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Extension of the Urinal Problem

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Abstract Given a restroom with n urinals aligned in a straight line. Kranakis and Krizanc investigated the problem of determining the optimal position that would maintain one's privacy for the longest time, using different models for the behavior of men in a restroom. We further extend that problem by defining the privacy that takes into account the distance between one and the nearest person. The average privacy along the time is considered. We explicitly obtain the formula of expected average privacy for each position which is a function of the Harmonic series, the maximum of which approaches $\log n$ asymptotically. From the formulae, one will obtain the most privacy if one chooses the first or the furthest urinals. The privacy will decrease when the chosen position is closer to the middle position. Moreover, we perform numerical simulations to confirm and illustrate the results.

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Keywords: Urinal problem; Random behavior; Strategic Decision

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1. Introduction

In 2010, Kranakis and Krizanc proposed the question about the privacy of a man in a public restroom [1]. The problem was stated that the restroom containing several urinals aligned along the wall with neighboring positions could easily see each other. To obtain privacy, one may choose not to pick the one whose neighboring positions are unoccupied. The problem is if the first person arrives at the restroom, which urinal should he pick. This leads us to the optimisation problem by choosing the choice that minimizes the chance someone will pick his neighbor and destroy his privacy. That paper considered a variety of behavior and gave us the best strategy to maintain privacy. This problem is similar to many problems that state about placing objects and leave some space between them, such as, Solution to An Unfriendly Seating Arrangement Problem [2] and Random Maximal Independent Sets and the Unfriendly Theater Arrangement Problem [3].

The mentioned problems consider the concept of privacy as a binary state, that is, only have and not have privacy. For the Urinal Problem, one will have privacy if no person is taking one's neighbor's position. In practice, however, even if there is no person next to one, if some person occupies a urinal close to one, one would feel a little discomfort.

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Additionally, the feeling of insecurity will increase if the closest person is more close to one. In this paper, we model this feeling by extend the definition of privacy from a binary state as in [1] to a multi-state according to the distance from one to the closest person. This problem is similar to the Obnoxious Facility Location Problem, mentioned in [4], which focus on placing object furthest from all other used positions. We consider the random behavior of the other people, find the average privacy of each urinal, and give an optimal strategy for choosing the best position. At last, we perform numerical experiments to illustrate our results.

2. Problem Statement

Consider a restroom with n urinals arranged in a straight line from the door to the back. Each urinal is labelled with an integer between 1 and n according to its proximity to the door, with 1 being the closest and n the furthest. Each person sequentially enter the restroom a choose their own urinal and stay there for an arbitrarily long time.

For $1 \leq i \leq n$, denote the ith person to enter the restroom by A_i. Then we say that they enter the restroom at the time $t = i$. Let p_i be the position of the urinal that A_i takes. Obviously, $(p_1, p_2, p_3, \dots, p_n)$ is a permutation of the set $\{1, 2, 3, \dots, n\}$. We will introduce the newly-defined concept of privacy of A_i.

Definition 2.1. The privacy of the ith person to enter the restroom with n urinals at the time t by

$$P(i,t) = \min_{j \neq i; j \leq t} d_{i,j}$$

for $t = \max(2, i), \dots, n$, where $d_{i,j} = |p_i - p_j|$ is the distance between A_i and A_j.

Intuitively, the privacy $P(i,t)$ is the distance between A_i and their closest neighbour at the time t. The value of $P(1,1)$ is not defined, since there is no neighbour of the first person at time $t = 1$. The function $P(i,t)$ is a decreasing function over t. Note that when the value of i is clear, we may simply write d_j to refer to the distance.

Furthermore, we are going to define the average privacy of the person A_i, which is the average of $P(i,t)$ from the time $t = i$ until $t = n$, except for A_1, which we start calculating the privacy at the time $t = 2$.

Definition 2.2. Define the average privacy of the ith person to enter the restroom by

$$P_{avg}(i) = \begin{cases} \dfrac{\sum_{k=2}^{n} P(i,k)}{n-1}, & \text{if } i = 1 \\ \dfrac{\sum_{k=i}^{n} P(i,k)}{n-t+1}, & \text{otherwise.} \end{cases}$$

Proposition 2.3. *We have that $P(i,t) \leq n - t + 1$.*

Proof. Suppose that there are some i and t such that $P(i,t) > n - t + 1$. This implies that there is a gap in the restroom of length at least $n - t + 1$. However, by the time t, there are t urinals that are filled, only $n - t$ remain vacant. Therefore, it is not possible to have a gap of length greater than $n - t$. Thus, $P(i,t) \leq n - t + 1$. ∎

Corollary 2.4. *We have that $1 \leq P_{avg}(i) \leq \frac{n}{2}$.*

Proof. Direct result from Proposition 2.3. ∎

3. Results

We assume that people have random behavior, that is, the probability of each person except A_i choosing the vacant positions is uniformly distributed. One can think of A_i as being oneself, having the liberty to choose any available position at the time i.

The following lemma is going to be useful in our computation later. A similar statement involving the size of the set containing the permutations of the set $\{1, 2, \ldots, n\}$ with x as the largest element in the cycle containing 1 is presented in [5] by Jerrold W. Grossman. Indeed, this result follows from [5], and vice versa.

Lemma 3.1. *The number of permutations of the set* $\{1, 2, \ldots, n\}$ *with* x *as the smallest element in the cycle containing* n *is* $\frac{n!}{x(x+1)}$.

Proof. Let A denotes the number of permutations of the set $\{1, 2, \ldots, n\}$ with x as the smallest element in the cycle containing n. Let C be the cycle containing n. Consider C with length $k+2$, with 2 elements that are fixed that are x and n. Then there are $\binom{n-x-1}{k}(k+1)!$ permutations of C. Obviously, there are $(n-k-2)!$ permutations for the remaining $n-k-2$ elements. So,

$$|A| = \sum_{k=0}^{n-x-1} \binom{n-x-1}{k} (k+1)!(n-k-2)!. \tag{3.1}$$

Solving equation 3.1 yields

$$|A| = \frac{n!}{x(x+1)}, \tag{3.2}$$

as desired. ∎

We first investigate the case that we are the first person to enter the restroom and can choose any position.

Theorem 3.2. *If the first person takes the outermost position, that is* $p_1 = 1$ *or* $p_1 = n$*, then*

$$\mathbb{E}(P_{avg}(1)) = \frac{n}{n-1}(H_n - 1)$$

where H_n *is the nth harmonic number, defined by* $H_n = \sum_{k=1}^{n} \frac{1}{k}$.

Proof. Obviously when $p_1 = 1$ and $p_1 = n$, the results would be the same. So, we may assume that $p_1 = 1$. Then, $(p_2, p_3, \ldots, p_n)$ is a permutation of $\{2, 3, \ldots, n\}$. Since there is no urinal to the left of p_1, $(d_2, d_3, \ldots, d_n)$ is a permutation of $\{1, 2, \ldots, n-1\}$. By definition of $P(1,t)$, it is equal to the minimum of $\{d_2, d_3, \ldots, d_t\}$. Although $d_2, d_3, \ldots, d_t$ is not necessarily decreasing, $P(1,t)$ is decreasing.

Let $\mathcal{S}_k$ be the set of all permutations of $\{1, 2, \ldots, k\}$, and $c_k \in \mathcal{S}_k$. Now, we define $f : \mathcal{S}_{n-1} \times \{2, 3, \ldots, n\} \to \{1, 2, \ldots, n\}$ which maps (c_{n-1}, t) to $P(1,t)$ when the sequence of the distance of the remaining $n-1$ people is c_{n-1}. Denote the sum of average privacy of all permutations by S. Then,

$$S = \sum_{c_{n-1} \in \mathcal{S}_{n-1}} \frac{1}{n-1} \sum_{k=2}^{n} f(c_{n-1}, k). \tag{3.3}$$

That is, S is the summation of all possible $f(c_{n-1}, t)$ divided by $n-1$. Then, by definition of expected value,

$$\mathbb{E}(P_{avg}(1)) = \frac{S}{(n-1)!}. \tag{3.4}$$

Let A_x denotes the set of the all ordered pairs (c_{n-1}, t) which satisfies the relation $f(c_{n-1}, t) = x$. Then, S can be alternatively written as

$$S = \frac{1}{n-1} \sum_{x=1}^{n-1} x|A_x|. \tag{3.5}$$

Additionally, we let B_x be the set of all permutation of $\{1, 2, \ldots, n\}$ that has x as the least element in the cycle containing n, and let $c_{n-1} = (d_2, d_3, \ldots, d_n)$. We define g_x as a map from $A_x \times \{2, 3, \ldots, n\}$ to $\mathcal{S}_n$ by

$$\begin{aligned} g_x(c_{n-1}, t) &= (r_1, r_2, \ldots, r_n) \\ &= [\quad]n, d_2, d_3, \ldots, d_t \begin{pmatrix} b_1 & b_2 & \ldots & b_{n-t} \\ d_{t+1} & d_{t+2} & \ldots & d_n \end{pmatrix} \end{aligned} \tag{3.6}$$

where $(b_1, b_2, \ldots, b_{t-1})$ is a permutation of $\{d_{t+1}, d_{t+2}, \ldots, d_n\}$ such that $b_1 < b_2 < \cdots < b_{t-1}$. Note that the matrix notation implies that $r_{b_1} = d_{t+1}, r_{b_2} = d_{t+2}, \ldots, r_{b_{n-t}} = d_n$. Obviously, since the product of permutations is unique, $(r_1, r_2, \ldots, r_n)$ is unique, g_x is a function.

Next, we will prove that g_x is bijection from A_x onto B_x. Recall that $(c_{n-1}, t) \in A_x$ if $f(c_{n-1}, t) = x$, in other words, if $x = \min_{2 \le j \le t} d_j$. So the cycle $[\quad]n, d_2, d_3, \ldots, d_t$ has x as its least element. Thus, $g_x(c_{n-1}, t) \in B_x$.

Let $a_1, a_2 \in \mathcal{S}_{n-1}$ and $t_1, t_2 \in \{2, 3, \ldots, n\}$ such that $g_x(a_1, t_1) = g_x(a_2, t_2)$. Let $g_x(a_1, t_1) = (r_{11}, r_{12}, r_{13}, \ldots, r_{1n})$ and $g_x(a_2, t_2) = (r_{21}, r_{22}, r_{23}, \ldots, r_{2n})$. Since $g_x(a_1, t_1) = g_x(a_2, t_2)$, we have $r_{1y} = r_{2y}$ for all $1 \le y \le n$.

Let C_1, C_2 be cycles that contains n in $g_x(a_1, t_1)$, and $g_x(a_2, t_2)$, respectively. Clearly, C_1 and M_1 are disjoint, as well as C_2 and M_2. Let M_1, M_2 be permutation matrices of the remaining element of $g_x(a_1, t_1)$, and $g_x(a_2, t_2)$, respectively. If $C_1 \ne C_2$, then there must be maps $n \mapsto r_{1n} \mapsto \cdots \mapsto r_{1y} \mapsto m_1$ and $n \mapsto r_{1n} \mapsto \cdots \mapsto r_{2y} \mapsto m_2$ for which $m_1 \ne m_2$. This implies that $r_{1r_{1y}} \ne r_{2r_{2y}}$. But since $r_{1y} = r_{2y}$, this implies that there is some z such that $r_{1z} \ne r_{2z}$, which is a contradiction. Therefore, $C_1 = C_2$, which implies that the first $t_1 = t_2$ entries of the permutations a_1 and a_2 are the same.

Since $C_1 = C_2$, the set of elements of M_1 must be equal to the set of elements of M_2 as well. Similarly, if $M_1 \ne M_2$, then there is some $r_{1y} \ne r_{2y}$, which contradicts $r_{1z} = r_{2z}$ for all $1 \le y \le n$. Thus, $M_1 = M_2$, and the last $n - t_1$ entries of a_1 is the same as that of a_2. We can then conclude that $(a_1, t_1) = (a_2, t_2)$. Therefore, g_x is one-to-one.

Lastly, let $g_x(\alpha, \beta) = \gamma$. Let C be the cycle containing n of γ, and since $g_x(\alpha, \beta)$ is defined, x is the least element in C, and C consists of t element. And let M be the permutation matrix of the remaining $n-t$ numbers of γ.

Let $C = [\quad]n, d_2, d_3, \ldots, d_t$ and $M = \begin{pmatrix} b_1 & b_2 & \ldots & b_{n-t} \\ d_{t+1} & d_{t+2} & \ldots & d_n \end{pmatrix}$, for $b_1 < b_2 < \cdots < b_{n-t}$. We can then see that $\alpha = (d_2, d_3, \ldots, d_n)$ and $\beta = t$. It is easy to see that $g_x(\alpha, \beta) = \gamma$. Therefore, g_x is an onto function, and therefore is a bijection.

Since g_x is a bijection, we get $|A_x| = |B_x|$. From Lemma 3.1, $|B_x| = \frac{n!}{x(x+1)}$, which we can substitute into equation 3.5,

$$S = \frac{1}{n-1}\sum_{k=1}^{n-1}\frac{n!}{k+1} = \frac{n!}{n-1}(H_n - 1). \tag{3.7}$$

Combining equation 3.4 with equation 3.7, we finally arrive at

$$\mathbb{E}(P_{avg}(1)) = \frac{n}{n-1}(H_n - 1), \tag{3.8}$$

when $p_1 = 1$ or $p_1 = n$. ■

From Theorem 3.2, we can see that as $n \to \infty$, $\mathbb{E}(P_{avg}(1)) \to \log n$. Next, we will extend the result to the case that first person takes the m position.

Theorem 3.3. *For $2 \le m \le \frac{n}{2}$, if the first person take the position m, that is $p_1 = m$, then*

$$\mathbb{E}(P_{avg}(1)) = \frac{n}{n-1}\left(H_{n-1} - \frac{1}{2}H_{m-1} - \frac{n-m}{n}\right).$$

Proof. The proof of this has similar approach to that of Theorem 3.2, but instead of considering the distance $d_{1,j}$, we consider the value v_j, which is defined by

$$v_j = \begin{cases} 2d_j - 1 & ;\ p_j < p_1 \\ 2d_j & ;\ p_1 < p_j \le 2p_1 - 1 \\ p_j - 1 & ;\ \text{otherwise.} \end{cases} \tag{3.9}$$

For example, when $p_1 = 3$ and $n = 8$, the value v_j assigned to each position is as in the following table.

p_j	1	2	3	4	5	6	7	8
v_j	3	1		2	4	5	6	7

We assign smaller numbers to the position with lower privacy. But because some values of privacy can be achieved in two positions, we have to alternate that value v_j from left to right, in order to implement Lemma 3.1.

Let $c_{n-1} = (v_2, v_3, \dots, v_n)$, so c_{n-1} is a permutation of $\{1, 2, \dots, n-1\}$. Let t be the time, so $2 \le t \le n$. In an analogous manner to Theorem 3.2, we can define a bijection from the set of all (c_{n-1}, t) that results in $P(1,t) = x$ to the set of the permutations of $\{1, 2, \dots, n\}$ with x as its smallest element in the cycle containing n, which we denote by A_x.

For privacy $p = P(1,t) < m$, there are two sets of A_x that correspond to the permutation that gives that privacy, which are A_{2p-1} and A_{2p}. For privacy $p = P(1,t) \ge m$, there is only one set of A_x that corresponds, i.e. A_{p+m-1}.

By Lemma 3.1, it holds that

$$|A_{2p-1}| + |A_{2p}| = \frac{n!}{(2p-1)(2p)} + \frac{n!}{(2p)(2p+1)} = \frac{2 \cdot n!}{(2p-1)(2p+1)}. \tag{3.10}$$

And also,

$$|A_{p+m-1}| = \frac{n!}{(p+m-1)(p+m)}. \tag{3.11}$$

Summing the privacy from all possible (c_{n-1}, t), and divided by the number of all possible variations, as

$$\mathbb{E}(P_{avg}(1)) = \frac{\sum_{k=1}^{m-1} k(|A_{2k-1}| + |A_{2k}|) + \sum_{k=m}^{n-m} k|A_{k+m-1}|}{(n-1)(n-1)!}. \tag{3.12}$$

We can rewrite the first term as a telescopic sum, and then add each term up as follow:

$$n! \sum_{k=1}^{m-1} \left(\frac{k}{2k-1} - \frac{k}{2k+1} \right) = n! \left(H_{2m-2} - \frac{1}{2} H_{m-1} - \frac{m-1}{2m-1} \right). \tag{3.13}$$

In a similar way, the second term is found to be equivalent to

$$n! \sum_{k=m}^{n-m} \left(\frac{k}{k+m-1} - \frac{k}{k+m} \right) = n! \left(\frac{m}{2m-1} + H_{n-1} - H_{2m-1} - \frac{n-m}{n} \right). \tag{3.14}$$

Adding up equation 3.13 and equation 3.14, and divide it by $(n-1)(n-1)!$, we have a nice result, that is

$$\mathbb{E}(P_{avg}(1)) = \frac{n}{n-1} \left(H_{n-1} - \frac{1}{2} H_{m-1} - \frac{n-m}{n} \right). \tag{3.15}$$

This concludes the proof. ∎

By symmetry, we can find the expected privacy of $p_1 = m > \frac{n}{2}$ simply by substituting m with $n+1-m$.

Corollary 3.4. *For $\frac{n}{2} \leq m \leq n-1$. If $p_1 = m$, then*

$$\mathbb{E}(P_{avg}(1)) = \frac{n}{n-1} \left(H_{n-1} - \frac{1}{2} H_{n-m} - \frac{m-1}{n} \right).$$

We can see that the value of $\mathbb{E}(P_{avg}(1))$ reaches the maximum when $m = 1$ of $m = n$. The function decreasing when m is close to $\frac{n}{2}$. It is intuitive that as we got closer to the middle urinal, our expected privacy would be reduced. This is proven as a consequence of Theorem 3.3, in the case of the first person to enter the restroom. Next, we will show that in the case that $p_t = 1$, the value $\mathbb{E}(P_{avg}(t))$ will decrease as t goes from 1 to n.

Lemma 3.5. *The number of permutations of the set $\{1, 2, \ldots, n\}$ whose cycle containing n has length ℓ and smallest element x is $\binom{n-x-1}{\ell-2}(\ell-1)!(n-\ell)!$.*

Proof. See the proof of Lemma 3.1, particularly, equation 3.1. Substitute k with $\ell - 2$. ∎

Theorem 3.6. *For $t \geq 2$, if $p_t = 1$ or $p_t = n$, then*

$$\mathbb{E}(P_{avg}(t)) = \frac{(n-t)!}{(n-1)!} \sum_{x=1}^{n-t+1} \frac{(n+tx-2x)(n-x-1)!}{(x+1)(n-t-x+1)!}.$$

Proof. The proof takes a similar approach to that of Theorem 3.2. We define an ordered pair (c_{n-1}, k), when c_{n-1} is the $(n-1)$-tuple of the distances from the position $p_t = 1$ or $p_t = n$ to $p_1, p_2, \ldots, p_{k-1}, p_{k+1}, \ldots, p_n$, which is a permutation of $\{1, 2, \ldots, n-1\}$, and $k \geq t$ is the time at which we consider the privacy of A_t.

Let $f(c_{n-1}, k)$ be the minimum of the first k numbers in c_{n-1}, i.e. the privacy at the time k if order of the positions is described by c_{n-1}. Let A_x denotes the set of (c_{n-1}, k) such that $f(c_{n-1}, k) = x$, and B_x denotes the set of permutations of $\{1, 2, \ldots, n\}$ whose cycle containing n has length of at least $t+1$ and smallest element x. Then, there is a bijection from A_x to B_x.

Consequently,

$$\mathbb{E}(P_{avg}(t)) = \frac{1}{(n-t+1)(n-1)!} \sum_{x=1}^{n-t+1} (x|B_x|). \tag{3.16}$$

By Lemma 3.5, we have

$$\sum_{x=1}^{n-t+1} (x|B_x|) = \sum_{x=1}^{n-t+1} \left[x \sum_{\ell=t}^{n-x+1} \binom{n-x-1}{\ell-2} (\ell-1)!(n-\ell)! \right], \tag{3.17}$$

which can be simplified to

$$\sum_{x=1}^{n-t+1} (x|B_x|) = \sum_{x=1}^{n-t+1} \frac{(n+tx-2x)(n-t+1)(n-t)!(n-x-1)!}{(x+1)(n-t-x+1)!}. \tag{3.18}$$

Thus, the expected average privacy of A_t is

$$\mathbb{E}(P_{avg}(t)) = \frac{(n-t)!}{(n-1)!} \sum_{x=1}^{n-t+1} \frac{(n+tx-2x)(n-x-1)!}{(x+1)(n-t-x+1)!}, \tag{3.19}$$

which completes the proof. ∎

Obviously, the function in Theorem 3.6 is a decreasing function of t.

4. Numerical Simulations

The purpose of this section is to perform experiments and demonstrate our results numerically. First, we simulate the situation in Theorem 3.2 by making the first person pick urinal position 1 from n urinals. Then let other $n-1$ people who afterwards arrive after him randomly take any empty position until all the urinals are full. When each person arrives, the privacy of the first person is calculated. After the last person arrives, we calculate the average privacy of the first person. We repeat this simulation T times and compute the empirical mean of the average privacy.

Table 1 presents the empirical means from simulations when $T = 100, 500, 1000$ and 5000, and compare it with the theoretical expected value from Theorem 3.2 for four numbers of urinals n which are $10, 50, 100$ and 500. We illustrate the result in the case where $n = 10$ in Figure 1.

The experiment shows that empirical means approach the theoretically expected value of average privacy when the number of iterations T increases. This confirms the result of Theorem 3.2.

For Theorem 3.3, we consider the effect of the choice of the first person. Now, we perform a similar simulation for each choice of the first person 1000 times, then compute the empirical mean and the standard deviation of average privacy. As we expect, Table 2

TABLE 1. Theoretical expected value and empirical mean of the average privacy of A_1 when $p_1 = 1$.

Number of urinals (n)	**Empirical**				**Theoretical**
	100	500	1000	5000	
10	2.00	2.13	2.12	2.16	2.14
50	3.71	3.47	3.61	3.60	3.57
100	4.24	4.26	4.13	4.20	4.23
500	5.80	5.84	5.77	5.83	5.80

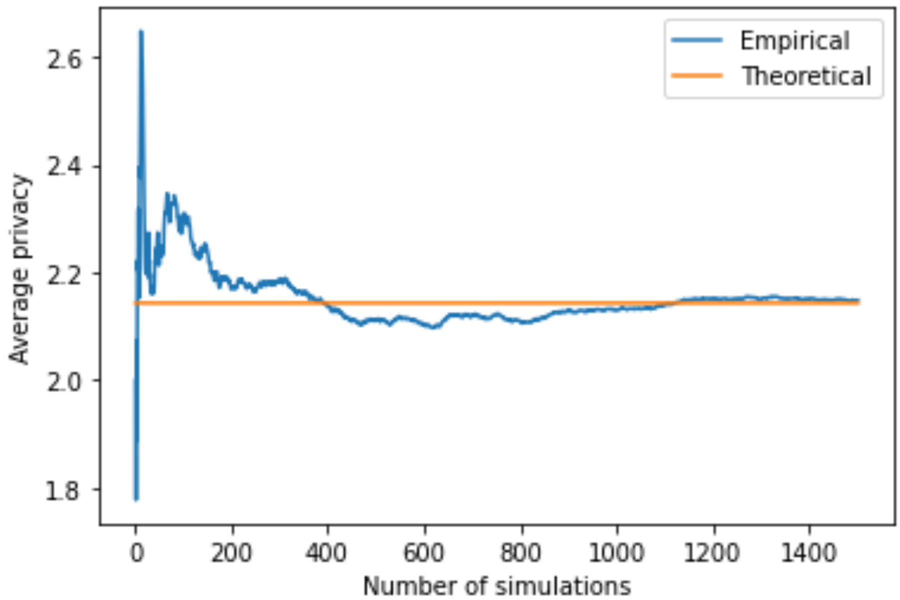

FIGURE 1. Theoretical expected value and empirical mean of the average privacy in the case that $n = 10$ and $p_1 = 1$.

and Figure 2 show that the empirical mean of privacy agrees to the one in Theorem 3.3. We can conclude that the average privacy reaches the maximum when one chooses the outermost urinal. Furthermore, this value will decrease when the choice is closer to the middle one.

Ultimately, for Theorem 3.6, since the expected average privacy of the first and the last person are the same, we simulate by fixing the position of A_t to be position 1, and then randomise the positions of the other $n - 1$ people. After that, we calculate the privacy of A_t at the time $t, t + 1, \ldots, n$, in order to compute the average privacy. We repeat the simulation 1000 times for each time, and then compute empirical mean and the standard deviation of average privacy as shown in Table 3 and Figure 3, along with theoretical average privacy which can be computed by Theorem 3.6.

TABLE 2. Theoretical expected value and empirical mean also with standard deviation of the average privacy for each choice of p_1 in the case that $n = 10$.

Urinal that the first person took	**Empirical**	**SD**	**Theoretical**
1	2.13	0.78	2.14
2	1.69	0.60	1.70
3	1.55	0.46	1.53
4	1.45	0.39	1.46
5	1.45	0.36	1.43
6	1.42	0.35	1.43
7	1.46	0.39	1.46
8	1.5	0.45	1.53
9	1.71	0.60	1.70
10	2.17	0.77	2.14

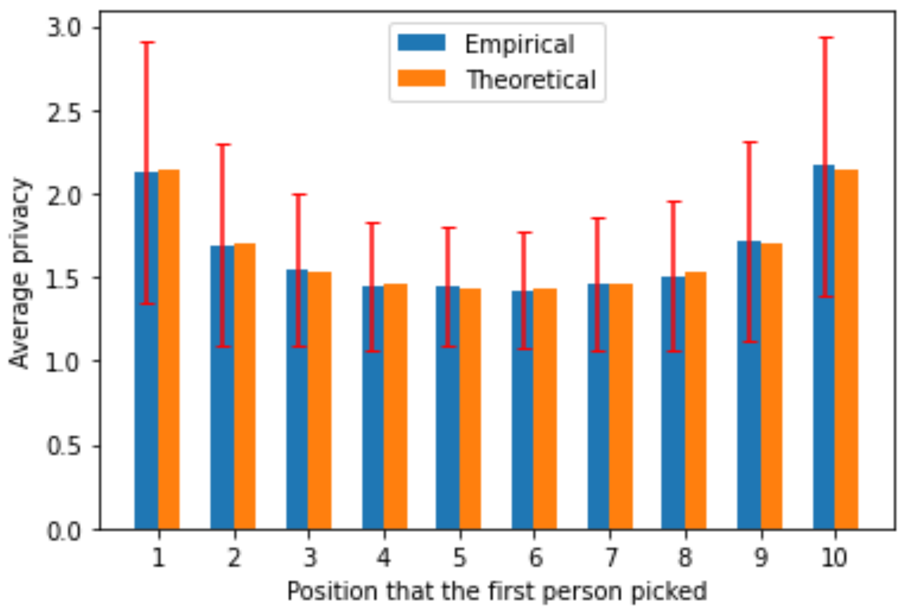

FIGURE 2. Theoretical expected value and empirical mean of the average privacy for each choice of p_1 in the case that $n = 10$.

The result is as anticipated, the average privacy of the person who pick the first urinal decreases as the time increases; so, the person who get into the restroom before tend to have more privacy, on average, than the later ones.

TABLE 3. Theoretical expected value and empirical mean also with standard deviation of the average privacy for each time t when the first urinal was picked in the case that $n = 10$.

Time that the first urinal was picked (t)	**Empirical**	**SD**	**Theoretical**
2	2.16	0.78	2.14
3	1.79	0.67	1.79
4	1.54	0.57	1.57
5	1.38	0.46	1.41
6	1.28	0.4	1.29
7	1.2	0.32	1.2
8	1.14	0.25	1.12
9	1.06	0.16	1.06

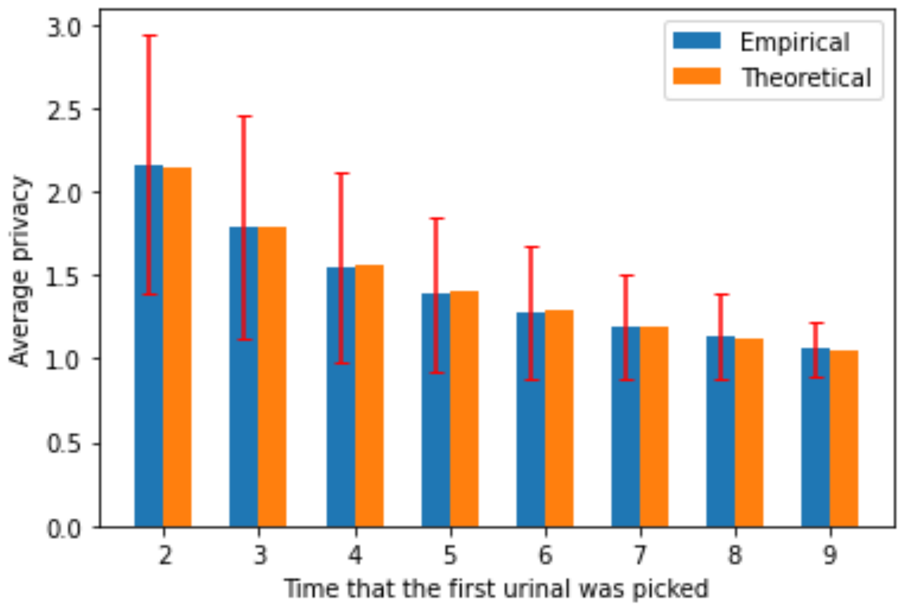

FIGURE 3. Theoretical expected value and empirical mean also with standard deviation of the average privacy for each time t when the first urinal was picked in the case that $n = 10$.

5. CONCLUSION

The paper extends the urinal problem from considering privacy as a binary state to a multi-state value. For the classic urinal problem, the expected time until the privacy was disturbed was considered. In practice, privacy is not binary, one can feel more insecure when the more people stand closer to him. So, instead of computing the expected time until the privacy was disturbed, we focus on the expected average privacy of his choice of position. We found that the most sided urinals are the best choice. Position 1 and n create the maximum expected average privacy. We obtained the explicit formula of the expected average privacy, which is a function of the Harmonic series. The formula shows that the expected average privacy decreases when the chosen position approaches the center. So we suggest one to choose the position of one's urinal as far from the center as much as possible for the sake of privacy. Our suggestion agrees with the result of [1].

Many variations and generalisations are left to be studied regarding this problem. For instance, it would be interesting to find the solution and convergence of the analogue of this problem in the nth-dimensional space. We also conjecture that $\mathbb{E}(P_{avg}(t))$ is optimised in the sides and decresing as we get closer to the center regardless of the distance function d.

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