain

$$\hat{U}(\omega,\tau) = C(\omega)\exp\big[-(\frac{1}{2}\sigma^2\omega^2 + i\omega(r-\frac{1}{2}\sigma^2)+r)\tau\big] \tag{2.8}$$

as the solution of (2.7). We need to find $C(\omega)$. We have from (2.4), $U(R,0) = (e^R - p)^+ = f(R)$ thus $\hat{U}(\omega,0) = \hat{f}(\omega)$. It follows that $C(\omega) = \hat{f}(\omega)$ from (2.8). Now, by (2.2) we have

$$\begin{aligned}
U(R,\tau) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega R}\hat{U}(\omega,\tau)d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega R}\hat{f}(\omega)\exp\big[-(\frac{1}{2}\sigma^2\omega^2 + i\omega(r-\frac{1}{2}\sigma^2)+r)\tau\big]d\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i\omega R}e^{-i\omega x}\exp\big[-(\frac{1}{2}\sigma^2\omega^2 + i\omega(r-\frac{1}{2}\sigma^2)+r)\tau\big]f(x)dxd\omega \\
&= \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\big[-(\frac{1}{2}\sigma^2\omega^2\tau + i(\tau(r-\frac{1}{2}\sigma^2) - R + x)\omega + r\tau)\big]f(x)dxd\omega \\
&= \frac{1}{2\pi}e^{-r\tau}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\sigma^2\tau\left(\omega^2 - \frac{2i((r-\frac{1}{2}\tau\sigma^2)-R+x)\omega}{\tau\sigma^2}\right)\right]f(x)dxd\omega \\
&= \frac{1}{2\pi}e^{-r\tau}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \exp\left[-\frac{1}{2}\sigma^2\tau\left(\omega - \frac{i((r-\frac{1}{2}\sigma^2)\tau-R+x)}{\tau\sigma^2}\right)^2\right. \\
&\qquad\qquad \left. - \frac{((r-\frac{1}{2}\sigma^2)\tau-R+x)^2}{2\tau\sigma^2}\right]f(x)dxd\omega \\
&= \frac{1}{2\pi}e^{-r\tau}\int_{-\infty}^{\infty}\exp\left[-\frac{((r-\frac{1}{2}\sigma^2)\tau-R+x)^2}{2\sigma^2\tau}\right]f(x)dx \\
&\quad \times \int_{-\infty}^{\infty}\exp\left[-\frac{1}{2}\sigma^2\tau\left(\omega - \frac{i(r-\frac{1}{2}\sigma^2)\tau-R+x}{\tau\sigma^2}\right)^2\right]d\omega.
\end{aligned}$$

Put $u = \sigma\sqrt{\frac{\tau}{2}}(\omega - \dfrac{i(r-\frac{1}{2}\tau\sigma^2) - R + x}{\tau\sigma^2})$ thus $du = \sigma\sqrt{\frac{\tau}{2}}d\omega$ that is $d\omega = \sqrt{\frac{2}{\tau}}\frac{1}{\sigma}du$ and because of $\int_{-\infty}^{\infty} e^{-u^2}du = \sqrt{\pi}$ then

$$\begin{aligned}
U(R,\tau) &= \frac{1}{2\pi}e^{-r\tau}\int_{-\infty}^{\infty}\exp\left[-\frac{((r-\frac{1}{2}\tau\sigma^2)-R+x)^2}{2\tau\sigma^2}\right]f(x)dx \\
&\times \frac{1}{\sigma}\sqrt{\frac{2}{\tau}}\int_{-\infty}^{\infty} e^{-u^2}du \\
&= \frac{1}{2\pi\sigma}e^{-r\tau}\sqrt{\frac{2}{\tau}}\sqrt{\pi}\int_{-\infty}^{\infty}\exp\left[-\frac{((r-\frac{1}{2}\tau\sigma^2)-R+x)^2}{2\tau\sigma^2}\right]f(x)dx \\
&= \frac{1}{\sqrt{2\pi\tau\sigma^2}}e^{-r\tau}\int_{-\infty}^{\infty}\exp\left[-\frac{((R-x)-(r-\frac{1}{2}\sigma^2)\tau)^2}{2\tau\sigma^2}\right]f(x)dx \\
&= K(R,\tau)*f(R).
\end{aligned}$$

Thus $U(R,\tau) = K(R,\tau)*f(R)$ where $K(R,\tau) = \dfrac{e^{-r\tau}}{\sqrt{2\pi\tau\sigma^2}}exp[-\frac{(R-(r-\frac{1}{2}\sigma^2)\tau)^2}{2\tau\sigma^2}]$ is the kernel of (2.3).

If $\tau = 0$ then $t = T$ and it can be shown that $\lim_{\tau\to 0} K(R,\tau) = \delta(R)$ where $\delta(R)$ is the Diract-delta distribution. Thus, from (2.5)

$$U(R,0) = \delta(R)*f(R) = f(R).$$

It follows that (2.4) holds. ■

We can relate $U(R,\tau)$ in (2.3) to $u(s_t,t)$ in (1.1). Since $U(R,\tau) = U(lns_t, T-t) = u(s_t,t)$ and (2.5) so that

$$\begin{aligned}
u(s_t,t) &= K(\ln s_t, T-t)*f(\ln s_t) \qquad (2.9)\\
&= \frac{e^{-r(T-t)}}{\sqrt{2\pi(T-t)\sigma^2}}\exp\left[-\frac{\left[\ln s_t - \left(r-\frac{\sigma^2}{2}\right)(T-t)\right]^2}{2\sigma^2(T-t)}\right]*(s_t-p)^+
\end{aligned}$$

is the solution of (1.1) in convolution form.

Definition 2.8. Let Ω be closed and bounded set and is called *the spectrum of function* $f(x)$ if $\Omega = supp\hat{f}(\omega)$ where $supp\hat{f}(\omega)$ is the support of fourier transform of f, see [6, p227].

Let $\Omega = [a,b]$, we define the spectrum of the solution $U(R,\tau)$ as

$$[a,b] = supp\hat{U}(\omega,\tau) \qquad (2.10)$$

where

$$\hat{U}(\omega,\tau) = \begin{cases} \hat{f}(\omega)\exp[-(\frac{\sigma^2}{2}\omega^2 + i\omega(r-\frac{\sigma^2}{2}) + r)\tau] & for\ \ \omega\in[a,b] \\ 0 & for\ \ \omega\notin[a,b] \end{cases} \qquad (2.11)$$

In addition, we also define the size of the spectrum $[a,b] = b-a$.

3. Main Results

Theorem 3.1. The solution of (2.3) with the initial condition (2.4) can be written by the convolution form $U(R,\tau) = K(R,\tau) * f(R)$ where $K(R,\tau)$ is defined by (2.6). Moreover $|U(R,\tau)| \leq \frac{MN}{2\pi}(b-a)$ where $M = \max_{\omega\in[a,b]} \left|\hat{f}(\omega)\right|$ and $N = \max_{\omega\in[a,b]} exp[-\left(\frac{\sigma^2}{2}\omega + r\right)\tau]$ and $b-a$ is the size of spectrum given by (2.10).

Proof. By Lemma 2.3, taking Fourier transform to (2.3) we obtain $U(R,\tau) = K(R,\tau) * f(R)$ which satisfies the condition (2.4). Now using (2.11) one has

$$\begin{aligned}
U(R,\tau) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i\omega R}\hat{U}(\omega,\tau)d\omega \\
&= \frac{1}{2\pi}\int_a^b e^{i\omega R}\hat{f}(\omega)exp\left[-(\frac{1}{2}\sigma^2\omega^2 + i\omega(r-\frac{\sigma^2}{2}) + r)\tau\right]d\omega \\
&= \frac{1}{2\pi}\int_a^b \hat{f}(\omega)\, e^{-\left(\frac{\sigma^2}{2}\omega + r\right)\tau} e^{i\omega\tau\left(\frac{R}{\tau}+r-\frac{\sigma^2}{2}\right)}d\omega.
\end{aligned}$$

Let $M = \max_{\omega\in[a,b]} \left|\hat{f}(\omega)\right|$ and $N = \max_{\omega\in[a,b]} exp[-\left(\frac{\sigma^2}{2}\omega + r\right)\tau]$. Thus

$$\begin{aligned}
|U(R,\tau)| &\leq \frac{1}{2\pi}\int_a^b \left|\hat{f}(\omega)\right|\left|e^{i\omega\tau\left(\frac{R}{\tau}+r-\frac{\sigma^2}{2}\right)}\right| e^{-\left(\frac{\sigma^2}{2}\omega+r\right)\tau}d\omega \\
&= \frac{1}{2\pi}\int_a^b \left|\hat{f}(\omega)\right| e^{-\left(\frac{\sigma^2}{2}\omega+r\right)\tau}d\omega \\
&\leq \frac{MN}{2\pi}\int_a^b d\omega = \frac{MN}{2\pi}(b-a).
\end{aligned}$$

It follow that $|U(R,\tau)| \leq \frac{MN}{2\pi}(b-a)$ as required. That mean $U(R,\tau)$ is bounded by some constant times the size of spectrum. ∎

Theorem 3.2. (The properties of kernel $K(R,\tau)$)
The kernel given by (2.6) of Lemma 2.3 has the following properties
(i) $K(R,\tau)$ satisfies equation (2.3) and $K(R,\tau) > 0$ for $0 \leq \tau \leq T$.
(ii) $\lim_{\tau\to 0} K(R,\tau) = \delta(R)$ where $\delta(R)$ is the Dirac-delta distribution.
(iii) $e^{r\tau}\int_{-\infty}^{\infty} K(R,\tau)dR = 1$.
(iv) $K(R,\tau)$ is Gaussian function with expected value $E(X) = e^{-r\tau}\tau\left(r - \frac{1}{2}\sigma^2\right)$ and variance

$$V(X) = \left[\tau^2\left(r - \tfrac{1}{2}\sigma^2\right)^2\left(1 - 2e^{-r\tau} + e^{-2r\tau}\right) + \tau\sigma^2\right]e^{-r\tau}.$$

Proof. (i) By computing directly, $K(R,\tau)$ satisfies (2.3) and $K(R,\tau) > 0$ for $0 \leq \tau \leq T$ is obvious.
(ii) Consider,

$$\lim_{\tau\to 0} K(R,\tau) = \lim_{\tau\to 0}\left(\frac{e^{\frac{-(R-(r-\frac{1}{2}\sigma^2)\tau)^2}{2\tau\sigma^2}}}{\sqrt{2\pi\tau\sigma^2}}\right) = \delta(R).$$

(iii) Let $v = \frac{R-m}{\sqrt{2\tau\sigma^2}}$ and $m = (r - \frac{1}{2}\sigma^2)\tau$ then $dR = \sqrt{2\tau\sigma^2}dv$ thus

$$\begin{aligned} e^{r\tau}\int_{-\infty}^{\infty} K(R,\tau)dR &= \frac{1}{\sqrt{2\pi\tau\sigma^2}}\int_{-\infty}^{\infty} e^{\frac{-(R-m)^2}{2\tau\sigma^2}}dR. \\ &= \frac{1}{\sqrt{2\pi\tau\sigma^2}}\sqrt{2\tau\sigma^2}\int_{-\infty}^{\infty} e^{-v^2}dv = \frac{1}{\sqrt{2\pi\tau\sigma^2}}\sqrt{2\pi\tau\sigma^2} = 1. \end{aligned}$$

(iv) Let X be real-valued random variable with the Gaussian function or normal distribution $K_\tau(R) := K(R,\tau)$, we have the expected value or mean value

$$\begin{aligned} E(X) &= \int_{-\infty}^{\infty} K_\tau(R)RdR = \frac{e^{-r\tau}}{\sqrt{2\pi\tau\sigma^2}}\int_{-\infty}^{\infty} \mathrm{e}^{-\frac{(R-m)^2}{2\tau\sigma^2}}RdR \\ &= \frac{e^{-r\tau}}{\sqrt{2\pi\tau\sigma^2}}\left[\sqrt{2\tau\sigma^2}\int_{-\infty}^{\infty} e^{-v^2}\left(m + v\sqrt{2\tau\sigma^2}\right)dv\right] \\ &= \frac{e^{-r\tau}}{\sqrt{2\pi\tau\sigma^2}}\left[\sqrt{2\tau\sigma^2}\left(m\int_{-\infty}^{\infty} e^{-v^2}dv + \sqrt{2\tau\sigma^2}\int_{-\infty}^{\infty} ve^{-v^2}dv\right)\right] \\ &= \frac{e^{-r\tau}}{\sqrt{2\pi\tau\sigma^2}}\left[\sqrt{2\tau\sigma^2}\left(m\sqrt{\pi} - \frac{1}{2}\sqrt{2\tau\sigma^2}\left[e^{-v^2}\right]_{-\infty}^{\infty}\right)\right] \\ &= \frac{e^{-r\tau}}{\sqrt{2\pi\tau\sigma^2}}\sqrt{2\pi\tau\sigma^2}m = e^{-r\tau}\tau\left(r - \tfrac{1}{2}\sigma^2\right) \end{aligned}$$

and variance

$$\begin{aligned} V(X) &= \int_{-\infty}^{\infty} [R - E(X)]^2 K_\tau(R)dR \\ &= \frac{e^{-r\tau}}{\sqrt{2\pi\tau\sigma^2}}\int_{-\infty}^{\infty}\left(R - me^{-r\tau}\right)^2 e^{-\frac{(R-m)^2}{2\tau\sigma^2}}dR \\ &= \frac{e^{-r\tau}}{\sqrt{2\pi\tau\sigma^2}}\left[\int_{-\infty}^{\infty} R^2 e^{-\frac{(R-m)^2}{2\tau\sigma^2}}dR - 2me^{-r\tau}\int_{-\infty}^{\infty} Re^{-\frac{(R-m)^2}{2\tau\sigma^2}}dR\right. \\ &\qquad\left. + m^2 e^{-2r\tau}\int_{-\infty}^{\infty} e^{-\frac{(R-m)^2}{2\tau\sigma^2}}dR\right] \\ &= \frac{e^{-r\tau}}{\sqrt{2\pi\tau\sigma^2}}\left[\sqrt{2\tau\sigma^2}\int_{-\infty}^{\infty}\left(m + v\sqrt{2\tau\sigma^2}\right)^2 e^{-v^2}dv\right. \\ &\qquad -2me^{-r\tau}\sqrt{2\tau\sigma^2}\int_{-\infty}^{\infty}\left(m + v\sqrt{2\tau\sigma^2}\right)e^{-v^2}dv \\ &\qquad\left. +m^2 e^{-2r\tau}\sqrt{2\tau\sigma^2}\int_{-\infty}^{\infty} e^{-v^2}dv\right] \\ &= \frac{e^{-r\tau}}{\sqrt{2\pi\tau\sigma^2}}\left[\left(m^2 + \tau\sigma^2\right)\sqrt{2\pi\tau\sigma^2} - 2m^2 e^{-r\tau}\sqrt{2\pi\tau\sigma^2} + m^2 e^{-2r\tau}\sqrt{2\pi\tau\sigma^2}\right] \\ &= \left[\tau^2\left(r - \tfrac{1}{2}\sigma^2\right)^2\left(1 - 2e^{-r\tau} + e^{-2r\tau}\right) + \tau\sigma^2\right]e^{-r\tau}. \end{aligned}$$

■

4. Conclusion

The Black-scholes formula given by (1.3) which is the solution of (1.1) is used widely in the real world applications. But, when (1.1) is transformed to (2.3) by changing the variables, we obtain (2.5) as a solution in convolution form. Such solution needs more mathematical concepts than (1.3), particularly in the part of main result shows the theorem concerning the boundedness of the option price which is bounded by the size of the spectrum. Moreover, some properties of the kernel are studied. Namely, the kernel is a positive solution of (2.3), the kernel will be the Dirac-delta distribution when the time close to the expiration time, improper integral of the kernel converges to some exponential function. Furthermore the expected value and variance formular which relate to the kernel are defined. However, this paper contains the new results which useful in the advanced research area of Financial Mathematics.

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