Thai Journal of Mathematics
Volume 19 Number 4 (2021)
Pages 1651–1659

http://thaijmath.in.cmu.ac.th

ISSN 1686-0209

Some Convergence Theorems for Monotone Nonexpansive Mappings in Hyperbolic Metric Spaces

Izhar Uddin[1], **Sajan Aggarwal**[1], **Khanitin Muangchoo-in**[2,3] **and Poom Kumam**[4,5,*]

[1] *Department of Mathematics, Jamia Millia Islamia, New Delhi 110025, India*
e-mail : izharuddin_amu@yahoo.co.in (I. Uddin); aggarwal.maths1993@gmail.com (S. Aggarwal)

[2] *Fixed Point Research Laboratory, Fixed Point Theory and Applications Research Group, Center of Excellence in Theoretical and Computational Science (TaCS-CoE), Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha Uthit Rd., Bang Mod, Thung Khru, Bangkok 10140, Thailand*

[3] *Department of Exercise and Sports Science, Faculty of Sports and Health Science, Thailand National Sports University Chaiyaphum Campus, 239, Moo. 4, Muang Chaiyaphum, Chaiyaphum 36000, Thailand*
e-mail : kanitin22@gmail.com (K. Muangchoo-in)

[4] *Center of Excellence in Theoretical and Computational Science (TaCS-CoE), Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha Uthit Rd., Bang Mod, Thung Khru, Bangkok 10140, Thailand*

[5] *Department of Medical Research, China Medical University Hospital, China Medical University, Taichung 40402, Taiwan*
e-mail : poom.kum@kmutt.ac.th (P. Kumam)

Abstract In this paper, we prove strong and $\Delta-$convergence theorem for monotone nonexpansive mapping in a partially ordered hyperbolic metric space using Mann iteration scheme. Moreover, we give an numerical example to illustrate the main result in this paper.

MSC: 47H09; 47H10

Keywords: fixed point; monotone nonexpansive mapping; Mann's iteration; hyperbolic metric spaces

Submission date: 28.09.2018 / Acceptance date: 05.06.2019

1. Introduction

Banach contraction principle is one of the most utilized result ever proved in nonlinear analysis. It's applications are enormous to different field of mathematics. Recently, Ran and Reuring [1] and Nieto and Rodriguez-Löpez [2] extended the Banach contraction principle in partially ordered metric space. Ran and Reuring [1] applied their results to prove the solution of matrix equations while Nieto and Rodriguez-Löpez [2] utilized that result to prove the existence of solution of differential equations. After the three basic discovery due to Browder [3, 4], Göhde [5] and Kirk [6], the fixed point theory of

*Corresponding author.

Published by The Mathematical Association of Thailand.

nonexpansive mapping has been attracting a number of researchers. Due to rich geometric and convex structure, Banach space provide a natural platform to study the existence and approximating results for nonexpansive mapping. The results given by Browder [3, 4], Gӧhde [5], Kirk [6], Edelstin [7], Mann [8], Ishikawa [9] and Halpern [10] are of central attraction.

It is always a challenge to extend the results of linear spaces (like Banach space or Hilbert space) to nonlinear spaces (like metric spaces and topological vector spaces). There are many hurdles in proving the existence and approximation the fixed point result due to absence of linear and convex structure in metric spaces. To overcome this problem, Takahasi [11] introduced the concept of convex metric spaces and proved some fixed point theorems for nonexpansive mappings. Reich and Shafrir [12] introduced the hyperbolic metric spaces and proved some convergence theorems for Mann iteration while Kohlenbach [13] gave another approach for the same. Recently, Dehaish and Khamsi [14] proved Browder fixed point theorem for a monotone nonexpansive mapping in a partial order Banach space. Buthania and Khamsi also studied the convergence behavior of Mann iteration scheme for monotone nonexpansive mappings in Banach spaces. In 2016, Song et al. [15] proved convergence theorem for Mann iteration under some different conditions in respect of monotone nonexpansive mapping in a partially ordered Banach space.

In this paper, we study the convergence behaviour of Mann iteration in hyperbolic spaces for monotone nonexpansive mapping. Our results generalize and improve the results of Dehaish and Khamsi [14] and Song et al. [15]. Moreover, we give an numerical example to illustrate the main result in this paper.

2. Preliminaries

To make our paper self contained, we collect some needed results and definitions. Let $(X, d, \leq)$ be a metric space with partial order. Let $T : K \to K$ be a map preserve monotone order if $x \leq y \Rightarrow T(x) \leq T(y)$, for any $x, y \in X$. A point $x \in X$ is said to be a fixed point of T whenever $Tx = x$. The set of fixed point of T is denoted by $F(T)$. T is said to be monotone nonexpansive if T is monotone and $d(Tx, Ty) \leq d(x, y)$ for any $x, y \in X$ such that x and y are comparable ($x \leq y$ or $y \leq x$). Let (X, d) be a metric space. Suppose there exist a family $\mathcal{F}$ of metric segments such that any two points x, y in X are endpoints of a unique metric segment $[x, y] \in \mathcal{F}([x, y])$ is an isometric image of the real line interval $[0, d(x, y)]$. We shall denote z by $\beta x \oplus (1 - \beta)y$ the unique point of $[x, y]$, which satisfies $d(x, z) = (1 - \beta)d(x, y)$ and $d(z, y) = \beta d(x, y)$, where $\beta \in [0, 1]$. Such metric space with a family $\mathcal{F}$ of metric segments are usually called *convex metric space*. Moreover, if we have

$$d(\alpha p \oplus (1 - \alpha)x, \alpha q \oplus (1 - \alpha)y) \leq \alpha d(p, q) + (1 - \alpha)d(x, y),$$

for all p, q, x, y in X and $\alpha \in [0, 1]$, then X is said to be a *hyperbolic metric space*. Linear example of *hyperbolic metric space* is normed linear space. Hadmard manifolds, the Hilbert open unit ball equipped with the hyperbolic metric and the CAT(0) spaces are example of nonlinear hyperbolic metric space (e.g. Kumam [16], [17], Pakkaranang [18], [19] and Thounthong [20]). The fixed point theory in nonlinear domain is rapidly growing (e.g. Kirk [21], Shukla et al. [22], Reich and Shafrir [12], Uddin et al. [23] etc).

Hyperbolic metric space endowed with partial order is called *partially ordered hyperbolic metric spaces*. In this paper, we will assume that order interval are convex and closed. Recall that an order interval is any of the subset $[a, \to) = \{x \in X; a \leq x\}$ and $(\leftarrow, b] =$

$\{x \in X; x \leq b\}$, for any $a, b \in X$.
The following notion of a uniformly convex hyperbolic metric space is

Definition 2.1. Let (X, d) be a hyperbolic metric space. We say that X is uniformly convex if for any $a \in X$, for every $r > 0$, and for each $\varepsilon > 0$

$$\delta(r, \varepsilon) = \inf\left\{1 - \frac{1}{r}d\left(\frac{1}{2}x \oplus \frac{1}{2}y, a\right); d(x,a) \leq r, d(y,a) \leq r, d(x,y) \geq r\varepsilon\right\} > 0.$$

In 1976, Lim [24] introduced the concept of $\Delta-$convergence in metric spaces.

Definition 2.2. A bounded sequence $\{x_n\}$ in X is said to $\Delta-$converge to $x \in X$ if x is the unique asymptotic center of every subsequence $\{u_n\}$ of $\{x_n\}$. We write $x_n \rightharpoonup x$ ($\{x_n\}$ $\Delta-$converges to x).

Lemma 2.3 ([25, 26])**.** *Let K be a nonempty, closed and convex subset of complete uniformly convex hyperbolic metric space X. Then every bounded sequence $\{x_n\} \in X$ has a unique asymptotic center with respect to K.*

Lemma 2.4 ([27])**.** *Let X be a uniformly convex hyperbolic space. Let $R \in [0, \infty)$ be such that $\limsup_{n\to\infty} d(x_n, a) \leq R$, $\limsup_{n\to\infty} d(y_n, a) \leq R$ and $\lim_{n\to\infty} d(a, \alpha_n x_n \oplus (1-\alpha_n)y_n) = R$ where $\alpha_n \in [a, b]$, with $0 < a \leq b < 1$. Then we have, $\lim_{n\to\infty} d(x_n, y_n) = 0$.*

Now, we present Mann iteration [8] in the setting of hyperbolic metric space is defined by

$$\begin{cases} x_1 \in K, \\ x_{n+1} = \beta_n x_n \oplus (1-\beta_n)Tx_n, \end{cases} \tag{2.1}$$

for each $n \geq 1$, where $\beta_n \in (0, 1)$.

In this paper, we prove some strong and $\Delta-$convergence for iteration scheme (2.1) in the setting of hyperbolic metric spaces.

3. Δ-Convergence and Strong Convergence Theorems

Let us start with following useful lemma.

Lemma 3.1. *Let K be a nonempty, closed and convex subset of a partially ordered hyperbolic metric space X and $T : K \to K$ be a monotone nonexpansive mapping. If $\{x_n\}$ is a sequence defined by (2.1) such that $x_1 \leq T(x_1)$ (or $T(x_1) \leq x_1$). Then,*
(i) $x_n \leq x_{n+1} \leq T(x_n)$ (or $T(x_n) \leq x_{n+1} \leq x_n$);
(ii) $x_n \leq p$ (or $p \leq x_n$), provided $\{x_n\}$ $\Delta-$converge to a point $p \in K$ for all $n \in \boldsymbol{N}$.

Proof. We shall use induction method to prove this result. Since by assumption we have $x_1 \leq T(x_1)$. By the convexity of order interval $[x_1, T(x_1)]$ and by (2.1), we have

$$x_1 \leq x_2 \leq Tx_1.$$

Thus, the result is true for $n = 1$. Now suppose that the result is true for n, that is,

$$x_n \leq x_{n+1} \leq Tx_n.$$

Since $x_n \leq x_{n+1}$, by monotonicity of T, we get

$$Tx_n \leq Tx_{n+1}.$$

Which implies

$$x_{n+1} \leq Tx_{n+1}.$$

Now, by convexity of ordered interval $[x_{n+1}, Tx_{n+1}]$,
we get

$$x_{n+1} \leq x_{n+2} \leq Tx_{n+1}.$$

By induction, we get first result, that is

$$x_n \leq x_{n+1} \leq Tx_n.$$

Suppose p is a $\Delta-$limit of $\{x_n\}$. As the sequence $\{x_n\}$ is monotone increasing and the order interval $[x_m, \rightarrow)$ is closed and convex. We claim that $p \in [x_m, \rightarrow)$ for a fixed $m \in \mathbf{N}$. If $p \notin [x_m \rightarrow)$, then the asymptotic center of subsequence $\{x_r\}$ of $\{x_n\}$ defined by leaving first $m-1$ terms of sequence $\{x_n\}$ will not equal to p which is a contradiction as p is a $\Delta-$limit of $\{x_n\}$. This completes the proof. ■

Lemma 3.2. *Let K be a nonempty, closed and convex subset of a uniformly convex partially ordered hyperbolic metric space X and let $T : K \to K$ be a monotone nonexpansive mapping with $F(T) \neq \phi$. Assume that the sequence $\{x_n\}$ is defined by (2.1) and $x_1 \leq Tx_1$ (or $Tx_1 \leq x_1$). If $p \leq x_1$ (or $x_1 \leq p$) for some $p \in F(T)$, then*

$$\lim_{n\to\infty} d(x_n, Tx_n) = 0.$$

Proof. It follows from the Lemma 3.1, that $p \leq x_1 \leq x_n$
by monotonicity

$$Tp \leq Tx_n.$$

For any $p \in F(T)$

$$\begin{aligned} d(x_{n+1}, p) &= d(\beta_n x_n \oplus (1-\beta_n)Tx_n, p) \\ &\leq \beta_n d(x_n, p) + (1-\beta_n)d(Tx_n, Tp) \\ &\leq d(x_n, p) \end{aligned}$$

which implies that $\{d(x_n, p)\}$ is a decreasing and bounded below sequence of real numbers. Hence, $\lim_{n\to\infty} d(x_n, p)$ exist. Let $\lim_{n\to\infty} d(x_n, p) = c$. As $d(Tx_n, p) \leq d(x_n, p)$ therefore $\limsup_{n\to\infty} d(Tx_n, p) \leq c$. Also, $\lim_{n\to\infty} d(\beta_n x_n \oplus (1-\beta_n)Tx_n, p) = \lim_{n\to\infty} d(x_{n+1}, p) = c$. In view of Lemma 2.4, $\lim_{n\to\infty} d(x_n, Tx_n) = 0$. ■

Theorem 3.3. *Let K be a nonempty, closed and convex subset of a complete uniformly convex partially ordered hyperbolic metric space X and let $T : K \to K$ be a monotone nonexpansive mapping with $F(T) \neq \phi$. If sequence $\{x_n\}$ is defined by (2.1) with $x_1 \leq Tx_1$ (or $Tx_1 \leq x_1$). If $p \leq x_1$ (or $x_1 \leq p$) for some $p \in F(T)$, then $\{x_n\}$ $\Delta-$ converges to a fixed point x^* of T.*

Proof. From Lemma 3.2 $\lim_{n\to\infty} d(x_n, p)$ exist for each $p \in F(T)$, so that the sequence is bounded and $\lim_{n\to\infty} d(x_n, Tx_n) = 0$.
By Lemma 2.3, $\{x_n\}$ have unique asymptotic center.
Let $A(x_n) = x^*$ and $\{u_n\}$ is any subsequence of $\{x_n\}$ such that $A(u_n) = u$. Now claim

$x^* = u$.
On contrary, suppose that $x^* \neq u$. Now

$$\begin{aligned}\limsup_{n\to\infty} d(u_n, u) &< \limsup_{n\to\infty} d(u_n, x^*)\\ &\leq \limsup_{n\to\infty} d(x_n, x^*)\\ &< \limsup_{n\to\infty} d(x_n, u)\\ &= \limsup_{n\to\infty} d(u_n, u)\end{aligned}$$

which is a contradiction and hence $\Delta - \lim_{n\to\infty} x_n = x^*$.
Now, we claim that $x^* \in F(T)$. As $x^* \leq x_n \Rightarrow Tx^* \leq Tx_n$

$$\begin{aligned}\limsup_{n\to\infty} d(Tx^*, x_n) &\leq \limsup_{n\to\infty} d(Tx^*, Tx_n) + \limsup_{n\to\infty} d(Tx_n, x_n)\\ &\leq \limsup_{n\to\infty} d(x^*, x_n).\end{aligned}$$

Since $\Delta - \lim_{n\to\infty} x_n = x^*$

$$\limsup_{n\to\infty} d(x^*, x_n) < \limsup_{n\to\infty} d(Tx^*, x_n).$$

Thus, we have $Tx^* = x^*$. ∎

Theorem 3.4. *Let K be a nonempty and closed convex subset of a complete uniformly convex partially ordered hyperbolic metric space X and $T : K \to K$ be a monotone nonexpansive mapping with $F(T) \neq \phi$. If sequence $\{x_n\}$ is defined by (2.1) with $x_1 \leq Tx_1$ (or $Tx_1 \leq x_1$). If $p \leq x_1$ (or $x_1 \leq p$) for some $p \in F(T)$, then $\{x_n\}$ converges to a fixed point of T iff $\liminf_{n\to\infty} d(x_n, F(T)) = 0$.*

Proof. It is easy to see that if $\{x_n\}$ converges to a point $x \in F(T)$ then $\liminf_{n\to\infty} d(x_n, F(T)) = 0$.
For converse part, suppose that $\liminf_{n\to\infty} d(x_n, F(T)) = 0$ From Lemma 3.2, for any $p \in F(T)$, we have

$$\inf_{p\in F(T)} d(x_{n+1}, p) \leq \inf_{p\in F(T)} d(x_n, p)$$

so that

$$d(x_{n+1}, F(T)) \leq d(x_n, F(T)).$$

Hence, $\lim_{n\to\infty} d(x_n, F(T))$ exist. As $\liminf_{n\to\infty} d(x_n, F(T)) = 0$, we get $\lim_{n\to\infty} d(x_n, F(T)) = 0$.
Now, we show that $\{x_n\}$ is a Cauchy sequence in K. Let $\varepsilon > 0$ be arbitrary chosen that there exist $n_0 \in \mathbf{N}$ such that

$$d(x_n, F(T)) < \frac{\varepsilon}{4}.$$

In particular

$$\inf\{d(x_{n_0}, p) : p \in F(T)\} < \frac{\varepsilon}{4}$$

then there must exist a $p \in F(T)$ such that

$$d(x_{n_0}, p) < \frac{\varepsilon}{2}.$$

Now, for $m, n \geq n_0$, we have

$$\begin{aligned} d(x_{m+n,x_n}) &\leq d(x_{n+m,p}) + d(p, x_n) \\ &< 2\frac{\varepsilon}{2} = \varepsilon. \end{aligned}$$

Hence, $\{x_n\}$ is a Cauchy sequence in closed subset K of X therefore it must converge to a point of K.
Let $\lim_{n\to\infty} x_n = q$. Now

$$\begin{aligned} d(q, Tq) &\leq d(q, x_n) + d(x_n, Tx_n) + d(Tx_n, Tq) \\ &\leq d(q, x_n) + d(x_n, Tx_n) + d(x_n, q) \\ &\to 0 \text{ as } n \to \infty. \end{aligned}$$

Thus, $Tq = q$. ■

Example 3.5. Let $T : [-2, 2] \to [-2, 2]$ be a mapping defined by

$$Tx = \frac{x}{5}$$

for any $x \in [-2, 2]$.
Now, we will show that T is a monotone nonexpensive mapping.
First, we prove that T is a monotone mapping. We divide the proof into two cases.
Case1 $x \leq y$, let $x = -2$ and $y \in [-2, 2]$ then $Tx = \frac{-2}{5} \leq \frac{y}{5} = Ty$ for all $y \in [-2, 2]$
Case2 $y \leq x$, let $y = -2$ and $x \in [-2, 2]$ then $Ty = \frac{-2}{5} \leq \frac{x}{5} = Ty$ for all $x \in [-2, 2]$
then T is a monotone mapping.
Second, We prove that T is a nonexpansive mapping.

$$d(Tx, Ty) = |Tx - Ty| = \left|\frac{x}{5} - \frac{y}{5}\right| = \frac{1}{5}|x - y| \leq |x - y| = d(x, y)$$

then T is a nonexpansive mapping. Hence, T is a monotone nonexpansive mapping. Also 0 is a fixed point of T. Define the sequence $\{\beta_n\} = \frac{n}{5n+1}$ the following table shows the numerical example for convergence theorem.

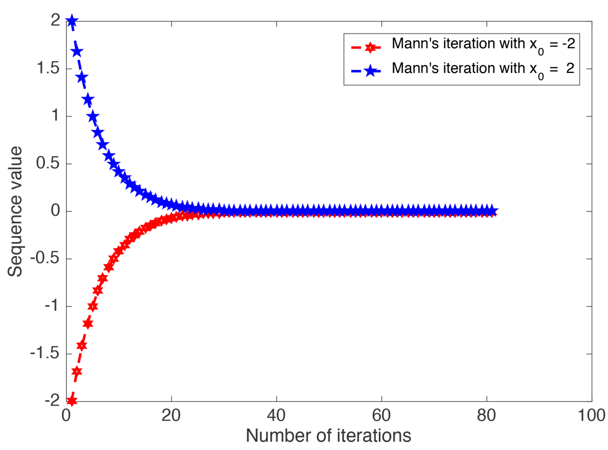

FIGURE 1. The numerical solution for $\beta_n = \frac{n}{5n+1}$, $x_0 = 2$ and $x_0 = -2$

TABLE 1. The convergent step of $\{x_n\}$ for Example 3.5 with $\{\beta_n\} = \frac{n}{5n+1}$

Number of iterations	Sequence value of Mann's iterations	
	$x_0 = 2$	$x_0 = -2$
1	2.000000	-2.000000
5	1.067178	-1.067178
10	0.458736	-0.458736
15	0.194904	-0.194904
20	0.082424	-0.082424
25	0.034768	-0.034768
30	0.014643	-0.014643
35	0.006160	-0.006160
40	0.002589	-0.002589
45	0.001087	-0.001087
50	0.000456	-0.000456
55	0.000191	-0.000191
60	0.000080	-0.000080
65	0.000033	-0.000033
70	0.000014	-0.000014
75	0.000005	-0.000005
80	0.000002	-0.000002

4. Conclusion

In this article, we proved strong and Δ-convergence theorems for monotone nonexpansive mapping in ordered hyperbolic metric space. In part of numerical example, we gave a example of monotone nonexpansive such the sequence defined by Mann iterative scheme to show the convergent behavior, we saw that behavior of two initial sequences are monotone to the fixed point of its.

Acknowledgements

The authors would like to thank the Theoretical and Computational Science (TaCS) Center under Computational and Applied Science for Smart Innovation Cluster (CLASSIC), Faculty of Science, KMUTT for financial support.

References

[1] A.C.M. Ran, M.C.B. Reuring, A fixed point theorems in partially ordered sets and some applications to matrix equations, Proc. Am. Math. Soc. 132 (2005) 1435–1443.

[2] J.J. Nieto, R. Rodriguez-Lopez, Contractive mapping theorems in partially ordered sets and applications to ordinary differential equation, Order 22 (2005) 223–239.

[3] F.E. Browder, Fixed point theorems for noncompact mappings in Hilbert Space, Proc. Nat. Acad. Sci. U.S.A. 53 (1965) 1272–1276.

[4] F.E. Browder, Nonexpansive nonlinear operators in a Banach Space, Proc. Nat. Acad. Sci. USA. 54 (1965) 1041–1044.

[5] D. Göhde, Zum Prinzip der kontraktiven Abbildung, Math. Nachr. 30 (1965) 251–258.

[6] W.A. Kirk, A fixed point theorem for mappings which do not increase distances, Amer. Math. Monthly 72 (1965) 1004–1006.

[7] M. Edelstain, An extension of Banach's contraction principle, Proc. Amer. Math. Soc. 12 (1961) 7–10.

[8] W.R. Mann, Mean value methods in iteration, Proc. Amer. Math. Soc. 4 (1953) 506–510.

[9] S. Ishikawa, Fixed points by a new iteration method, Proc. Amer. Math. Soc. 59 (1976) 65–71.

[10] B. Halpern, Fixed points of nonexpansive maps, Bull. Amer. Math. Soc. 73 (1967) 957–961.

[11] W. Takahashi, A convexity in metric space and nonexpansive mappings, I, Kodai Math. Sem. Rep. 22 (1970) 142–149.

[12] S. Reich, I. Shafrir, Nonexpansive iterations in hyperbolic spaces, Nonlinear Anal. 15 (1990) 537–558.

[13] U. Kohlenbach, Some logical metatheorems with application in functional analysis, Trans. Amer. Math. Soc. 357 (2005) 89–128.

[14] B.A.B, Dehaish, M.A. khamsi, Browder and Gӧhde fixed point theorem for monotone nonexpansive mappings, Fixed Point Theory Appl. 2016 (2016) Article no. 20.

[15] Y. Song, P. Kumam, Y.J. Cho, Fixed point theorems and iterative approximation for monotone nonexpansive mapping in ordered Banach spaces, Fixed Point Theory Appl. 2016 (2016) Article no. 73.

[16] W. Kumam, N. Pakkaranang, P. Kumam, One-step iteration scheme for multivalued nonexpansive mappings in CAT(0) spaces, Mediterranean Journal of Mathematics 13 (2016) 1211–1225.

[17] W. Kumam, N. Pakkaranang, P. Kumam, Convergence analysis of modied Picard-S hybrid iterative algorithms for total asymptotically nonexpansive mappings in Hadamard spaces, International Journal of Computer Mathematics 97 (2020) 175–188.

[18] N. Pakkaranang, P. Kumam, Y.J. Cho, P. Saipara, A. Padcharoen, C. Khaofong, Strong convergence of modied viscosity implicit approximation methods for asymptotically nonexpansive mappings in complete CAT(0) spaces, Journal of Mathematics and Computer Science 17 (3) (2017) 345–354.

[19] N. Pakkaranang, P. Kumam, Y.J. Cho, Proximal point algorithms for solving convex minimization problem and common fixed points problem of asymptotically quasi-nonexpansive mappings in CAT(0) spaces with convergence analysis, Numerical Algorithms (3) 78 (2018) 827–845.

[20] P. Thounthong, N. Pakkaranang, Y.J. Cho, W. Kumam, P. Kumam, The numerical reckoning of modied proximal point methods for monimization problems in non-positive curvature metric spaces, International Journal of Computer Mathematics 97 (2020) 245–262.

[21] W.A. Kirk, Fixed point theorems in CAT(0) spaces and **R**-Trees, Fixed Point Theory and Applicaation 4 (2004) 309–316.

[22] R. Shukla, R. Pant, P. Kumam, On the $\alpha-$nonexpansive mapping in partially ordered hyperbolic metric spaces, Journal of Mathematical Analysis 8 (1) (2017) 1–15.

[23] I. Uddin, J.J. Nieto, J. Ali, One-step iteration scheme for multivalued nonexpansive mappings in CAT(0) spaces, Mediterranean Journal of Mathematics 13 (2016) 1211–1225.

[24] T.C. Lim, Remarks on some fixed point theorems, Proc. Am. Math. Soc. 60 (1976) 179–182.

[25] M.A. Khamsi, A.R. Khan, Inequalities in metric spaces with applications, Nonlinear Anal. 74 (2011) 4036–4045.

[26] H. Fukhar-Ud-Din, Existence and approximation of fixed points in convex metric spaces, Carpath. J. Math. 2 (2014) 175–185.

[27] H. Fukhar-Ud-Din, M.A. Khamsi, Approximating common fixed points in hyperbolic spaces, Fixed Point Theory and Appl. 2014 (2014) Article no. 113.