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Transmission and reflection probabilities and quasinormal frequencies of perfect fluid spheres in various coordinates

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Abstract Einstein's field equation describes gravity as the result of spacetime curvature due to mass and energy. Perfect fluid spheres are the solution to Einsteins field equation. We use perfect fluid spheres in modeling black holes. The most commonly used coordinates for perfect fluid spheres are the Schwarzschild coordinates and isotropic coordinates. Thus far, we have obtained the general potentials of these two coordinates. The results show that the general potentials of black holes in the form of perfect fluid spheres in these two coordinates are functions that depend on the radius. In this paper, we are interested in perfect fluid spheres in various coordinates; namely, general diagonal coordinates, Gaussian polar coordinates, Buchdahl coordinates, Synge isothermal coordinates, and exponential coordinates. We calculate the general potentials of perfect fluid spheres in various coordinates using the concept of general potential of the Schwarzschild black hole developed by Ngampitipan, and then use the general potentials to obtain the transmission and reflection probabilities using the bogoliubov coefficients. Finally, we calculate the quasinormal frequencies of perfect fluid spheres in various coordinates using the WKB approximation method.

MSC: 83C15; 81R20; 34E20; 35J10
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1. Introduction

In the previous research, we were interested in perfect fluid black holes in Schwarzschild and isotropic coordinates and derived the potentials of these perfect fluid black holes developed by Ngampitipan [1] and Kunlapat, et al [2]. The commonly used coordinates for perfect fluid spheres are Schwarzschild coordinates (about 55%), isotropic coordinates (about 35%), and 10% are other coordinates [3]. There is currently only a small number of research focusing on other coordinates. Therefore, we are interested in perfect fluid spheres in other coordinates, and in calculating the general potentials of perfect fluid spheres in other coordinates. In this paper, we are interested in perfect fluid spheres in various coordinates, which is a more complicated metric and different from the metric in previous research. In order to know the parameters $\zeta(r)$ and $B(r)$ in the metric of perfect fluid spheres in various coordinates and calculate the transmission and reflection probabilities of perfect fluid spheres in various coordinates, we must match the Schwarzschild exterior metric in Schwarzschild coordinates and various coordinates by coordinate transformation.

2. Perfect fluid spheres in various coordinates

In this paper, we are interested in perfect fluid spheres in various coordinates; namely, general diagonal coordinates, Gaussian polar coordinates, Buchdahl coordinates, Synge isothermal coordinates, and exponential coordinates. Perfect fluid spheres in various coordinates are presented as follows;

2.1. General diagonal coordinates

The metric is given by [3, 4]

$$ds^2 = -\zeta(r)^2 dt^2 + \frac{dr^2}{B(r)} dr^2 + R(r)^2 d\Omega^2. \tag{2.1}$$

The system of the Einstein field equations for the line element becomes [5]

$$\rho = \frac{1}{R(r)^2}(1 - B(r)) + \frac{B(r)'}{R(r)}, \tag{2.2}$$

$$p_r = -\frac{1}{R(r)^2}(1 - B(r)) - \frac{2B(r)}{R(r)}\frac{\zeta(r)'}{\zeta(r)}, \tag{2.3}$$

$$p_t = B(r)\left[\frac{\zeta(r)''}{\zeta(r)} - 2\left(\frac{\zeta(r)'}{\zeta(r)}\right)^2 + 2\left(\frac{\zeta(r)'}{\zeta(r)}\right)\left(\frac{1}{R(r)} + \frac{B(r)'}{B(r)}\right) + \frac{B(r)'}{2R(r)B(r)}\right], \tag{2.4}$$

where ρ is the density, p_r is the radial pressure and p_t is the transverse pressure. One of the properties of the perfect fluid sphere is isotropy ($p_r = p_t$).
From the metric in the general coordinates, we calculate

$$g_{\mu\nu} = \begin{pmatrix} -\zeta(r)^2 & 0 & 0 & 0 \\ 0 & B(r)^{-1} & 0 & 0 \\ 0 & 0 & R(r)^2 & 0 \\ 0 & 0 & 0 & R(r)^2 \sin^2\theta \end{pmatrix}, \tag{2.5}$$

$$g^{\mu\nu} = \begin{pmatrix} -\zeta(r)^{-2} & 0 & 0 & 0 \\ 0 & B(r) & 0 & 0 \\ 0 & 0 & R(r)^{-2} & 0 \\ 0 & 0 & 0 & R(r)^{-2}\sin^{-2}\theta \end{pmatrix}, \tag{2.6}$$

$$g = -\frac{\zeta(r)^2}{B(r)}R(r)^4\sin^2\theta, \tag{2.7}$$

$$\sqrt{-g} = \sqrt{\frac{\zeta(r)^2}{B(r)}}R(r)^2\sin\theta. \tag{2.8}$$

where $g_{\mu\nu}$ is the metric tensor, $g^{\mu\nu}$ is the inverse of the metric tensor, and g is the determinant of the metric tensor.
We then substitute these equations into the Klein-Gordon equation.

2.2. Gaussian polar coordinates

The metric is given by [3, 4]

$$ds^2 = -\zeta(r)^2dt^2 + dr^2 + R(r)^2d\Omega^2. \tag{2.9}$$

The system of the Einstein field equations for the line element becomes [5]

$$\rho = 0, \tag{2.10}$$

$$p_r = -\frac{2}{R(r)}\frac{\zeta(r)'}{\zeta(r)}, \tag{2.11}$$

$$p_t = \left[\frac{\zeta(r)''}{\zeta(r)} - 2\left(\frac{\zeta(r)'}{\zeta(r)}\right)^2 + 2\left(\frac{\zeta(r)'}{\zeta(r)}\right)\left(\frac{1}{R(r)}\right)\right], \tag{2.12}$$

with one of the properties of the perfect fluid sphere being isotropy ($p_r = p_t$).
From the metric in the Gaussian polar coordinates, we calculate

$$g_{\mu\nu} = \begin{pmatrix} -\zeta(r)^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & R(r)^2 & 0 \\ 0 & 0 & 0 & R(r)^2\sin^2\theta \end{pmatrix}, \tag{2.13}$$

$$g^{\mu\nu} = \begin{pmatrix} -\zeta(r)^{-2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & R(r)^{-2} & 0 \\ 0 & 0 & 0 & R(r)^{-2}\sin^{-2}\theta \end{pmatrix}, \tag{2.14}$$

$$g = -\zeta(r)^2R(r)^4\sin^2\theta, \tag{2.15}$$

$$\sqrt{-g} = \zeta(r)R(r)^2\sin\theta. \tag{2.16}$$

We then substitute these equations into the Klein-Gordon equation.

2.3. Buchdahl coordinates

The metric is given by [3, 4]

$$ds^2 = -\zeta(r)^{-1}dt^2 + \zeta(r)^2dr^2 + \zeta(r)R(r)^2d\Omega^2. \tag{2.17}$$

The system of the Einstein field equations for the line element becomes [5]

$$\rho = \frac{1}{\zeta(r)R(r)^2}(1-\zeta(r)^{-2}) + \frac{2\zeta(r)^{-5/2}\zeta(r)'}{R(r)}, \tag{2.18}$$

$$p_r = -\frac{1}{\zeta(r)R(r)^2}(1-\zeta(r)^{-2}) - \frac{\zeta(r)^{-5/2}\zeta(r)'}{R(r)}, \tag{2.19}$$

$$p_t = \zeta(r)^{-2}\left[\frac{1}{2}\left(\frac{\zeta(r)'}{\zeta(r)}\right)^2 - \frac{3}{2}\frac{\zeta(r)'}{\sqrt{\zeta(r)}R(r)\zeta(r)}\right], \tag{2.20}$$

with one of the properties of the perfect fluid sphere being isotropy ($p_r = p_t$). From the metric in the Buchdahl coordinates, we calculate

$$g_{\mu\nu} = \begin{pmatrix} -\zeta(r)^{-1} & 0 & 0 & 0 \\ 0 & \zeta(r)^2 & 0 & 0 \\ 0 & 0 & \zeta(r)R(r)^2 & 0 \\ 0 & 0 & 0 & \zeta(r)R(r)^2\sin^2\theta \end{pmatrix}, \tag{2.21}$$

$$g^{\mu\nu} = \begin{pmatrix} -\zeta(r) & 0 & 0 & 0 \\ 0 & \zeta(r)^{-2} & 0 & 0 \\ 0 & 0 & \zeta(r)^{-1}R(r)^{-2} & 0 \\ 0 & 0 & 0 & \zeta(r)^{-1}R(r)^{-2}\sin^{-2}\theta \end{pmatrix}, \tag{2.22}$$

$$g = -\zeta(r)^3R(r)^4\sin^2\theta, \tag{2.23}$$

$$\sqrt{-g} = \sqrt{\zeta(r)^3}R(r)^2\sin\theta. \tag{2.24}$$

We then substitute these equations into the Klein-Gordon equation.

2.4. Synge isothermal coordinates

The metric is given by [3, 4]

$$ds^2 = -\zeta(r)^{-2}\{dt^2 - dr^2\} + \{\zeta(r)^{-2}R(r)^2d\Omega^2\}. \tag{2.25}$$

The system of the Einstein field equations for the line element becomes [5]

$$\rho = \frac{1}{\zeta(r)^{-2}R(r)^2}(1-\zeta(r)^2) + \frac{2\zeta(r)^2\zeta(r)'}{R(r)}, \tag{2.26}$$

$$p_r = -\frac{1}{\zeta(r)^{-2}R(r)^2}(1-\zeta(r)^2) + \frac{2\zeta(r)^2\zeta(r)'}{R(r)}, \tag{2.27}$$

$$p_t = \zeta(r)^2\left[\left(\frac{\zeta(r)'}{\zeta(r)}\right)^2 - \frac{\zeta(r)''}{\zeta(r)}\right], \tag{2.28}$$

with one of the properties of the perfect fluid sphere being isotropy ($p_r = p_t$).

From the metric in the Synge coordinates, we calculate

$$g_{\mu\nu} = \begin{pmatrix} -\zeta(r)^{-2} & 0 & 0 & 0 \\ 0 & \zeta(r)^{-2} & 0 & 0 \\ 0 & 0 & \zeta(r)^{-2}R(r)^2 & 0 \\ 0 & 0 & 0 & \zeta(r)^{-2}R(r)^2\sin^2\theta \end{pmatrix}, \tag{2.29}$$

$$g^{\mu\nu} = \begin{pmatrix} -\zeta(r)^{2} & 0 & 0 & 0 \\ 0 & \zeta(r)^{2} & 0 & 0 \\ 0 & 0 & \zeta(r)^{2}R(r)^{-2} & 0 \\ 0 & 0 & 0 & \zeta(r)^{2}R(r)^{-2}\sin^{-2}\theta \end{pmatrix}, \tag{2.30}$$

$$g = -\zeta(r)^{-8}R(r)^4\sin^2\theta, \tag{2.31}$$

$$\sqrt{-g} = \zeta(r)^{-4}R(r)^2\sin\theta. \tag{2.32}$$

We then substitute these equations into the Klein-Gordon equation.

2.5. Exponential coordinates

The metric is given by [3, 4]

$$ds^2 = -\exp(-2r)dt^2 + \exp(+2r)\left\{\frac{dr^2}{B(r)} + R(r)^2 d\Omega^2\right\}. \tag{2.33}$$

The system of the Einstein field equations for the line element becomes [5]

$$\rho = \frac{1}{\exp(+2r)R(r)^2}\left(1 - \frac{B(r)}{\exp(+2r)}\right) + \frac{2B(r)}{\exp(+3r)R(r)}\left(1 - \frac{B(r)'}{2B(r)}\right), \tag{2.34}$$

$$p_r = -\frac{1}{\exp(+2r)R(r)^2}\left(1 - \frac{B(r)}{\exp(+2r)}\right) - \frac{2B(r)}{\exp(+3r)R(r)}, \tag{2.35}$$

$$p_t = \frac{B(r)}{\exp(+2r)}\left[2 - \frac{2}{\exp(+r)R(r)} + \frac{B(r)'}{2B(r)}\left(\frac{1}{\exp(+r)R(r)} - 1\right)\right], \tag{2.36}$$

with one of the properties of the perfect fluid sphere being isotropy ($p_r = p_t$). From the metric in the exponential coordinates, we calculate

$$g_{\mu\nu} = \begin{pmatrix} -\exp(-2r) & 0 & 0 & 0 \\ 0 & \frac{\exp(+2r)}{B(r)} & 0 & 0 \\ 0 & 0 & \exp(+2r)R(r)^2 & 0 \\ 0 & 0 & 0 & \exp(+2r)R(r)^2\sin^2\theta \end{pmatrix}, \tag{2.37}$$

$$g^{\mu\nu} = \begin{pmatrix} -\exp(+2r) & 0 & 0 & 0 \\ 0 & \frac{B(r)}{\exp(+2r)} & 0 & 0 \\ 0 & 0 & \exp(-2r)R(r)^{-2} & 0 \\ 0 & 0 & 0 & \exp(-2r)R(r)^{-2}\sin^{-2}\theta \end{pmatrix}, \tag{2.38}$$

$$g = -\frac{\exp(+4r)}{B(r)} R(r)^4 \sin^2\theta, \tag{2.39}$$

$$\sqrt{-g} = \sqrt{\frac{\exp(+4r)}{B(r)}} R(r)^2 \sin\theta. \tag{2.40}$$

We then substitute these equations into the Klein-Gordon equation.

We will calculate the general potentials of perfect fluid spheres in various coordinates using the concept of general potentials of the Schwarzschild black hole developed by Ngampitipan [1].

3. Potentials in general coordinates

We will find the general potentials of perfect fluid spheres in various coordinates by beginning with the Klein-Gordon equation, and after we derive the equation, we obtain the Regge-Wheeler equation.

3.1. Klein-Gordon equation and Regge-Wheeler equation

The Klein-Gordon equation is given by [1]

$$\frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu \psi = 0, \tag{3.1}$$

where $g_{\mu\nu}$ is the metric tensor, $g^{\mu\nu}$ is the inverse of the metric tensor, and g is the determinant of the metric tensor. After we derive this equation, it transforms to the Regge-Wheeler equation.

The Regge-Wheeler equation is given by [1]

$$\frac{d^2\Psi}{dr_*^2} + [\omega^2 - V(r)]\Psi(r) = 0, \tag{3.2}$$

where ω is the wave's energy, $\Psi(r)$ is the wave function and $V(r)$ is the general potential of perfect fluid spheres in various coordinates.

3.2. General Potentials of Perfect fluid spheres

Perfect fluid spheres	**General potentials**
General diagonal coordinates	$V(r) = \frac{l(l+1)\zeta(r)^2}{R(r)^2} - \frac{R(r)}{\sqrt{B(r)}} \frac{d}{dr}(\frac{B(r)'\zeta(r)R(r)}{2B(r)} - R(r)'\zeta(r)))$
Gaussian polar coordinates	$V(r) = \frac{l(l+1)\zeta(r)^2}{R(r)^2} - R(r)^{-3} \frac{d}{dr}(R(r)^2 R(r)'\zeta(r))$
Buchdahl coordinates	$V(r) = \frac{l(l+1)}{\zeta(r)^2 R(r)^2} - R(r)^{-1}\zeta(r)^{-\frac{7}{2}} \frac{d}{dr}(\frac{\zeta(r)'R(r)}{2\zeta(r)} - R(r)')$
Synge isothermal coordinates	$V(r) = \frac{l(l+1)}{R(r)^3} - \zeta(r)^{-2} \frac{d}{dr}(2\frac{\zeta(r)'R(r)}{\zeta(r)} - R(r)')$
Exponential coordinates	$V(r) = \frac{l(l+1)}{R(r)^2} + R(r)^{-1} e^{R^2} \frac{d}{dr}(B(r)^{\frac{1}{2}} R(r)' e^{-R^2}(1+2R^2))$

Table 1. General potentials of perfect fluid spheres in various coordinates

General potentials of perfect fluid spheres in various coordinates as shown in Table 1 are functions that depend on the radius of perfect fluid spheres, where r is the radius of perfect fluid sphere and l is the angular momentum. After that, we will use these general potentials to obtain the transmission and reflection probabilities of perfect fluid spheres in various coordinates.

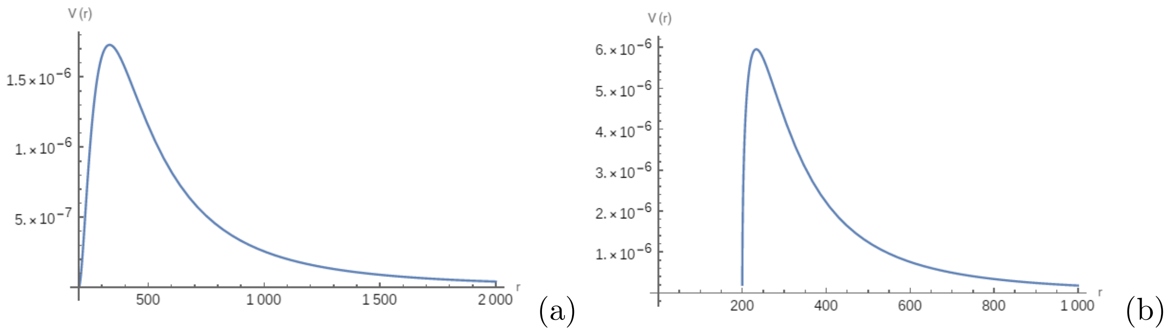

Figure 1. Plotting of the general potentials of perfect fluid spheres in various coordinates, (a) general diagonal coordinates and (b) Gaussian polar coordinates. In these two subfigures of the general potentials, the results show that the potentials depend on r and tend to decrease when r increases.

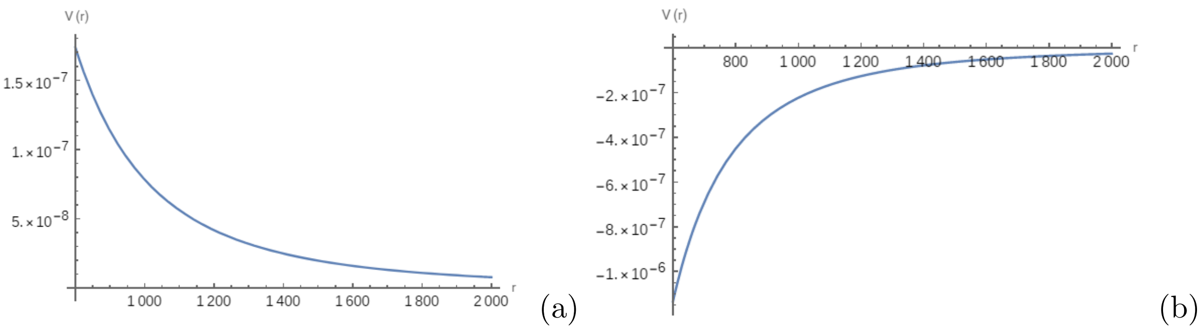

Figure 2. Plotting of the general potentials of perfect fluid spheres in various coordinates, (a) Buchdahl coordinates and (b) Synge isothermal coordinates. In subfigure (a), the results show that the potentials depend on r and tend to decrease when r increases, while the potential of subfigure (b) tend to increase.

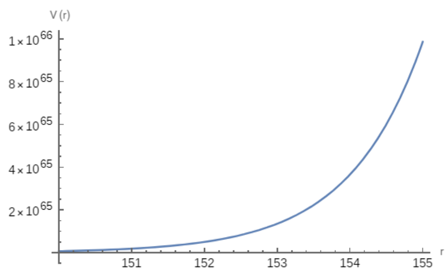

Figure 3. Plotting of the general potentials of perfect fluid spheres in exponential coordinates; the results show that the potentials depend on r and tend to increase when r increases.

4. Transmission and Reflection Probabilities of Perfect fluid spheres

The transmission and reflection probabilities can be calculated using the bogoliubov coefficients [6] which is highly accurate in obtaining the lower and upper bounds on the transmission and reflection probabilities, respectively. The rigorous bound of the transmission and reflection probabilities are given by [7]

$$T \geq \operatorname{sech}^2 \frac{1}{2\omega} \int_{-\infty}^{\infty} |V(r)| dr_*, \tag{4.1}$$

and

$$R \leq \tanh^2 \frac{1}{2\omega} \int_{-\infty}^{\infty} |V(r)| dr_*. \tag{4.2}$$

We obtained the transmission and reflection probabilities of perfect fluid spheres in various coordinates as shown in Tables 2 and 3, respectively. In addition, these parameters were also obtained; $p = \sqrt{\frac{(-2m+R-r_1)^2}{(R-r_1)^2}}$, $q = 1 - \frac{2m}{R-r_4}$, $s = \sqrt{\exp(+2(R-r_5))}l(l+1)$, $t = 2+4m^2+2m(R-r_5-1)-2(R-r_5)+(R-r_5)^2$, $u = Eiz(2R-2r_5)$ and $v = Eiz(R-r_5-2m)$.

Perfect fluid spheres	Transmission probabilities
General diagonal coordinates	$T \geq \operatorname{sech}^2 \left[\frac{1}{2\omega} \frac{3l(l+1)(R-r_1)^2 - 8m^2\sqrt{p} + 6m(R-r_1)\left(-l-l^2+\sqrt{p}\right)}{3(R-r_1)^3\sqrt{p}} \right]$
Gaussian polar coordinates	$T \geq \operatorname{sech}^2 \left[\frac{1}{2\omega} \frac{l(l+1)\left(1-\frac{2m}{R-r_2}\right)^{3/2}}{3m} - \frac{m}{(R-r_2)^2} \right]$
Buchdahl coordinates	$T \geq \operatorname{sech}^2 \left[\frac{1}{2\omega} \int_{-\infty}^{\infty} \frac{2l(l+1)+\zeta(r)^{4/5}\zeta'(r)R(r)}{2\zeta(r)^2 R(r)^2} dr \right]$
Synge isothermal coordinates	$T \geq \operatorname{sech}^2 \left[\frac{1}{2\omega} \frac{1}{m} \frac{l(l+1)}{4} q^2 + \frac{q^{5/2}}{5} - \frac{q^{3/2}}{3} \right]$
Exponential coordinates	$T \geq \operatorname{sech}^2 \left[st + 2u + 8\exp(2m - R + r_5) s m^3 v \right]$

Table 2. Transmission probabilities of perfect fluid spheres in various coordinates.

Perfect fluid spheres	Reflection probabilities
General diagonal coordinates	$R \leq \tanh^2 \left[\frac{1}{2\omega} \frac{3l(l+1)(R-r_1)^2 - 8m^2\sqrt{p} + 6m(R-r_1)\left(-l-l^2+\sqrt{p}\right)}{3(R-r_1)^3\sqrt{p}} \right]$
Gaussian polar coordinates	$R \leq \tanh^2 \left[\frac{1}{2\omega} \frac{l(l+1)\left(1-\frac{2m}{R-r_2}\right)^{3/2}}{3m} - \frac{m}{(R-r_2)^2} \right]$
Buchdahl coordinates	$R \leq \tanh^2 \left[\frac{1}{2\omega} \int_{-\infty}^{\infty} \frac{2l(l+1)+\zeta(r)^{4/5}\zeta'(r)R(r)}{2\zeta(r)^2 R(r)^2} dr \right]$
Synge isothermal coordinates	$R \leq \tanh^2 \left[\frac{1}{2\omega} \frac{1}{m} \frac{l(l+1)}{4} q^2 + \frac{q^{5/2}}{5} - \frac{q^{3/2}}{3} \right]$
Exponential coordinates	$R \leq \tanh^2 \left[st + 2u + 8\exp(2m - R - r_5) s m^3 v \right]$

Table 3. Reflection probabilities of perfect fluid spheres in various coordinates.

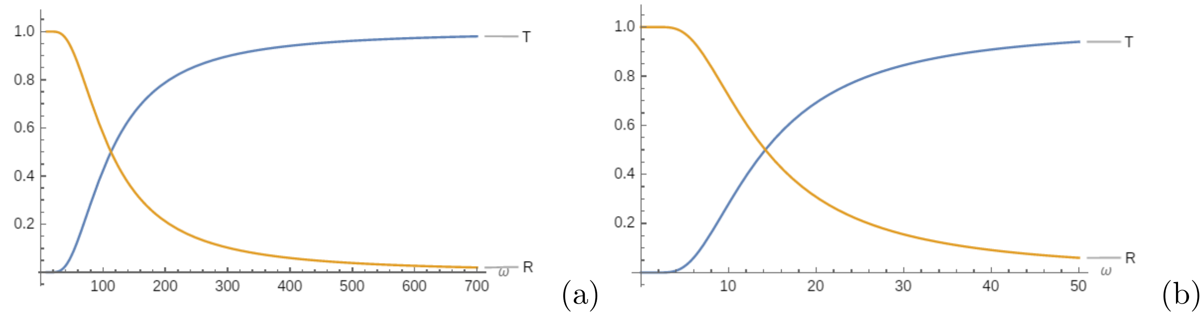

FIGURE 4. Transmission and reflection probabilities of perfect fluid spheres in various coordinates, (a) general diagonal coordinates and (b) Gaussian polar coordinates.

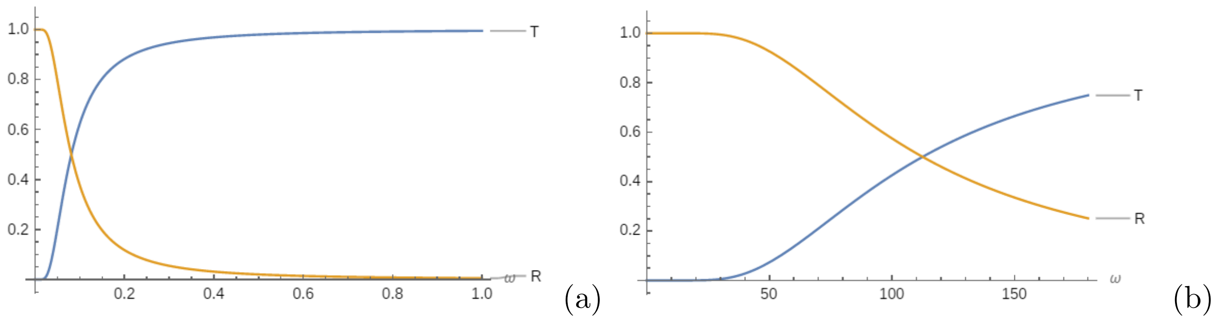

FIGURE 5. Transmission and reflection probabilities of perfect fluid spheres in various coordinates, (a) Buchdahl coordinates and (b) Synge isothermal coordinates.

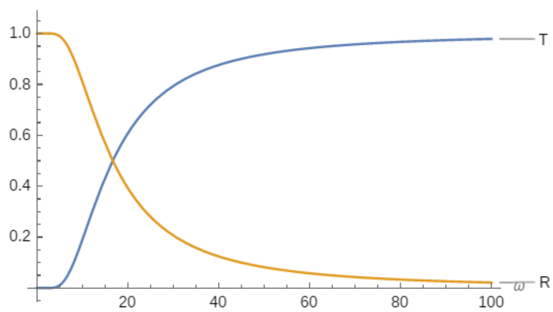

FIGURE 6. Transmission and reflection probabilities of perfect fluid spheres in exponential coordinates.

Figures 4, 5 and 6 represent the relation between the transmission and the reflection probabilities. The results show that the transmission probabilities of perfect fluid spheres in various coordinates as shown in Figures 4, 5 and 6 tend to increase when ω increases, while the reflection probabilities decrease.

5. Quasinormal frequencies of Perfect fluid spheres

Quasinormal frequencies explain the perturbation of perfect fluid spheres that contain the real and imaginary parts of the frequencies, which can be calculated using the WKB method [8] that is known in many cases to be more accurate than what we might expect and can approximate the solution of the one-dimensional Schrodinger equation. The Quasinormal frequencies of the perfect fluid spheres are defined by [8]

$$\omega^2 = V(r_0) + i\sqrt{\frac{2d^2Q}{dr_*^2}}\left(n+\frac{1}{2}\right), \tag{5.1}$$

where r_0 is the peak of $-Q(r)$ and Q is frequency dependent.
The Quasinormal frequency of the general diagonal coordinates is defined by [8]

$$\omega^2 = V(r_0) + \sqrt{-2}i\zeta(r_0)B(r_0)\left(n+\frac{1}{2}\right)$$

$$\sqrt{\zeta(r_0)^2V''(r_0) + B(r_0)^{-1}V'(r_0)(\zeta(r_0)^2B(r_0))'}, \tag{5.2}$$

where $\frac{dr_*}{dr} = \frac{1}{\sqrt{\zeta(r)^2B(r)}}$, r_0 is the peak of $-Q(r)$ and Q is frequency dependent.
The Quasinormal frequency of the Guassian polar coordinates is defined by [8]

$$\omega^2 = V(r_0) + \sqrt{-2}i\zeta(r_0)\sqrt{V''(r_0) + \zeta(r_0)^{-1}V'(r_0)\zeta(r_0)'}\left(n+\frac{1}{2}\right), \tag{5.3}$$

where $\frac{dr_*}{dr} = \frac{1}{\zeta(r)}$, r_0 is the peak of $-Q(r)$ and Q is frequency dependent.

The Quasinormal frequency of the Buchdahl coordinates is defined by [8]

$$\omega^2 = V(r_0) + \sqrt{-2}i\sqrt{\zeta(r_0)V''(r_0) + \frac{1}{2}V'(r_0)\zeta(r_0)'}\left(n+\frac{1}{2}\right), \tag{5.4}$$

where $\frac{dr_*}{dr} = \frac{1}{\sqrt{\zeta(r)}}$, r_0 is the peak of $-Q(r)$ and Q is frequency dependent.

The Quasinormal frequency of the Synge isothermal coordinates is defined by [8]

$$\omega^2 = V(r_0) + \sqrt{-2}i\zeta(r_0)^2\sqrt{V''(r_0) + 2V'(r_0)\zeta(r_0)^{-1}\zeta(r_0)'}\left(n+\frac{1}{2}\right), \tag{5.5}$$

where $\frac{dr_*}{dr} = \frac{1}{\zeta(r)^2}$, r_0 is the peak of $-Q(r)$ and Q is frequency dependent.

The Quasinormal frequency of the exponential coordinates is defined by [8]

$$\omega^2 = V(r_0) + \frac{\sqrt{-2}i}{\exp(+4r)}\sqrt{B(r_0)V''(r_0) + V'(r_0)\left(\frac{1}{2}B(r)' - 4B(r)\right)}\left(n+\frac{1}{2}\right), \tag{5.6}$$

where $\frac{dr_*}{dr} = \frac{\exp(+4r)}{\sqrt{B(r)}}$, r_0 is the peak of $-Q(r)$ and Q is frequency dependent.

The Quasinormal frequencies of the perfect fluid spheres in various coordinates are the frequencies that include the real part, which is the general potentials of perfect fluid spheres that are a function that depends on the peak frequency, and the imaginary part, which depends on the second derivative of the frequency.

6. Conclusion

In this paper, we were interested in perfect fluid spheres in general diagonal coordinates, Gaussian polar coordinates, Buchdahl coordinates, Synge isothermal coordinates, and exponential coordinates. First, we calculated the general potentials of perfect fluid spheres in various coordinates and the results show that the general potentials are a function that depends on the radius, which is the same as the results from the previous research and from plotting the graphs of the general potentials in various coordinates. We then obtained the transmission and reflection probabilities of perfect fluid spheres in various coordinates, while the plotted graphs show the relation between transmission and reflection probabilities. In various coordinates, the transmission probabilities increase and the reflection probabilities decrease when the wave's energy increases. Pertaining to physics, the results are independent of the coordinates. Finally, we calculated the quasinormal frequencies of perfect fluid spheres in various coordinates using the WKB approximation method, where the frequencies are in the form of complex numbers.

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References

[1] T. Ngampitipan, Rigorous bounds on greybody factors for various types of black holes, Ph.D. thesis, Chulalongkorn University, 2014.

[2] K. Sansuk, Petarpa Boonserm, T. Ngampitipan, Greybody factors and quasinormal frequencies of perfect fluid black holes in isotropic coordinate, AMM2021 proceedings. (2021) 88-98.

[3] P. Boonserm, Some Exact Solutions in General Relativity, Masters thesis, Victoria University of Wellington, 2006.

[4] P. Boonserm, M. Visser, and S. Weinfurtner, Generating perfect fluid spheres in general relativity, Phys.Rev. D71 (2005), 124037.

[5] K.Komathiraj, Exact models for anisotropic fluid sphere, IntSym. (2015) 174-178.

[6] P. Boonserm, Rigorous Bounds on Transmission, Reflection, and Bogoliubov Coefficients, Ph. D. Thesis, Victoria University of Wellington, 2009.

[7] T. Ngampitipan, and P. Boonserm, Bounding the greybody factors for the Reissner-Nordstrm black holes, J. Phys. Conf. Ser. 435(2013), 1-6.

[8] B. F. Schutz and C. M. Will, Black hole normal modes: a semianalytic approach, Astrophys. J. 291(1985), L33-L36.